

Dislocations in the Spacetime Continuum: Framework for Quantum Physics

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This paper provides a framework for the physical description of physical processes at the quantum level based on dislocations in the spacetime continuum within STCED (Spacetime Continuum Elastodynamics). In this framework, photon and particle self-energies and interactions are mediated by the strain energy density of the dislocations, replacing the role played by virtual particles in QED. We postulate that the spacetime continuum has a granularity characterized by a length b_0 corresponding to the smallest *STC* elementary Burgers dislocation-displacement vector. Screw dislocations corresponding to transverse displacements are identified with photons, and edge dislocations corresponding to longitudinal displacements are identified with particles. Mixed dislocations give rise to wave-particle duality. The strain energy density of the dislocations are calculated and proposed to explain the QED problem of mass renormalization.

1 Introduction

In a previous paper [1], the deformable medium properties of the spacetime continuum (*STC*) led us to expect dislocations, disclinations and other defects to be present in the *STC*. The effects of such defects would be expected to manifest themselves mostly at the microscopic level. In this paper, we present a framework to show that dislocations in the spacetime continuum are the basis of quantum physics. This paper lays the framework to develop a theory of the physical processes that underlie Quantum Electrodynamics (QED). The theory does not result in the same formalism as QED, but rather results in an alternative formulation that provides a physical description of physical processes at the quantum level. This framework allows the theory to be fleshed out in subsequent investigations.

1.1 Elastodynamics of the Spacetime Continuum

As shown in a previous paper [1], General Relativity leads us to consider the spacetime continuum as a deformable continuum, which allows for the application of continuum mechanical methods and results to the analysis of its deformations. The Elastodynamics of the Spacetime Continuum (*STCED*) [1–7] is based on analyzing the spacetime continuum within a continuum mechanical and general relativistic framework.

The combination of all spacetime continuum deformations results in the geometry of the *STC*. The geometry of the spacetime continuum of General Relativity resulting from the energy-momentum stress tensor can thus be seen to be a representation of the deformation of the spacetime continuum resulting from the strains generated by the energy-momentum stress tensor.

As shown in [1], for an isotropic and homogeneous spacetime continuum, the *STC* is characterized by the stress-strain relation

$$2\bar{\mu}_0\epsilon^{\mu\nu} + \bar{\lambda}_0 g^{\mu\nu}\epsilon = T^{\mu\nu} \quad (1)$$

where $T^{\mu\nu}$ is the energy-momentum stress tensor, $\epsilon^{\mu\nu}$ is the resulting strain tensor, and

$$\epsilon = \epsilon^\alpha{}_\alpha \quad (2)$$

is the trace of the strain tensor obtained by contraction. The volume dilatation ϵ is defined as the change in volume per original volume [8, see pp. 149–152] and is an invariant of the strain tensor. $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the spacetime continuum: $\bar{\mu}_0$ is the shear modulus and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus:

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \bar{\mu}_0/2 \quad (3)$$

in a four-dimensional continuum.

As shown in [1], energy propagates in the spacetime continuum as wave-like deformations which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape of the spacetime continuum without a change in volume and are thus massless. Thus deformations propagate in the spacetime continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements.

This provides a natural explanation for wave-particle duality, with the transverse mode corresponding to the wave aspects of the deformation and the longitudinal mode corresponding to the particle aspects of the deformation [7]. The rest-mass energy density of the longitudinal mode is given by [1, see Eq.(32)]

$$\rho c^2 = 4\bar{\kappa}_0\epsilon \quad (4)$$

where ρ is the rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC* (the resistance of the spacetime continuum to *dilatations*), and ϵ is the volume dilatation.

This equation demonstrates that rest-mass energy density arises from the volume dilatation of the spacetime continuum.

The rest-mass energy is equivalent to the energy required to dilate the volume of the spacetime continuum. It is a measure of the energy stored in the spacetime continuum as mass. The volume dilatation is an invariant, as is the rest-mass energy density.

This is an important result as it demonstrates that mass is not independent of the spacetime continuum, but rather mass is part of the spacetime continuum fabric itself. Mass results from the dilatation of the *STC* in the longitudinal propagation of energy-momentum in the spacetime continuum. Matter does not warp spacetime, but rather, matter *is* warped spacetime (i.e. dilated spacetime). The universe consists of the spacetime continuum and energy-momentum that propagates in it by deformation of its (*STC*) structure.

Note that in this paper, we denote the *STCED* spacetime continuum constants $\bar{\kappa}_0, \bar{\lambda}_0, \bar{\mu}_0, \bar{\rho}_0$ with a diacritical mark over the symbols to differentiate them from similar symbols used in other fields of Physics. This allows us to retain existing symbols such as μ_0 for the electromagnetic permeability of free space, compared to the Lamé elastic constant $\bar{\mu}_0$ used to denote the spacetime continuum shear modulus.

1.2 Defects in the Spacetime Continuum

As discussed in [1], given that the spacetime continuum behaves as a deformable medium, there is no reason not to expect dislocations, disclinations and other defects to be present in the *STC*. Dislocations in the spacetime continuum represent the fundamental displacement processes that occur in its structure. These fundamental displacement processes should thus correspond to basic quantum phenomena and provide a framework for the description of quantum physics in *STCED*.

Defect theory has been the subject of investigation since the first half of the XXth century and is a well-developed discipline in continuum mechanics [9–14]. The recent formulation of defects in solids is based on gauge theory [15, 16].

The last quarter of the XXth century has seen the investigation of spacetime defects in the context of string theory, particularly cosmic strings [17, 18], and cosmic expansion [20, 21]. Teleparallel spacetime with defects [18, 22, 23] has resulted in a differential geometry of defects, which can be folded into the Einstein-Cartan Theory (ECT) of gravitation, an extension of Einstein's theory of gravitation that includes torsion [19, 20]. Recently, the phenomenology of spacetime defects has been considered in the context of quantum gravity [24–26].

In this paper, we investigate dislocations in the spacetime continuum in the context of *STCED*. The approach followed till now by investigators has been to use Einstein-Cartan differential geometry, with dislocations (translational deformations) impacting curvature and disclinations (rotational deformations) impacting torsion. The dislocation itself is modelled via the line element ds^2 [17]. In this paper, we investigate spacetime continuum dislocations using the underlying dis-

placements u^ν and the energy-momentum stress tensor. We thus work from the RHS of the general relativistic equation (the stress tensor side) rather than the LHS (the geometric tensor side). It should be noted that the general relativistic equation used can be the standard Einstein equation or a suitably modified version, as in Einstein-Cartan or Teleparallel formulations.

In Section 2 of this paper, we review the basic physical characteristics and dynamics of dislocations in the spacetime continuum. The energy-momentum stress tensor is considered in Section 2.2. This is followed by a detailed review of stationary and moving screw and edge dislocations in Sections 3, 4 and 5, along with their strain energy density as calculated from *STCED*. The framework of quantum physics, based on dislocations in the spacetime continuum is covered in Section 6. Screw dislocations in quantum physics are considered in Section 6.2 and edge dislocations are covered in Section 6.3. Section 7 covers dislocation interactions in quantum physics, and Section 8 provides physical explanations of QED phenomena provided by dislocations in the *STC*. Section 9 summarizes the framework presented in this paper for the development of a physical description of physical processes at the quantum level, based on dislocations in the spacetime continuum within the theory of the Elastodynamics of the Spacetime Continuum (*STCED*).

2 Dislocations in the Spacetime Continuum

A dislocation is characterized by its dislocation-displacement vector, known as the *Burgers vector*, b^μ in a four-dimensional continuum, defined positive in the direction of a vector ξ^μ tangent to the dislocation line in the spacetime continuum [14, see pp.17–24].

A *Burgers circuit* encloses the dislocation. A similar reference circuit can be drawn to enclose a region free of dislocation (see Fig. 1). The Burgers vector is the vector required to make the Burgers circuit equivalent to the reference circuit (see Fig. 2). It is a measure of the displacement between the initial and final points of the circuit due to the dislocation.

It is important to note that there are two conventions used to define the Burgers vector. In this paper, we use the convention used by Hirth [14] referred to as the local Burgers vector. The local Burgers vector is equivalently given by the line integral

$$b^\mu = \oint_C \frac{\partial u^\mu}{\partial s} ds \quad (5)$$

taken in a right-handed sense relative to ξ^μ , where u^μ is the displacement vector.

A dislocation is thus characterized by a line direction ξ^μ and a Burgers vector b^μ . There are two types of dislocations: an *edge dislocation* for which $b^\mu \xi_\mu = 0$ and a *screw dislocation* which can be right-handed for which $b^\mu \xi_\mu = b$, or left-handed for which $b^\mu \xi_\mu = -b$, where b is the magnitude of the Burgers vector. Arbitrary mixed dislocations can be decom-

Fig. 1: A reference circuit in a region free of dislocation, S: start, F: finish

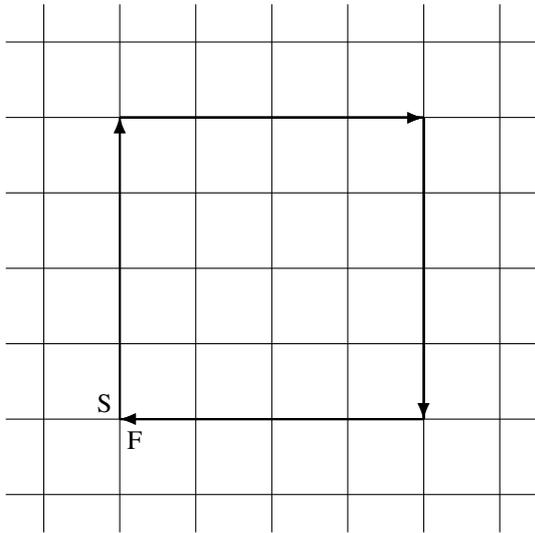
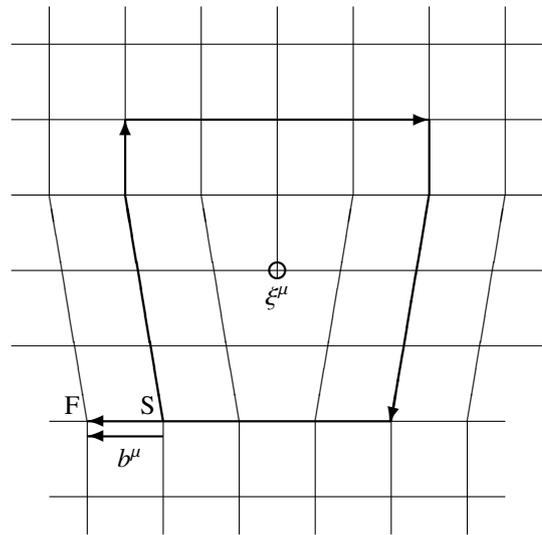


Fig. 2: A dislocation showing the Burgers vector b^μ , direction vector ξ^μ which points into the paper and the Burgers circuit, S: start, F: finish



posed into a screw component, along vector ξ^μ , and an edge component, perpendicular to vector ξ^μ .

