Question of Planckian "Action" in Gravitational Wave Detection Experiments

Joseph F. Messina

Topical Group in Gravitation, American Physical Society, P.O. Box 130520,
The Woodlands, TX 77393, USA. E-mail: jfmessina77@yahoo.com

It is shown that in the absence of a *purely* gravitational measurement of Planck's constant one *cannot* at present rule out the possibility that the ripples in the curvature of the *fabric* of spacetime may be *scaled* by a more diminutive "action" whose detection requires sensitivities beyond the standard quantum limit. An experiment that could unequivocally test this possibility is suggested.

1 Introduction

The search for gravitational waves, one of the centerpieces of general relativity, has been a work in progress for over five decades. Two main forms of detectors are currently in use worldwide. The first, pioneered by Weber [1] in the 1960s, is based on the expectation that a passing gravitational wave will induce a mechanical oscillation in a cryogenically cooled cylindrical bar whose resonance can then be amplified and recorded. The second method, using lasers, is designed to measure spacetime geometry variations between mirrors suspended in vacuum using interferometry in a Michelson configuration.

Despite the ever increasing sensitivity of these detectors these ripples in the curvature of the fabric of spacetime have yet to be detected. After these many years of experimentation one may therefore be justified in questioning whether the failure to detect these perturbations is symptomatic of yet to be discovered physics beyond the standard quantum limit.

It should be observed that if we examine this question from a quantum mechanical perspective we are inevitably struck by the fact that the role of Planck's constant in gravitational wave phenomena has always been taken for granted without questions regarding the possible limits of its applicability being asked, which is somewhat perplexing since no *purely* gravitational measurement of Planck's constant exists. As will be shown in this paper, if pursued, this element of uncertainty gives rise to the possibility that gravitational quanta may not be *scaled* by Planck's constant.

2 Scaling of gravitational quanta

It should be emphasized from the outset that any discussion of this possibility has as its foundation the irrefutable fact that nature has made available *two* immutable elementary "actions" in the context of the framework of quantum mechanics. That is, Planck's familiar constant, h, which has been shown experimentally to play an indispensable role in the microphysical realm, and a second, more diminutive "action" formed from two of the fundamental constants of quantum mechanics, namely, e^2/c – the ratio of the square of the elementary charge to the velocity of light, which has the value

7.6957×10^{-37} J s.

In what follows I shall put forward an experimentally verifiable hypothesis in favor of a dynamical interpretation of the fabric of spacetime. That is, we shall allow for the possibility that this more diminutive "action" is an *intrinsic* property of the fabric of spacetime; the *size* of the gravitational quanta being always *scaled* in terms of e^2/c. Implicit in this conceptualization is the widely held expectation that spacetime should play a dynamic role in its own right, rather than being a passive observer.

3 Possible experimental test

Clearly, the most direct way of verifying if this hypothesis corresponds to reality is to measure the *vibrational* displacement induced in a resonant detector by a passing gravitational wave. To give an illustration, let us assume, using the "action" constant e^2/c, that a gravitational quantum of *angular* frequency ω has an energy

$$E = \left(\frac{e^2}{2\pi c} \right) \omega \,. \tag{1}$$

We can then profit from the fact that the *vibrational* energy induced in a *resonant* detector, by a gravitational wave, can be converted to the fractional change in *vibrational* displacement by making use of the relation between amplitude x_0, energy E and the total mass M for a harmonic oscillator, in the familiar form

$$E = \frac{1}{2} M \omega^2 x_0^2 \,. \tag{2}$$

If we now take as an example Weber's seminal experiment, which used as an antenna a 1400 kg cylindrical aluminum bar that had a natural *resonance* frequency ν_0 of 1660 Hz, we can readily compute the *vibrational* displacement, x, caused by a *single* quantum of gravitational radiation of *angular* frequency $\omega = 2\pi\nu_0$, and energy $(e^2/2\pi c)\,\omega$. Combining Eqs. (1) and (2) and then substituting these values, we obtain

$$\begin{aligned} x &= \sqrt{\frac{2}{M\omega} \frac{e^2}{2\pi c}} \\ &\approx 1.3 \times 10^{-22} \text{ m}. \end{aligned} \tag{3}$$

Needless to say, such extraordinarily small displacements could not be measured with the technology available in Weber's day. Indeed, even today such a feat remains out of reach since there are no resonant-mass antennas in operation that have the required sensitivity.

Fortunately, since Weber's pioneering work in the 1960s numerous projects have been undertaken in an effort to enhance detector sensitivity. One of the more innovative of these efforts has been the development of the Schenberg *spherical* resonant-mass telescope in Brazil [2], which has the advantage of being omnidirectional. When fully operational it will provide information regarding a wave's amplitude, polarization, and direction of source. The detector program, which we shall presently exploit, uses an 1150 kg spherical resonant-mass made of a copper-aluminum alloy, and has a *resonance* frequency ν_0 of 3200 Hz. The *vibrational* displacement caused by a *single* quantum of gravitational radiation of *angular* frequency $\omega = 2\pi\nu_0$ can easily be computed by direct substitution of these values in Eq. (3). We thus obtain

$$x \approx 1.0 \times 10^{-22} \text{ m.} \tag{4}$$

Verification of this result is contingent on the Schenberg surpassing the standard quantum limit by *squeezing* the signal, which should result in a ten-fold increase in sensitivity. Clearly, in the absence of a physical law that prohibits an elementary "action" smaller than Planck's this result must be taken seriously.

4 Summary

The possibility was raised that gravitational quanta may not be scaled by Planck's constant. It was shown that in the absence of a *purely* gravitational measurement of Planck's constant one *cannot* at present rule out the possibility that gravitational quanta may be *scaled* by the more diminutive of nature's two *elementary* "actions", namely, e^2/c, which was conjectured to be an *intrinsic* property of the fabric of spacetime. A possible experiment requiring sensitivities beyond the standard quantum limit was suggested.

Acknowledgments

References

1. Weber J. Evidence for Discovery of Gravitational Radiation. *Phys. Rev. Lett.*, 1969, v. 22, 1320–1324.

2. Aguiar O. D. The Brazilian Spherical Detector: Progress and Plans. *Class. Quantum Grav.*, 2004, v. 21, 457–463.

Appendix

The recognition of the "action" e^2/c as an intrinsic property of the fabric of spacetime inevitably leads to quantum uncertainty at a more fundamental level than Planck's constant, in the analogous form

$$(\Delta x)(\Delta p) \approx \frac{e^2}{c} \tag{1}$$

where, as usual, x is uncertainty of position, and p the uncertainty in momentum. Its implication for the *temporal* events that make up the big bang can be simply illustrated in terms of the *sub-Planckian* unit of time, T_0, analogous to the Planck time $T_P = \sqrt{\hbar G/c^5}$, in the form

$$
\begin{aligned}
T_0 &= \sqrt{\frac{e^2}{2\pi c}\frac{G}{c^5}} \\
&= 1.837 \times 10^{-45} \text{ s}
\end{aligned}
\tag{2}
$$

where $(e^2/2\pi c)$ is the reduced *sub-Planckian* "action" constant, G is the Newtonian gravitational constant, and c is the velocity of light. Unfortunately, because of the *sub-Planckian* uncertainty principle, Eq. (1), we are prevented from speculating on times shorter than 10^{-44} seconds after the big bang, which is an order of magnitude prior to the Planck era (10^{-43} seconds). The disparity in this temporal sequence of events is, needless to say, cosmologically significant since it implies that a *sub-Planckian* era *preceded* the Planck era in the nascent universe, which should be discernible from its gravitational signature.