The edge dislocation was first proposed by Orowan [27], Polanyi [28] and Taylor [29] in 1934, while the screw dislocation was proposed by Burgers [30] in 1939. In this paper, we extend the concept of dislocations to the elastodynamics of the spacetime continuum. Edge dislocations correspond to *dilatations* (longitudinal displacements) and hence have an associated rest-mass energy, while screw dislocations correspond to *distortions* (transverse displacements) and are massless [1].

2.1 Dislocation dynamics

In three-dimensional space, the dynamic equation is written as [31, see pp. 88–89],

$$T^{ij}_{;j} = -X^i + \bar{\rho}_0 \ddot{u}^i \tag{6}$$

where $\bar{\rho}_0$ is the spacetime continuum density, X^i is the volume (or body) force, the comma (,) represents differentiation and \dot{u} denotes the derivative with respect to time. Substituting for $\varepsilon^{\mu\nu} = \frac{1}{2}(u^{\mu;\nu} + u^{\nu;\mu})$ in (1), using (2) and $u^{\mu}_{;\mu} = \varepsilon^\mu_\mu = \varepsilon$ in this equation, we obtain

$$\bar{\mu}_0 \vec{\nabla}^2 u^i + (\bar{\mu}_0 + \bar{\lambda}_0) \varepsilon^{;i} = -X^i + \bar{\rho}_0 \ddot{u}^i \tag{7}$$

which, upon converting the time derivative to indicial notation and rearranging, is written as

$$\bar{\mu}_0 \vec{\nabla}^2 u^i - \bar{\rho}_0 c^2 u^i_{;00} + (\bar{\mu}_0 + \bar{\lambda}_0) \varepsilon^{;i} = -X^i. \tag{8}$$

We use the arrow above the nabla symbol to indicate the 3-dimensional gradient whereas the 4-dimensional gradient is

written with no arrow. Using the relation [1]

$$c = \sqrt{\frac{\bar{\mu}_0}{\bar{\rho}_0}} \tag{9}$$

in the above, (8) becomes

$$\bar{\mu}_0 (\vec{\nabla}^2 u^i - u^i_{;00}) + (\bar{\mu}_0 + \bar{\lambda}_0) \varepsilon^{;i} = -X^i \tag{10}$$

and, combining the space and time derivatives, we obtain

$$\bar{\mu}_0 \nabla^2 u^i + (\bar{\mu}_0 + \bar{\lambda}_0) \varepsilon^{;i} = -X^i. \tag{11}$$

This equation is the space portion of the *STCED displacement wave equation* (51) of [1]

$$\bar{\mu}_0 \nabla^2 u^\nu + (\bar{\mu}_0 + \bar{\lambda}_0) \varepsilon^{;\nu} = -X^\nu. \tag{12}$$

Hence the dynamics of the spacetime continuum is described by the dynamic equation (12), which includes the accelerations from the applied forces.

In this analysis, we consider the simpler problem of dislocations moving in an isotropic continuum with no volume force. Then (12) becomes

$$\bar{\mu}_0 \nabla^2 u^\nu + (\bar{\mu}_0 + \bar{\lambda}_0) \varepsilon^{;\nu} = 0, \tag{13}$$

where ∇^2 is the four-dimensional operator and the semi-colon (;) represents covariant differentiation.

Separating u^ν into its longitudinal (irrotational) component u^ν_\parallel and its transverse (solenoidal) component u^ν_\perp using the Helmholtz theorem in four dimensions [32] according to

$$u^\nu = u^\nu_\parallel + u^\nu_\perp, \tag{14}$$

(12) can be separated into a screw dislocation displacement (transverse) equation

$$\bar{\mu}_0 \nabla^2 u_{\perp}^{\nu} = 0 \tag{15}$$

and an edge dislocation displacement (longitudinal) equation

$$\nabla^2 u_{\parallel}^{\nu} = -\frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0} \varepsilon^{;\nu}. \tag{16}$$

2.2 The energy-momentum stress tensor

The components of the energy-momentum stress tensor are given by [33]:

$$\begin{aligned} T^{00} &= H \\ T^{0j} &= s^j \\ T^{i0} &= g^i \\ T^{ij} &= \sigma^{ij} \end{aligned} \tag{17}$$

where H is the total energy density, s^j is the energy flux vector, g^i is the momentum density vector, and σ^{ij} is the Cauchy stress tensor which is the i^{th} component of force per unit area at x^j .

From the stress tensor $T^{\mu\nu}$, we can calculate the strain tensor $\varepsilon^{\mu\nu}$ and then calculate the strain energy density of the dislocations. As shown in [3], for a general anisotropic continuum in four dimensions, the spacetime continuum is approximated by a deformable linear elastic medium that obeys Hooke’s law [31, see pp. 50–53]

$$E^{\mu\nu\alpha\beta} \varepsilon_{\alpha\beta} = T^{\mu\nu} \tag{18}$$

where $E^{\mu\nu\alpha\beta}$ is the elastic moduli tensor. For an isotropic and homogeneous medium, the elastic moduli tensor simplifies to [31]:

$$E^{\mu\nu\alpha\beta} = \bar{\lambda}_0 (g^{\mu\nu} g^{\alpha\beta}) + \bar{\mu}_0 (g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}). \tag{19}$$

For the metric tensor $g_{\mu\nu}$, we use the flat spacetime diagonal metric $\eta_{\mu\nu}$ with signature $(-+++)$ as the *STC* is locally flat at the microscopic level. Substituting for (19) into (18) and expanding, we obtain

$$\begin{aligned} T^{00} &= (\bar{\lambda}_0 + 2\bar{\mu}_0) \varepsilon^{00} - \bar{\lambda}_0 \varepsilon^{11} - \bar{\lambda}_0 \varepsilon^{22} - \bar{\lambda}_0 \varepsilon^{33} \\ T^{11} &= -\bar{\lambda}_0 \varepsilon^{00} + (\bar{\lambda}_0 + 2\bar{\mu}_0) \varepsilon^{11} + \bar{\lambda}_0 \varepsilon^{22} + \bar{\lambda}_0 \varepsilon^{33} \\ T^{22} &= -\bar{\lambda}_0 \varepsilon^{00} + \bar{\lambda}_0 \varepsilon^{11} + (\bar{\lambda}_0 + 2\bar{\mu}_0) \varepsilon^{22} + \bar{\lambda}_0 \varepsilon^{33} \\ T^{33} &= -\bar{\lambda}_0 \varepsilon^{00} + \bar{\lambda}_0 \varepsilon^{11} + \bar{\lambda}_0 \varepsilon^{22} + (\bar{\lambda}_0 + 2\bar{\mu}_0) \varepsilon^{33} \\ T^{\mu\nu} &= 2\bar{\mu}_0 \varepsilon^{\mu\nu}, \quad \mu \neq \nu. \end{aligned} \tag{20}$$

In terms of the stress tensor, the inverse of (20) is given by

$$\begin{aligned} \varepsilon^{00} &= \frac{1}{4\bar{\mu}_0(2\bar{\lambda}_0 + \bar{\mu}_0)} \left[(3\bar{\lambda}_0 + 2\bar{\mu}_0) T^{00} + \bar{\lambda}_0 (T^{11} + T^{22} + T^{33}) \right] \\ \varepsilon^{11} &= \frac{1}{4\bar{\mu}_0(2\bar{\lambda}_0 + \bar{\mu}_0)} \left[(3\bar{\lambda}_0 + 2\bar{\mu}_0) T^{11} + \bar{\lambda}_0 (T^{00} - T^{22} - T^{33}) \right] \\ \varepsilon^{22} &= \frac{1}{4\bar{\mu}_0(2\bar{\lambda}_0 + \bar{\mu}_0)} \left[(3\bar{\lambda}_0 + 2\bar{\mu}_0) T^{22} + \bar{\lambda}_0 (T^{00} - T^{11} - T^{33}) \right] \\ \varepsilon^{33} &= \frac{1}{4\bar{\mu}_0(2\bar{\lambda}_0 + \bar{\mu}_0)} \left[(3\bar{\lambda}_0 + 2\bar{\mu}_0) T^{33} + \bar{\lambda}_0 (T^{00} - T^{11} - T^{22}) \right] \\ \varepsilon^{\mu\nu} &= \frac{1}{2\bar{\mu}_0} T^{\mu\nu}, \quad \mu \neq \nu. \end{aligned} \tag{21}$$

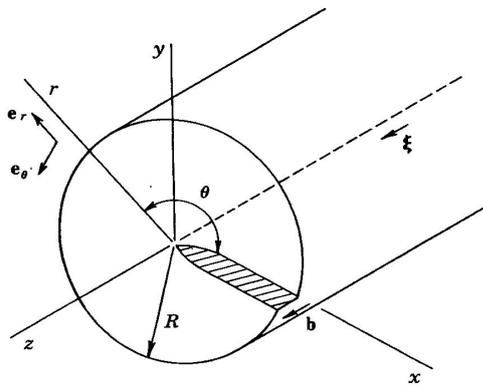
where $T^{ij} = \sigma^{ij}$. We calculate $\varepsilon = \varepsilon^{\alpha}_{\alpha}$ from the values of (21). Using $\eta_{\mu\nu}$, (3) and $T^{\alpha}_{\alpha} = \rho c^2$ from [2], we obtain (4) as required. This confirms the validity of the strain tensor in terms of the energy-momentum stress tensor as given by (21).

Eshelby [34–36] introduced an elastic field energy-momentum tensor for continuous media to deal with cases where defects (such as dislocations) lead to changes in configuration. The displacements u^{ν} are considered to correspond to a field defined at points x^{μ} of the spacetime continuum. This tensor was first derived by Morse and Feshback [37] for an isotropic elastic medium, using dyadics. The energy flux vector s_j and the field momentum density vector g_i are then given by [34, 37]:

$$\begin{aligned} s_j &= -\dot{u}_k \sigma_{kj} \\ g_i &= \bar{\rho}_0 u_{k,i} \dot{u}_k \\ b_{ij} &= L \delta_{ij} - u_{k,i} \sigma_{kj} \end{aligned} \tag{22}$$

where $\bar{\rho}_0$ is the density of the medium, in this case the spacetime continuum, L is the Lagrangian equal to $K - W$ where W is the strain energy density and K is the kinetic energy density ($H = K + W$), and b_{ij} is known as the Eshelby stress tensor [38, see p. 27]. If the energy-momentum stress tensor is symmetric, then $g^i = s^i$. In this paper, we consider the case where there are no changes in configuration, and use the energy-momentum stress tensor given by (17) and (20).

Fig. 3: A stationary screw dislocation in cartesian (x, y, z) and cylindrical polar (r, θ, z) coordinates [14, see p. 60].



3 Screw dislocation

3.1 Stationary screw dislocation

We consider a stationary screw dislocation in the spacetime continuum, with cylindrical polar coordinates (r, θ, z) , with the dislocation line along the z -axis (see Fig. 3). Then the Burgers vector is along the z -axis and is given by $b_r = b_\theta = 0$, $b_z = b$, the magnitude of the Burgers vector. The only non-zero component of the deformations is given by [14, see pp. 60–61] [13, see p. 51]

$$u_z = \frac{b}{2\pi} \theta = \frac{b}{2\pi} \tan^{-1} \frac{y}{x}. \quad (23)$$

This solution satisfies the screw dislocation displacement equation (15).

Similarly, the only non-zero components of the stress and strain tensors are given by

$$\begin{aligned} \sigma_{\theta z} &= \frac{b}{2\pi} \frac{\bar{\mu}_0}{r} \\ \varepsilon_{\theta z} &= \frac{b}{4\pi} \frac{1}{r} \end{aligned} \quad (24)$$

respectively.

3.2 Moving screw dislocation

We now consider the previous screw dislocation, moving along the x -axis, parallel to the dislocation, at a constant speed $v_x = v$. Equation (13) then simplifies to the wave equation for massless transverse shear waves for the displacements u_z along the z -axis, with speed $c_t = c$ given by (9), where c_t is the speed of the transverse waves corresponding to c the speed of light.

If coordinate system (x', y', z', t') is attached to the uniformly moving screw dislocation, then the transformation between the stationary and the moving screw dislocation is given by [14]

$$\begin{aligned} x' &= \frac{x - vt}{(1 - v^2/c^2)^{1/2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - vx/c^2}{(1 - v^2/c^2)^{1/2}}. \end{aligned} \quad (25)$$

which is the special relativistic transformation.

The only non-zero component of the deformation in cartesian coordinates is given by [14, see pp. 184–185]

$$u_z = \frac{b}{2\pi} \tan^{-1} \frac{\gamma y}{x - vt}, \quad (26)$$

where

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}. \quad (27)$$

This solution also satisfies the screw dislocation displacement equation (15). It simplifies to the case of the stationary screw dislocation when the speed $v = 0$.

Similarly, the only non-zero components of the stress tensor in cartesian coordinates are given by [14]

$$\begin{aligned} \sigma_{xz} &= -\frac{b\bar{\mu}_0}{2\pi} \frac{\gamma y}{(x - vt)^2 + \gamma^2 y^2} \\ \sigma_{yz} &= \frac{b\bar{\mu}_0}{2\pi} \frac{\gamma(x - vt)}{(x - vt)^2 + \gamma^2 y^2}. \end{aligned} \quad (28)$$

The only non-zero components of the strain tensor in cartesian coordinates are derived from $\varepsilon^{\mu\nu} = \frac{1}{2}(u^{\mu;\nu} + u^{\nu;\mu})$ [1, see Eq.(41)]:

$$\begin{aligned} \varepsilon_{xz} &= -\frac{b}{4\pi} \frac{\gamma y}{(x - vt)^2 + \gamma^2 y^2} \\ \varepsilon_{yz} &= \frac{b}{4\pi} \frac{\gamma(x - vt)}{(x - vt)^2 + \gamma^2 y^2}, \end{aligned} \quad (29)$$

in an isotropic continuum.

Non-zero components involving time are given by

$$\begin{aligned} \varepsilon_{tz} = \varepsilon_{zt} &= \frac{1}{2} \left(\frac{\partial u_z}{\partial(ct)} + \frac{\partial u_t}{\partial z} \right) \\ \varepsilon_{tz} &= \frac{b}{4\pi} \frac{v}{c} \frac{\gamma y}{(x - vt)^2 + \gamma^2 y^2} \end{aligned} \quad (30)$$

where $u_t = 0$ has been used. This assumes that the screw dislocation is fully formed and moving with velocity v as described. Using (20), the non-zero stress components involving time are given by

$$\sigma_{tz} = \sigma_{zt} = \frac{b\bar{\mu}_0}{2\pi} \frac{v}{c} \frac{\gamma y}{(x - vt)^2 + \gamma^2 y^2}. \quad (31)$$

Screw dislocations are thus found to be Lorentz invariant.

3.3 Screw dislocation strain energy density

We consider the stationary screw dislocation in the spacetime continuum of Section 3.1, with cylindrical polar coordinates (r, θ, z) , with the dislocation line along the z -axis and the Burgers vector along the z -axis $b_z = b$.

Then the strain energy density of the screw dislocation is given by the transverse distortion energy density [1, see Eq. (74)]

$$\mathcal{E}_\perp = \bar{\mu}_0 e^{\alpha\beta} e_{\alpha\beta} \tag{32}$$

where from [1, see Eq. (33)],

$$e^{\alpha\beta} = \varepsilon^{\alpha\beta} - e_s g^{\alpha\beta} \tag{33}$$

where $e_s = \frac{1}{4} \varepsilon^\alpha_\alpha$ is the dilatation which for a screw dislocation is equal to 0. The screw dislocation is thus massless ($\mathcal{E}_\parallel = 0$).

The non-zero components of the strain tensor are as defined in (24). Hence

$$\mathcal{E}_\perp = \bar{\mu}_0 (\varepsilon_{\theta z}^2 + \varepsilon_{z\theta}^2). \tag{34}$$

Substituting from (24),

$$\mathcal{E}_\perp = \frac{\bar{\mu}_0 b^2}{8\pi^2} \frac{1}{r^2} = \mathcal{E}. \tag{35}$$

We now consider the more general case of the moving screw dislocation in the spacetime continuum of Section 3.2, with cartesian coordinates (x, y, z) . The non-zero components of the strain tensor are as defined in (29) and (30). Substituting in (32), the equation becomes [1, see Eqs.(114–115)]

$$\mathcal{E}_\perp = 2\bar{\mu}_0 (-\varepsilon_{tz}^2 + \varepsilon_{xz}^2 + \varepsilon_{yz}^2). \tag{36}$$

Substituting from (29) and (30) into (36), the screw dislocation strain energy density becomes

$$\mathcal{E}_\perp = \frac{\bar{\mu}_0 b^2}{8\pi^2} \frac{\gamma^2}{(x - vt)^2 + \gamma^2 y^2} = \mathcal{E}. \tag{37}$$

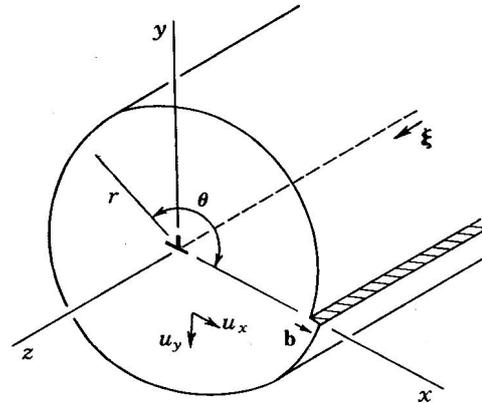
This equation simplifies to (35) in the case where $v = 0$, as expected. In addition, the energy density (which is quadratic in energy as per [1, see Eq.(76)]) is multiplied by the special relativistic γ factor.

4 Edge dislocation

4.1 Stationary edge dislocation

We consider a stationary edge dislocation in the spacetime continuum in cartesian coordinates (x, y, z) , with the dislocation line along the z -axis and the Burgers vector $b_x = b, b_y = b_z = 0$ (see Fig. 4). Then the non-zero components of the deformations are given in cartesian coordinates by [14, see

Fig. 4: A stationary edge dislocation in cartesian (x, y, z) and cylindrical polar (r, θ, z) coordinates [14, see p. 74].



p. 78]

$$\begin{aligned} u_x &= \frac{b}{2\pi} \left(\tan^{-1} \frac{y}{x} + \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{xy}{x^2 + y^2} \right) \\ u_y &= -\frac{b}{2\pi} \left(\frac{1}{2} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \log(x^2 + y^2) + \frac{1}{2} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{x^2 - y^2}{x^2 + y^2} \right). \end{aligned} \tag{38}$$

This solution results in a non-zero R.H.S. of the edge dislocation displacement equation (16) as required. Equation (16) can be evaluated to give a value of ε in agreement with the results of Section 4.3 as shown in that section.

The cylindrical polar coordinate description of the edge dislocation is more complex than the cartesian coordinate description. We thus use cartesian coordinates in the following sections, transforming to polar coordinate expressions as warranted. The non-zero components of the stress tensor in cartesian coordinates are given by [14, see p. 76]

$$\begin{aligned} \sigma_{xx} &= -\frac{b\bar{\mu}_0}{\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y(3x^2 + y^2)}{(x^2 + y^2)^2} \\ \sigma_{yy} &= \frac{b\bar{\mu}_0}{\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y(x^2 - y^2)}{(x^2 + y^2)^2} \\ \sigma_{zz} &= \frac{1}{2} \frac{\bar{\lambda}_0}{\bar{\mu}_0 + \bar{\lambda}_0} (\sigma_{xx} + \sigma_{yy}) \\ &= -\frac{b\bar{\mu}_0}{\pi} \frac{\bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y}{x^2 + y^2} \\ \sigma_{xy} &= \frac{b\bar{\mu}_0}{\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}. \end{aligned} \tag{39}$$

The non-zero components of the strain tensor in cartesian coordinates are derived from $\varepsilon^{\mu\nu} = \frac{1}{2}(u^{\mu;\nu} + u^{\nu;\mu})$ [1, see

Eq.(41)]:

$$\begin{aligned} \varepsilon_{xx} &= -\frac{b}{2\pi} \frac{y}{x^2 + y^2} \left(1 + \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{x^2 - y^2}{x^2 + y^2} \right) \\ &= -\frac{by}{2\pi} \frac{(3\bar{\mu}_0 + 2\bar{\lambda}_0)x^2 + \bar{\mu}_0 y^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)(x^2 + y^2)^2} \\ \varepsilon_{yy} &= -\frac{b}{2\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y}{x^2 + y^2} \left(1 - \frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0} \frac{2x^2}{x^2 + y^2} \right) \quad (40) \\ &= \frac{by}{2\pi} \frac{(\bar{\mu}_0 + 2\bar{\lambda}_0)x^2 - \bar{\mu}_0 y^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)(x^2 + y^2)^2} \\ \varepsilon_{xy} &= \frac{b}{2\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2} \end{aligned}$$

in an isotropic continuum.

4.2 Moving edge dislocation

We now consider the previous edge dislocation, moving along the x -axis, parallel to the z -axis, along the slip plane $x-z$, at a constant speed $v_x = v$. The solutions of (13) for the moving edge dislocation then include both longitudinal and transverse components. The only non-zero components of the deformations in cartesian coordinates are given by [11, see pp. 39–40] [39, see pp. 218–219]

$$\begin{aligned} u_x &= \frac{bc^2}{\pi v^2} \left(\tan^{-1} \frac{\gamma_l y}{x - vt} - \alpha^2 \tan^{-1} \frac{\gamma y}{x - vt} \right) \\ u_y &= \frac{bc^2}{2\pi v^2} \left(\gamma_l \log \left[(x - vt)^2 + \gamma_l^2 y^2 \right] - \right. \quad (41) \\ &\quad \left. - \frac{\alpha^2}{\gamma} \log \left[(x - vt)^2 + \gamma^2 y^2 \right] \right), \end{aligned}$$

where

$$\alpha = \sqrt{1 - \frac{v^2}{2c^2}}, \quad (42)$$

$$\gamma_l = \sqrt{1 - \frac{v^2}{c_l^2}} \quad (43)$$

and c_l is the speed of longitudinal deformations given by

$$c_l = \sqrt{\frac{2\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\rho}_0}}. \quad (44)$$

This solution again results in a non-zero R.H.S. of the edge dislocation displacement equation (16) as required, and (16) can be evaluated to give a value of ε as in Section 4.3. This solution simplifies to the case of the stationary edge dislocation when the speed $v = 0$.

The non-zero components of the stress tensor in cartesian coordinates are given by [14, see pp. 189–190] [11, see

pp. 39–40]

$$\begin{aligned} \sigma_{xx} &= \frac{bc^2 y}{\pi v^2} \left(\frac{\bar{\lambda}_0 \gamma_l^3 - (2\bar{\mu}_0 + \bar{\lambda}_0) \gamma_l}{(x - vt)^2 + \gamma_l^2 y^2} + \right. \\ &\quad \left. + \frac{2\bar{\mu}_0 \alpha^2 \gamma}{(x - vt)^2 + \gamma^2 y^2} \right) \\ \sigma_{yy} &= \frac{bc^2 y}{\pi v^2} \left(\frac{(2\bar{\mu}_0 + \bar{\lambda}_0) \gamma_l^3 - \bar{\lambda}_0 \gamma_l}{(x - vt)^2 + \gamma_l^2 y^2} - \right. \\ &\quad \left. - \frac{2\bar{\mu}_0 \alpha^2 \gamma}{(x - vt)^2 + \gamma^2 y^2} \right) \\ \sigma_{zz} &= \frac{1}{2} \frac{\bar{\lambda}_0}{\bar{\mu}_0 + \bar{\lambda}_0} (\sigma_{xx} + \sigma_{yy}) \quad (45) \\ &= \frac{\bar{\lambda}_0 b}{\pi} \frac{c^2}{c_l^2} \frac{-\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} \\ &= \frac{b}{\pi} \frac{\bar{\lambda}_0 \bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{-\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} \\ \sigma_{xy} &= \frac{\bar{\mu}_0 bc^2 (x - vt)}{\pi v^2} \left(\frac{2\gamma_l}{(x - vt)^2 + \gamma_l^2 y^2} - \right. \\ &\quad \left. - \frac{\alpha^2 (\gamma + 1/\gamma)}{(x - vt)^2 + \gamma^2 y^2} \right). \end{aligned}$$

It is important to note that for a screw dislocation, the stress on the plane $x - vt = 0$ becomes infinite at $v = c$. This sets an upper limit on the speed of screw dislocations in the spacetime continuum, and provides an explanation for the speed of light limit. This upper limit also applies to edge dislocations, as the shear stress becomes infinite everywhere at $v = c$, even though the speed of longitudinal deformations c_l is greater than that of transverse deformations c [14, see p. 191] [11, see p. 40].

The non-zero components of the strain tensor in cartesian coordinates are derived from $\varepsilon^{\mu\nu} = \frac{1}{2}(u^{\mu;\nu} + u^{\nu;\mu})$ [1, see Eq.(41)]:

$$\begin{aligned} \varepsilon_{xx} &= \frac{bc^2 y}{\pi v^2} \left(\frac{-\gamma_l}{(x - vt)^2 + (\gamma_l y)^2} + \frac{\alpha^2 \gamma}{(x - vt)^2 + (\gamma y)^2} \right) \\ \varepsilon_{yy} &= \frac{bc^2 y}{\pi v^2} \left(\frac{\gamma_l^3}{(x - vt)^2 + (\gamma_l y)^2} - \frac{\alpha^2 \gamma}{(x - vt)^2 + (\gamma y)^2} \right) \quad (46) \\ \varepsilon_{xy} &= \frac{bc^2 (x - vt)}{2\pi v^2} \left(\frac{2\gamma_l}{(x - vt)^2 + (\gamma_l y)^2} - \right. \\ &\quad \left. - \frac{\alpha^2 (\gamma + 1/\gamma)}{(x - vt)^2 + (\gamma y)^2} \right) \end{aligned}$$

in an isotropic continuum.

Non-zero components involving time are given by

$$\begin{aligned} \varepsilon_{tx} = \varepsilon_{xt} &= \frac{1}{2} \left(\frac{\partial u_x}{\partial(ct)} + \frac{\partial u_t}{\partial x} \right) \\ \varepsilon_{ty} = \varepsilon_{yt} &= \frac{1}{2} \left(\frac{\partial u_y}{\partial(ct)} + \frac{\partial u_t}{\partial y} \right) \\ \varepsilon_{tx} &= \frac{b}{2\pi} \frac{c}{v} \left(\frac{\gamma l y}{(x-vt)^2 + \gamma_l^2 y^2} - \right. \\ &\quad \left. - \alpha^2 \frac{\gamma y}{(x-vt)^2 + \gamma^2 y^2} \right) \\ \varepsilon_{ty} &= -\frac{b}{2\pi} \frac{c}{v} \left(\frac{\gamma_l(x-vt)}{(x-vt)^2 + \gamma_l^2 y^2} - \right. \\ &\quad \left. - \frac{\alpha^2}{\gamma^2} \frac{\gamma(x-vt)}{(x-vt)^2 + \gamma^2 y^2} \right) \end{aligned} \tag{47}$$

where $u_t = 0$ has been used. This assumes that the edge dislocation is fully formed and moving with velocity v as described. Using (20), the non-zero stress components involving time are given by

$$\begin{aligned} \sigma_{tx} &= \frac{b\bar{\mu}_0}{\pi} \frac{c}{v} \left(\frac{\gamma l y}{(x-vt)^2 + \gamma_l^2 y^2} - \right. \\ &\quad \left. - \alpha^2 \frac{\gamma y}{(x-vt)^2 + \gamma^2 y^2} \right) \\ \sigma_{ty} &= -\frac{b\bar{\mu}_0}{\pi} \frac{c}{v} \left(\frac{\gamma_l(x-vt)}{(x-vt)^2 + \gamma_l^2 y^2} - \right. \\ &\quad \left. - \frac{\alpha^2}{\gamma^2} \frac{\gamma(x-vt)}{(x-vt)^2 + \gamma^2 y^2} \right). \end{aligned} \tag{48}$$

4.3 Edge dislocation strain energy density

As we have seen in Section 3.3, the screw dislocation is massless as $\varepsilon = 0$ and hence $\mathcal{E}_{||} = 0$ for the screw dislocation: it is a pure distortion, with no dilatation. In this section, we evaluate the strain energy density of the edge dislocation.

As seen in [1, see Section 8.1], the strain energy density of the spacetime continuum is separated into two terms: the first one expresses the dilatation energy density (the mass longitudinal term) while the second one expresses the distortion energy density (the massless transverse term):

$$\mathcal{E} = \mathcal{E}_{||} + \mathcal{E}_{\perp} \tag{49}$$

where

$$\mathcal{E}_{||} = \frac{1}{2} \bar{\kappa}_0 \varepsilon^2 \equiv \frac{1}{32\bar{\kappa}_0} (\rho c^2)^2 \equiv \frac{1}{2\bar{\kappa}_0} t_s^2 \tag{50}$$

where ε is the volume dilatation and ρ is the mass energy density of the edge dislocation, and

$$\mathcal{E}_{\perp} = \bar{\mu}_0 e^{\alpha\beta} e_{\alpha\beta} \equiv \frac{1}{4\bar{\mu}_0} t^{\alpha\beta} t_{\alpha\beta}. \tag{51}$$

where from [1, see Eq. (36)] the energy-momentum stress tensor $T^{\alpha\beta}$ is decomposed into a stress deviation tensor $t^{\alpha\beta}$ and a scalar t_s , according to

$$t^{\alpha\beta} = T^{\alpha\beta} - t_s g^{\alpha\beta} \tag{52}$$

where $t_s = \frac{1}{4} T^{\alpha}_{\alpha}$. Then the dilatation strain energy density of the edge dislocation is given by the (massive) longitudinal dilatation energy density (50) and the distortion (massless) strain energy density of the edge dislocation is given by the transverse distortion energy density (51).

4.3.1 Stationary edge dislocation energy density

We first consider the case of the stationary edge dislocation of Section 4.1. The volume dilatation ε for the stationary edge dislocation is given by

$$\varepsilon = \varepsilon^{\alpha}_{\alpha} = \varepsilon_{xx} + \varepsilon_{yy} \tag{53}$$

where the non-zero diagonal elements of the strain tensor are obtained from (40). Substituting for ε_{xx} and ε_{yy} from (40), we obtain

$$\varepsilon = -\frac{b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y}{x^2 + y^2}. \tag{54}$$

In cylindrical polar coordinates, (54) is expressed as

$$\varepsilon = -\frac{b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\sin \theta}{r}. \tag{55}$$

We can disregard the negative sign in (54) and (55) as it can be eliminated by using the **FS/RH** convention instead of the **SF/RH** convention for the Burgers vector [14, see p. 22]).

As mentioned in Section 4.1, the volume dilatation ε can be calculated from the edge dislocation displacement (longitudinal) equation (16), viz.

$$\nabla^2 u_{||}^y = -\frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0} \varepsilon^{,y}.$$

For the x -component, this equation gives

$$\nabla^2 u_x = \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} = -\frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0} \varepsilon_{,x}. \tag{56}$$

Substituting for u_x from (38), we obtain

$$\nabla^2 u_x = -\frac{2b}{\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{xy}{(x^2 + y^2)^2} = -\frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0} \varepsilon_{,x}. \tag{57}$$

Hence

$$\varepsilon_{,x} = \frac{2b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{xy}{(x^2 + y^2)^2} \tag{58}$$

and

$$\varepsilon = \frac{2b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \int \frac{xy}{(x^2 + y^2)^2} dx. \tag{59}$$

Evaluating the integral [40], we obtain

$$\varepsilon = -\frac{b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y}{x^2 + y^2} \quad (60)$$

in agreement with (54).

Similarly for the y -component, substituting for u_y from (38), the equation

$$\nabla^2 u_y = \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} = -\frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0} \varepsilon_{,y} \quad (61)$$

gives

$$\varepsilon_{,y} = -\frac{b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{x^2 - y^2}{(x^2 + y^2)^2}. \quad (62)$$

Evaluating the integral [40]

$$\varepsilon = -\frac{b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \int \frac{x^2 - y^2}{(x^2 + y^2)^2} dy, \quad (63)$$

we obtain

$$\varepsilon = -\frac{b}{\pi} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y}{x^2 + y^2} \quad (64)$$

again in agreement with (54).

The mass energy density is calculated from (4)

$$\rho c^2 = 4\bar{\kappa}_0 \varepsilon = 2(2\bar{\lambda}_0 + \bar{\mu}_0) \varepsilon \quad (65)$$

where (3) has been used. Substituting for ε from (54), the mass energy density of the stationary edge dislocation is given by

$$\rho c^2 = \frac{4b}{\pi} \frac{\bar{\kappa}_0 \bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{y}{x^2 + y^2}. \quad (66)$$

In cylindrical polar coordinates, (66) is expressed as

$$\rho c^2 = \frac{4b}{\pi} \frac{\bar{\kappa}_0 \bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\sin \theta}{r}. \quad (67)$$

Using (54) in (50), the stationary edge dislocation longitudinal dilatation strain energy density is then given by

$$\mathcal{E}_{||} = \frac{b^2}{2\pi^2} \frac{\bar{\kappa}_0 \bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \frac{y^2}{(x^2 + y^2)^2}. \quad (68)$$

In cylindrical polar coordinates, (68) is expressed as

$$\mathcal{E}_{||} = \frac{b^2}{2\pi^2} \frac{\bar{\kappa}_0 \bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \frac{\sin^2 \theta}{r^2}. \quad (69)$$

The distortion strain energy density is calculated from (51). The expression is expanded using the non-zero elements of the strain tensor (40) to give

$$\mathcal{E}_{\perp} = \bar{\mu}_0 (e_{xx}^2 + e_{yy}^2 + e_{xy}^2 + e_{yx}^2). \quad (70)$$

As seen previously in (33),

$$e^{\alpha\beta} = \varepsilon^{\alpha\beta} - e_s g^{\alpha\beta} \quad (71)$$

where $e_s = \frac{1}{4} \varepsilon$ is the volume dilatation calculated in (54) and

$$e^{\alpha\beta} e_{\alpha\beta} = \left(\varepsilon^{\alpha\beta} - \frac{1}{4} \varepsilon g^{\alpha\beta} \right) \left(\varepsilon_{\alpha\beta} - \frac{1}{4} \varepsilon g_{\alpha\beta} \right). \quad (72)$$

For $g^{\alpha\beta} = \eta^{\alpha\beta}$, the off-diagonal elements of the metric tensor are 0, the diagonal elements are 1 and (70) becomes

$$\mathcal{E}_{\perp} = \bar{\mu}_0 \left[\left(\varepsilon_{xx} - \frac{1}{4} \varepsilon \right)^2 + \left(\varepsilon_{yy} - \frac{1}{4} \varepsilon \right)^2 + 2\varepsilon_{xy}^2 \right]. \quad (73)$$

Expanding the quadratic terms and making use of (53), (73) becomes

$$\mathcal{E}_{\perp} = \bar{\mu}_0 \left(\varepsilon_{xx}^2 + \varepsilon_{yy}^2 - \frac{3}{8} \varepsilon^2 + 2\varepsilon_{xy}^2 \right) \quad (74)$$

and finally

$$\mathcal{E}_{\perp} = \bar{\mu}_0 \left(\frac{5}{8} \varepsilon^2 - 2\varepsilon_{xx} \varepsilon_{yy} + 2\varepsilon_{xy}^2 \right). \quad (75)$$

Substituting from (40) and (54) in the above,

$$\begin{aligned} \mathcal{E}_{\perp} = & \frac{5}{8} \frac{b^2 \bar{\mu}_0}{\pi^2} \left(\frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \right)^2 \frac{y^2}{(x^2 + y^2)^2} + \frac{b^2 \bar{\mu}_0}{2\pi^2} \\ & \frac{y^2 \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0)x^4 - 2\bar{\mu}_0^2 x^2 y^2 - \bar{\mu}_0^2 y^4 \right]}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2 (x^2 + y^2)^4} + \\ & + \frac{b^2 \bar{\mu}_0}{2\pi^2} \left(\frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \right)^2 \frac{x^2 (x^2 - y^2)^2}{(x^2 + y^2)^4}. \end{aligned} \quad (76)$$

which becomes

$$\begin{aligned} \mathcal{E}_{\perp} = & \frac{b^2}{2\pi^2} \frac{\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \frac{1}{(x^2 + y^2)^4} \\ & \left\{ \frac{5}{4} \bar{\mu}_0^2 y^2 (x^2 + y^2)^2 - \right. \\ & - y^2 \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0)x^4 - 2\bar{\mu}_0^2 x^2 y^2 - \bar{\mu}_0^2 y^4 \right] + \\ & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 x^2 (x^2 - y^2)^2 \right\}. \end{aligned} \quad (77)$$

In cylindrical polar coordinates, (77) is expressed as

$$\begin{aligned} \mathcal{E}_{\perp} = & \frac{b^2}{2\pi^2} \frac{\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \left\{ \frac{5}{4} \bar{\mu}_0^2 \frac{\sin^2 \theta}{r^2} - \frac{\sin^2 \theta}{r^2} \right. \\ & \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0) \cos^4 \theta - \right. \\ & - 2\bar{\mu}_0^2 \cos^2 \theta \sin^2 \theta - \bar{\mu}_0^2 \sin^4 \theta \left. \right] + \\ & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 \frac{\cos^2 \theta}{r^2} (\cos^2 \theta - \sin^2 \theta)^2 \right\} \end{aligned} \quad (78)$$

or

$$\begin{aligned} \mathcal{E}_\perp = & \frac{b^2}{2\pi^2} \frac{\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \left\{ \frac{5}{4} \bar{\mu}_0^2 \frac{\sin^2 \theta}{r^2} - \right. \\ & - \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0) \cos^4 \theta \frac{\sin^2 \theta}{r^2} - \right. \\ & - 2\bar{\mu}_0^2 \cos^2 \theta \frac{\sin^4 \theta}{r^2} - \bar{\mu}_0^2 \frac{\sin^6 \theta}{r^2} \left. \right] + \\ & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 \cos^2 2\theta \frac{\cos^2 \theta}{r^2} \right\}. \end{aligned} \tag{79}$$

4.3.2 Moving edge dislocation energy density

We next consider the general case of the moving edge dislocation in the spacetime continuum of Section 4.2, with cartesian coordinates (x, y, z) . We first evaluate the volume dilatation ε for the moving edge dislocation. The volume dilatation is given by

$$\varepsilon = \varepsilon^\alpha_\alpha = \varepsilon_{xx} + \varepsilon_{yy} \tag{80}$$

where the non-zero diagonal elements of the strain tensor are obtained from (46). Substituting for ε_{xx} and ε_{yy} from (46) in (80), we notice that the transverse terms cancel out, and we are left with the following longitudinal term:

$$\varepsilon = \frac{bc^2y}{\pi v^2} \frac{\gamma_l^3 - \gamma_l}{(x - vt)^2 + \gamma_l^2 y^2} \tag{81}$$

This equation can be further reduced to

$$\varepsilon = \frac{bc^2}{\pi v^2} \frac{v^2}{c_l^2} \frac{\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} \tag{82}$$

and finally, using $c^2/c_l^2 = \bar{\mu}_0/(2\bar{\mu}_0 + \bar{\lambda}_0)$ (see (9) and (44)),

$$\varepsilon(x_i, t) = \frac{b}{2\pi} \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2}. \tag{83}$$

As seen previously, the mass energy density is calculated from (65):

$$\rho c^2 = 4\bar{\kappa}_0 \varepsilon = 2(2\bar{\lambda}_0 + \bar{\mu}_0) \varepsilon. \tag{84}$$

Substituting for ε from (83), the mass energy density of an edge dislocation is given by

$$\rho(x_i, t) c^2 = \frac{b}{2\pi} \frac{8\bar{\kappa}_0 \bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2}. \tag{85}$$

Using (83) in (50), the edge dislocation longitudinal dilatation strain energy density is then given by

$$\mathcal{E}_\parallel = \frac{1}{2} \bar{\kappa}_0 \left(\frac{b}{2\pi} \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} \right)^2. \tag{86}$$

The distortion strain energy density is calculated from (51). The expression is expanded using the non-zero elements of the strain tensor (46) and (47) and, from (71) and (72), we obtain [1, see Eqs.(114–115)]

$$\begin{aligned} \mathcal{E}_\perp = & \bar{\mu}_0 \left[\left(\varepsilon_{xx} - \frac{1}{4} \varepsilon \right)^2 + \left(\varepsilon_{yy} - \frac{1}{4} \varepsilon \right)^2 \right. \\ & \left. - 2\varepsilon_{tx}^2 - 2\varepsilon_{ty}^2 + 2\varepsilon_{xy}^2 \right]. \end{aligned} \tag{87}$$

Expanding the quadratic terms and making use of (53) as in (74), (87) becomes

$$\mathcal{E}_\perp = \bar{\mu}_0 \left(\varepsilon_{xx}^2 + \varepsilon_{yy}^2 - \frac{3}{8} \varepsilon^2 - 2\varepsilon_{tx}^2 - 2\varepsilon_{ty}^2 + 2\varepsilon_{xy}^2 \right). \tag{88}$$

Substituting from (46), (47) and (82),

$$\begin{aligned} \mathcal{E}_\perp = & \bar{\mu}_0 \left(\frac{b}{2\pi} \frac{c^2}{v^2} \right)^2 \left\{ -\frac{3}{8} \left(2 \frac{v^2}{c_l^2} \frac{\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} \right)^2 + \right. \\ & + 4 \left(\frac{-\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} + \frac{\alpha^2 \gamma y}{(x - vt)^2 + \gamma^2 y^2} \right)^2 + \\ & + 4 \left(\frac{\gamma_l^3 y}{(x - vt)^2 + \gamma_l^2 y^2} - \frac{\alpha^2 \gamma y}{(x - vt)^2 + \gamma^2 y^2} \right)^2 - \\ & - 2 \frac{v^2}{c^2} \left(\frac{\gamma_l y}{(x - vt)^2 + \gamma_l^2 y^2} - \alpha^2 \frac{\gamma y}{(x - vt)^2 + \gamma^2 y^2} \right)^2 - \\ & - 2 \frac{v^2}{c^2} \left(\frac{-\gamma_l(x - vt)}{(x - vt)^2 + \gamma_l^2 y^2} + \frac{\alpha^2}{\gamma^2} \frac{\gamma(x - vt)}{(x - vt)^2 + \gamma^2 y^2} \right)^2 + \\ & \left. + 2 \left(\frac{2\gamma_l(x - vt)}{(x - vt)^2 + \gamma_l^2 y^2} - \frac{\alpha^2(\gamma + 1/\gamma)(x - vt)}{(x - vt)^2 + \gamma^2 y^2} \right)^2 \right\} \end{aligned} \tag{89}$$

which simplifies to

$$\begin{aligned} \mathcal{E}_\perp = & \bar{\mu}_0 \frac{b^2}{2\pi^2} \frac{c^4}{v^4} \left\{ \frac{\alpha^4 (3 + \gamma^2)}{(x - vt)^2 + \gamma^2 y^2} - \right. \\ & - 2\alpha^2 \frac{\left(3 + \frac{1}{\gamma^2} \right) \gamma_l \gamma (x - vt)^2 + \left(2\gamma_l^2 - \frac{v^2}{c^2} \right) \gamma_l \gamma y^2}{\left((x - vt)^2 + \gamma_l^2 y^2 \right) \left((x - vt)^2 + \gamma^2 y^2 \right)} + \\ & \left. + \frac{\left(3 + \gamma^2 \right) \gamma_l^2 (x - vt)^2 + 2 \left(\alpha^2 + \gamma_l^4 - \frac{3}{8} \frac{v^4}{c_l^4} \right) \gamma_l^2 y^2}{\left((x - vt)^2 + \gamma_l^2 y^2 \right)^2} \right\}. \end{aligned} \tag{90}$$

We consider the above equations for the moving edge dislocation in the limit as $v \rightarrow 0$. Then the terms

$$\frac{\gamma y}{(x - vt)^2 + \gamma^2 y^2} \rightarrow \frac{\sin \theta}{r} \tag{91}$$

and

$$\frac{x - vt}{(x - vt)^2 + \gamma^2 y^2} \rightarrow \frac{\cos \theta}{r} \tag{92}$$

in cylindrical polar coordinates. Similarly for the same terms with γ_l instead of γ .

The volume dilatation obtained from (83) is then given in cylindrical polar coordinates (r, θ, z) by

$$\varepsilon \rightarrow \frac{b}{2\pi} \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\sin \theta}{r}. \tag{93}$$

The mass energy density is obtained from (85) to give

$$\rho c^2 \rightarrow \frac{b}{2\pi} \frac{8\bar{\kappa}_0\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\sin \theta}{r}. \tag{94}$$

From (86), the edge dislocation dilatation strain energy density is then given by

$$\mathcal{E}_{\parallel} \rightarrow \frac{b^2}{2\pi^2} \frac{\bar{\kappa}_0\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \frac{\sin^2 \theta}{r^2}. \tag{95}$$

These equations are in agreement with (55), (67) and (69) respectively.

The edge dislocation distortion strain energy density in the limit as $v \rightarrow 0$ is obtained from (89) by making use of (91) and (92) as follows:

$$\begin{aligned} \mathcal{E}_{\perp} \rightarrow & \bar{\mu}_0 \frac{b^2 c^4}{4\pi^2 v^4} \left\{ -\frac{3}{2} \frac{v^4}{c_l^4} \frac{\sin^2 \theta}{r^2} + \right. \\ & + 4 \left(-\frac{\sin \theta}{r} + \alpha^2 \frac{\sin \theta}{r} \right)^2 + 4 \left(\gamma_l^2 \frac{\sin \theta}{r} - \alpha^2 \frac{\sin \theta}{r} \right)^2 - \\ & - 2 \frac{v^2}{c^2} \left(\frac{\sin \theta}{r} - \alpha^2 \frac{\sin \theta}{r} \right)^2 - \\ & - 2 \frac{v^2}{c^2} \left(-\gamma_l \frac{\cos \theta}{r} + \frac{\alpha^2}{\gamma} \frac{\cos \theta}{r} \right)^2 + \\ & \left. + 2 \left(2\gamma_l \frac{\cos \theta}{r} - \alpha^2 \left(\gamma + \frac{1}{\gamma} \right) \frac{\cos \theta}{r} \right)^2 \right\}. \end{aligned} \tag{96}$$

Simplifying,

$$\begin{aligned} \mathcal{E}_{\perp} \rightarrow & \bar{\mu}_0 \frac{b^2 c^4}{4\pi^2 v^4} \left\{ -\frac{3}{2} \frac{v^4}{c_l^4} \frac{\sin^2 \theta}{r^2} + \right. \\ & + 4(-1 + \alpha^2)^2 \frac{\sin^2 \theta}{r^2} + 4(\gamma_l^2 - \alpha^2)^2 \frac{\sin^2 \theta}{r^2} - \\ & - 2 \frac{v^2}{c^2} (1 - \alpha^2)^2 \frac{\sin^2 \theta}{r^2} - \\ & - 2 \frac{v^2}{c^2} \left(-\gamma_l + \frac{\alpha^2}{\gamma} \right)^2 \frac{\cos^2 \theta}{r^2} + \\ & \left. + 2 \left(2\gamma_l - \alpha^2 \left(\gamma + \frac{1}{\gamma} \right) \right)^2 \frac{\cos^2 \theta}{r^2} \right\}. \end{aligned} \tag{97}$$

Using the definitions of γ^2 , γ_l^2 and α^2 from (27), (42) and (43) respectively, using the first term of the Taylor expansion for

γ and γ_l as $v \rightarrow 0$, and neglecting the terms multiplied by $-2v^2/c^2$ in (97) as they are of order v^6/c^6 , (97) becomes

$$\begin{aligned} \mathcal{E}_{\perp} \rightarrow & \bar{\mu}_0 \frac{b^2 c^4}{4\pi^2 v^4} \left\{ \left[-\frac{3}{2} \frac{v^4}{c_l^4} + \frac{v^4}{c^4} + \right. \right. \\ & \left. \left. + 4 \left(1 - \frac{v^2}{c_l^2} - 1 + \frac{v^2}{2c^2} \right)^2 \right] \frac{\sin^2 \theta}{r^2} + \right. \\ & \left. + 4 \left[1 - \frac{1}{2} \frac{v^2}{c_l^2} - 1 + \frac{v^2}{2c^2} \right]^2 \frac{\cos^2 \theta}{r^2} \right\}. \end{aligned} \tag{98}$$

Squaring and simplifying, we obtain

$$\begin{aligned} \mathcal{E}_{\perp} \rightarrow & \bar{\mu}_0 \frac{b^2 c^4}{4\pi^2 v^4} \left\{ \left(\frac{5}{2} \frac{v^4}{c_l^4} + 2 \frac{v^4}{c^4} + 4 \frac{v^4}{c_l^2 c^2} \right) \frac{\sin^2 \theta}{r^2} + \right. \\ & \left. + \left(\frac{v^4}{c_l^4} + \frac{v^4}{c^4} - 2 \frac{v^4}{c_l^2 c^2} \right) \frac{\cos^2 \theta}{r^2} \right\} \end{aligned} \tag{99}$$

and further

$$\begin{aligned} \mathcal{E}_{\perp} \rightarrow & \bar{\mu}_0 \frac{b^2}{2\pi^2} \left\{ \left(1 + 2 \frac{c^2}{c_l^2} + \frac{5}{4} \frac{c^4}{c_l^4} \right) \frac{\sin^2 \theta}{r^2} + \right. \\ & \left. + \frac{1}{2} \left(1 - 2 \frac{c^2}{c_l^2} + \frac{c^4}{c_l^4} \right) \frac{\cos^2 \theta}{r^2} \right\}. \end{aligned} \tag{100}$$

Using $c^2/c_l^2 = \bar{\mu}_0/(2\bar{\mu}_0 + \bar{\lambda}_0)$ (see (9) and (44)), (100) becomes

$$\begin{aligned} \mathcal{E}_{\perp} \rightarrow & \bar{\mu}_0 \frac{b^2}{2\pi^2} \left\{ \left(1 + \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \right. \right. \\ & \left. \left. + \frac{5}{4} \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \frac{\sin^2 \theta}{r^2} + \frac{1}{2} \left(1 - \right. \right. \\ & \left. \left. - \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \frac{\cos^2 \theta}{r^2} \right\}. \end{aligned} \tag{101}$$

This equation represents the impact of the time terms included in the calculation of (87) and the limit operation $v \rightarrow 0$ used in (89).

5 Curved dislocations

In this section, we consider the equations for generally curved dislocations generated by infinitesimal elements of a dislocation. These allow us to handle complex dislocations that are encountered in the spacetime continuum.

5.1 The Burgers displacement equation

The Burgers displacement equation for an infinitesimal element of a dislocation $d\mathbf{l} = \boldsymbol{\xi} dl$ in vector notation is given by [14, see p. 102]

$$\begin{aligned} \mathbf{u}(\mathbf{r}) = & \frac{\mathbf{b}}{4\pi} \int_A \frac{\hat{\mathbf{R}} \cdot d\mathbf{A}}{R^2} - \frac{1}{4\pi} \oint_C \frac{\mathbf{b} \times d\mathbf{l}'}{R} + \\ & + \frac{1}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \nabla \left[\oint_C \frac{(\mathbf{b} \times \mathbf{R}) \cdot d\mathbf{l}'}{R} \right] \end{aligned} \tag{102}$$

where \mathbf{u} is the displacement vector, \mathbf{r} is the vector to the displaced point, \mathbf{r}' is the vector to the dislocation infinitesimal element $d\mathbf{l}'$, $\mathbf{R} = \mathbf{r}' - \mathbf{r}$, \mathbf{b} is the Burgers vector, and closed loop C bounds the area A .

In tensor notation, (102) is given by

$$\begin{aligned} u_\mu(r^\nu) = & -\frac{1}{8\pi} \int_A b_\mu \frac{\partial}{\partial x'^\alpha} (\nabla'^2 R) dA^\alpha - \\ & -\frac{1}{8\pi} \oint_C b^\beta \epsilon_{\mu\beta\gamma} \nabla'^2 R dx'^\gamma - \\ & -\frac{1}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \oint_C b_\beta \epsilon^{\beta\alpha\gamma} \frac{\partial^2 R}{\partial x'^\mu \partial x'^\alpha} dx'_\gamma \end{aligned} \quad (103)$$

where $\epsilon^{\alpha\beta\gamma}$ is the permutation symbol, equal to 1 for cyclic permutations, -1 for anti-cyclic permutations, and 0 for permutations involving repeated indices. As noted by Hirth [14, see p. 103], the first term of this equation gives a discontinuity $\Delta \mathbf{u} = \mathbf{b}$ over the surface A , while the two other terms are continuous except at the dislocation line. This equation is used to calculate the displacement produced at a point \mathbf{r} by an arbitrary curved dislocation by integration over the dislocation line.

5.2 The Peach and Koehler stress equation

The Peach and Koehler stress equation for an infinitesimal element of a dislocation is derived by differentiation of (103) and substitution of the result in (20) [14, see p. 103–106]. In this equation, the dislocation is defined continuous except at the dislocation core, removing the discontinuity over the surface A and allowing to express the stresses in terms of line integrals alone.

$$\begin{aligned} \sigma_{\mu\nu} = & -\frac{\bar{\mu}_0}{8\pi} \oint_C b^\alpha \epsilon_{\beta\alpha\mu} \frac{\partial}{\partial x'^\beta} (\nabla'^2 R) dx'_\nu - \\ & -\frac{\bar{\mu}_0}{8\pi} \oint_C b^\alpha \epsilon_{\beta\alpha\nu} \frac{\partial}{\partial x'^\beta} (\nabla'^2 R) dx'_\mu - \\ & -\frac{\bar{\mu}_0}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \oint_C b_\alpha \epsilon^{\beta\alpha\gamma} \\ & \left(\frac{\partial^3 R}{\partial x'^\beta \partial x'^\mu \partial x'^\nu} - \delta_{\mu\nu} \frac{\partial}{\partial x'^\beta} (\nabla'^2 R) \right) dx'_\gamma. \end{aligned} \quad (104)$$

This equation is used to calculate the stress field of an arbitrary curved dislocation by line integration.

6 Framework for quantum physics

In a solid, dislocations represent the fundamental displacement processes that occur in its atomic structure. A solid viewed in electron microscopy or other microscopic imaging techniques is a tangle of screw and edge dislocations [10, see p. 35 and accompanying pages]. Similarly, dislocations in the spacetime continuum are taken to represent the fundamental displacement processes that occur in its structure. These fundamental displacement processes should thus correspond to

basic quantum phenomena and provide a framework for the description of quantum physics in *STCED*.

We find that dislocations have fundamental properties that reflect those of particles at the quantum level. These include self-energy and interactions mediated by the strain energy density of the dislocations. The role played by virtual particles in Quantum Electrodynamics is replaced by the interaction of the energy density of the dislocations. This theory is not perturbative as in QED, but rather calculated from analytical expressions. The analytical equations can become very complicated, and in some cases, perturbative techniques are used to simplify the calculations, but the availability of analytical expressions permit a better understanding of the fundamental processes involved.

Although the existence of virtual particles in QED is generally accepted, there are physicists who still question this interpretation of QED perturbation expansions. Weingard [41] “argues that if certain elements of the orthodox interpretation of states in QM are applicable to QED, then it must be concluded that virtual particles cannot exist. This follows from the fact that the transition amplitudes correspond to superpositions in which virtual particle type and number are not sharp. Weingard argues further that analysis of the role of measurement in resolving the superposition strengthens this conclusion. He then demonstrates in detail how in the path integral formulation of field theory no creation and annihilation operators need appear, yet virtual particles are still present. This analysis shows that the question of the existence of virtual particles is really the question of how to interpret the propagators which appear in the perturbation expansion of vacuum expectation values (scattering amplitudes).” [42]

The basic Feynman diagrams can be seen to represent screw dislocations as photons, edge dislocations as particles, and their interactions. The exchange of virtual particles in interactions can be taken as the forces resulting from the overlap of the dislocations’ strain energy density, with suitably modified diagrams. The perturbative expansions are also replaced by finite analytical expressions.

6.1 Quantization

The Burgers vector as defined by expression (5) has similarities to the Bohr-Sommerfeld quantization rule

$$\oint_C p dq = nh \quad (105)$$

where q is the position canonical coordinate, p is the momentum canonical coordinate and h is Planck’s constant. This leads us to consider the following quantization rule for the *STC*: at the quantum level, we assume that the spacetime continuum has a granularity characterized by a length b_0 corresponding to the smallest elementary Burgers dislocation-displacement vector possible in the *STC*. The idea that the existence of a shortest length in nature would lead to a natural cut-off to generate finite integrals in QED has been raised

before [43]. The smallest elementary Burgers dislocation-displacement vector introduced here provides a lower bound as shown in the next section. Then the magnitude of a Burgers vector can be expressed as a multiple of the elementary Burgers vector:

$$b = nb_0. \tag{106}$$

We find that b is usually divided by 2π in dislocation equations, and hence we define

$$\tilde{b} = \frac{b}{2\pi}, \tag{107}$$

and similarly for the elementary Burgers dislocation-displacement vector b_0 ,

$$\tilde{b}_0 = \frac{b_0}{2\pi}. \tag{108}$$

6.2 Screw dislocations in quantum physics

Screw dislocations in the spacetime continuum are identified with massless, transverse deformations, specifically photons. Consider the displacement of a stationary screw dislocation as derived in Section 3.1:

$$u_z = \frac{b}{2\pi} \theta = \tilde{b} \theta. \tag{109}$$

Taking the derivative with respect to time, we obtain

$$\dot{u}_z = v_z = \frac{b}{2\pi} \dot{\theta} = \frac{b}{2\pi} \omega. \tag{110}$$

The speed of the transverse displacement is c , the speed of light. Substituting for $\omega = 2\pi\nu$, (110) becomes

$$c = b\nu. \tag{111}$$

Hence

$$b = \lambda, \tag{112}$$

the wavelength of the screw dislocation. This result is illustrated in Fig. 5. It is important to note that this relation applies only to screw dislocations.

The strain energy density of the screw dislocation is given by the transverse distortion energy density derived in Section 3.3. For a stationary screw dislocation, substituting (107) into (35),

$$\mathcal{E}_\perp = \frac{\bar{\mu}_0 \tilde{b}^2}{2} \frac{1}{r^2}. \tag{113}$$

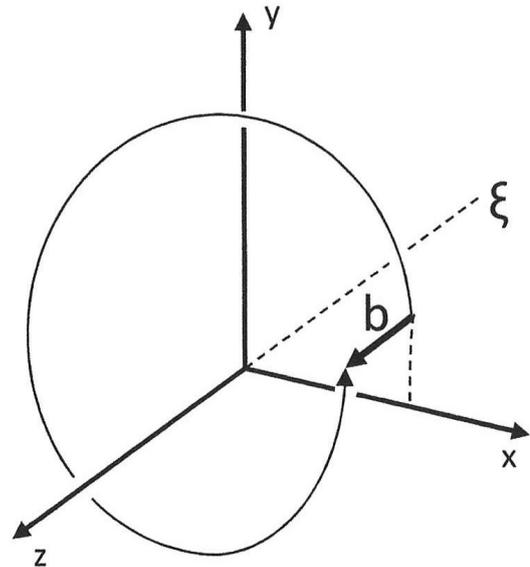
The total strain energy of the screw dislocation is then given by

$$W_\perp = \int_V \mathcal{E}_\perp dV \tag{114}$$

where the volume element dV in cylindrical polar coordinates is given by $rdr d\theta dz$. Substituting for \mathcal{E}_\perp from (113), (114) becomes

$$W_\perp = \int_V \frac{\bar{\mu}_0 \tilde{b}^2}{2r^2} r dr d\theta dz. \tag{115}$$

Fig. 5: A wavelength of a screw dislocation.



From (106), \tilde{b} can be taken out of the integral to give

$$W_\perp = \frac{\bar{\mu}_0 \tilde{b}^2}{2} \int_b^\Lambda \frac{1}{r} dr \int_\theta d\theta \int_z dz \tag{116}$$

where Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

The strain energy per wavelength is then given by

$$\frac{W_\perp}{\lambda} = \frac{\bar{\mu}_0 \tilde{b}^2}{2} \log \frac{\Lambda}{b} \int_0^{2\pi} d\theta \tag{117}$$

and finally

$$\frac{W_\perp}{\lambda} = \frac{\bar{\mu}_0 b^2}{4\pi} \log \frac{\Lambda}{b}. \tag{118}$$

The implications of the total strain energy of the screw dislocation are discussed further in comparison to Quantum Electrodynamics (QED) in Section 7.

6.3 Edge dislocations in quantum physics

The strain energy density of the edge dislocation is derived in Section 4.3. The dilatation (massive) strain energy density of the edge dislocation is given by the longitudinal strain energy density (50) and the distortion (massless) strain energy density of the edge dislocation is given by the transverse strain energy density (51).

For the stationary edge dislocation of (79), using (107)

into (79), we have

$$\begin{aligned} \mathcal{E}_\perp = & \frac{2\bar{b}^2\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \left\{ \frac{5}{4}\bar{\mu}_0^2 \frac{\sin^2 \theta}{r^2} - \right. \\ & - \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0) \cos^4 \theta \frac{\sin^2 \theta}{r^2} - \right. \\ & - 2\bar{\mu}_0^2 \cos^2 \theta \frac{\sin^4 \theta}{r^2} - \bar{\mu}_0^2 \frac{\sin^6 \theta}{r^2} \left. \right] + \\ & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 \cos^2 2\theta \frac{\cos^2 \theta}{r^2} \right\}. \end{aligned} \quad (119)$$

The distortion strain energy of the edge dislocation is then given by

$$W_\perp = \int_V \mathcal{E}_\perp dV \quad (120)$$

where the volume element dV in cylindrical polar coordinates is given by $rdr d\theta dz$. Substituting for \mathcal{E}_\perp from (119) and taking \bar{b} out of the integral, (120) becomes

$$\begin{aligned} W_\perp = & \frac{2\bar{b}^2\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \int_z \int_\theta \int_{b_0}^\Lambda \left\{ \frac{5}{4}\bar{\mu}_0^2 \frac{\sin^2 \theta}{r^2} - \right. \\ & - \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0) \cos^4 \theta \frac{\sin^2 \theta}{r^2} - \right. \\ & - 2\bar{\mu}_0^2 \cos^2 \theta \frac{\sin^4 \theta}{r^2} - \bar{\mu}_0^2 \frac{\sin^6 \theta}{r^2} \left. \right] + \\ & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 \cos^2 2\theta \frac{\cos^2 \theta}{r^2} \right\} r dr d\theta dz \end{aligned} \quad (121)$$

where again Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

Evaluating the integral over r ,

$$\begin{aligned} W_\perp = & \frac{2\bar{b}^2\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \log \frac{\Lambda}{b_0} \int_z \int_0^{2\pi} \left\{ \frac{5}{4}\bar{\mu}_0^2 \sin^2 \theta - \right. \\ & - \left[(3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0) \cos^4 \theta \sin^2 \theta - \right. \\ & - 2\bar{\mu}_0^2 \cos^2 \theta \sin^4 \theta - \bar{\mu}_0^2 \sin^6 \theta \left. \right] + \\ & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 \cos^2 2\theta \cos^2 \theta \right\} d\theta dz. \end{aligned} \quad (122)$$

Evaluating the integral over θ [40], we obtain (123) at the top of the next page. Applying the limits of the integration, both the coefficients of $\bar{\lambda}_0^2$ and $\bar{\mu}_0\bar{\lambda}_0$ are equal to 0 and only the coefficient of $\bar{\mu}_0^2$ is non-zero. Equation (123) then becomes

$$W_\perp = \frac{2\bar{b}^2\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \log \frac{\Lambda}{b_0} \int_0^\ell \frac{9\pi}{4} \bar{\mu}_0^2 dz. \quad (124)$$

where ℓ is the length of the edge dislocation.

Evaluating the integral over z , we obtain the stationary edge dislocation transverse strain energy per unit length

$$\frac{W_\perp}{\ell} = \frac{9\pi}{2} \bar{b}^2 \bar{\mu}_0 \left(\frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \right)^2 \log \frac{\Lambda}{b_0}. \quad (125)$$

We find that the stationary edge dislocation transverse strain energy per unit length (where we have added the label E)

$$\frac{W_\perp^E}{\ell} = \frac{9}{8\pi} \bar{b}^2 \bar{\mu}_0 \left(\frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \right)^2 \log \frac{\Lambda}{b_0} \quad (126)$$

is similar to the stationary screw dislocation transverse strain energy per unit length

$$\frac{W_\perp^S}{\ell} = \frac{1}{4\pi} \bar{b}^2 \bar{\mu}_0 \log \frac{\Lambda}{b_0} \quad (127)$$

except for the proportionality constant.

Similarly, the longitudinal strain energy of the stationary edge dislocation is given by

$$W_\parallel^E = \int_V \mathcal{E}_\parallel dV. \quad (128)$$

Substituting for \mathcal{E}_\parallel from (69), this equation becomes

$$W_\parallel^E = \int_V \frac{b^2}{2\pi^2} \frac{\bar{\kappa}_0 \bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \frac{\sin^2 \theta}{r^2} dV. \quad (129)$$

Similarly to the previous derivation, this integral gives

$$\frac{W_\parallel^E}{\ell} = \frac{1}{2\pi} \bar{b}^2 \bar{\kappa}_0 \left(\frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \right)^2 \log \frac{\Lambda}{b_0}. \quad (130)$$

The total strain energy of the stationary screw and edge dislocations have similar functional forms, with the difference residing in the proportionality constants. This is due to the simpler nature of the stationary dislocations and their cylindrical polar symmetry. This similarity is not present for the general case of moving dislocations as evidenced in equations (37), (86) and (90).

For the moving edge dislocation in the limit as $v \rightarrow 0$, substituting for (101) in (120) and using (107), we have

$$\begin{aligned} W_\perp^E \rightarrow & 2\bar{b}^2\bar{\mu}_0 \int_z \int_\theta \int_{b_0}^\Lambda r dr d\theta dz \\ & \left\{ \left(1 + \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{5}{4} \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \frac{\sin^2 \theta}{r^2} + \right. \\ & \left. + \frac{1}{2} \left(1 - \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \frac{\cos^2 \theta}{r^2} \right\} \end{aligned} \quad (131)$$

where again Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

$$\begin{aligned}
 W_{\perp} = & \frac{2\bar{b}^2\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \log \frac{\Lambda}{b_0} \int_z \left[\frac{5}{4} \bar{\mu}_0^2 \left(\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right) - \right. \\
 & - (3\bar{\mu}_0 + 2\bar{\lambda}_0)(\bar{\mu}_0 + 2\bar{\lambda}_0) \left(\frac{\theta}{16} + \frac{1}{64} \sin 2\theta - \frac{1}{64} \sin 4\theta - \frac{1}{192} \sin 6\theta \right) + \\
 & + 2\bar{\mu}_0^2 \left(\frac{\theta}{16} - \frac{1}{64} \sin 2\theta - \frac{1}{64} \sin 4\theta + \frac{1}{192} \sin 6\theta \right) + \\
 & + \bar{\mu}_0^2 \left(\frac{5\theta}{16} - \frac{15}{64} \sin 2\theta + \frac{3}{64} \sin 4\theta - \frac{1}{192} \sin 6\theta \right) + \\
 & \left. + (\bar{\mu}_0 + \bar{\lambda}_0)^2 \left(\frac{\theta}{4} + \frac{3}{16} \sin 2\theta + \frac{1}{16} \sin 4\theta + \frac{1}{48} \sin 6\theta \right) \right]_0^{2\pi} dz
 \end{aligned} \tag{123}$$

Evaluating the integral over r ,

$$\begin{aligned}
 W_{\perp}^E \rightarrow & 2\bar{b}^2\bar{\mu}_0 \log \frac{\Lambda}{b_0} \int_z \int_0^{2\pi} d\theta dz \\
 & \left\{ \left(1 + \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{5}{4} \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \sin^2 \theta + \right. \\
 & \left. + \frac{1}{2} \left(1 - \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \cos^2 \theta \right\}.
 \end{aligned} \tag{132}$$

Evaluating the integral over θ [40] and applying the limits of the integration, we obtain

$$\begin{aligned}
 W_{\perp}^E \rightarrow & 2\bar{b}^2\bar{\mu}_0 \log \frac{\Lambda}{b_0} \int_0^{\ell} dz \\
 & \left\{ \left(1 + \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{5}{4} \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) (\pi) + \right. \\
 & \left. + \frac{1}{2} \left(1 - \frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) (\pi) \right\}
 \end{aligned} \tag{133}$$

and evaluating the integral over z , we obtain the moving edge dislocation transverse strain energy per unit length in the limit as $v \rightarrow 0$

$$\begin{aligned}
 \frac{W_{\perp}^E}{\ell} \rightarrow & \frac{3}{4\pi} \bar{b}^2\bar{\mu}_0 \left(1 + \frac{2}{3} \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} + \right. \\
 & \left. + \frac{7}{6} \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \right) \log \frac{\Lambda}{b_0}
 \end{aligned} \tag{134}$$

where ℓ is the length of the edge dislocation.

6.4 Strain energy of moving dislocations

In the general case of moving dislocations, the derivation of the screw dislocation transverse strain energy and the edge dislocation transverse and longitudinal strain energies is more difficult. In this section, we provide an overview discussion of the topic.

6.4.1 Screw dislocation transverse strain energy

The transverse strain energy of a moving screw dislocation, which also corresponds to its total strain energy, is given by

$$W_{\perp}^S = \int_V \mathcal{E}_{\perp}^S dV \tag{135}$$

where the strain energy density \mathcal{E}_{\perp}^S is given by (113), viz.

$$\mathcal{E}_{\perp}^S = \frac{1}{2} \bar{b}^2 \bar{\mu}_0 \frac{\gamma^2}{(x - vt)^2 + \gamma^2 y^2} \tag{136}$$

and V is the 4-dimensional volume of the screw dislocation. The volume element dV in cartesian coordinates is given by $dx dy dz d(ct)$.

Substituting for \mathcal{E}_{\perp}^S , (135) becomes

$$W_{\perp}^S = \int_V \frac{1}{2} \bar{b}^2 \bar{\mu}_0 \frac{\gamma^2}{(x - vt)^2 + \gamma^2 y^2} dx dy dz d(ct). \tag{137}$$

As before, \bar{b} is taken out of the integral from (106), and the integral over z is handled by considering the strain energy per unit length of the dislocation:

$$\frac{W_{\perp}^S}{\ell} = \frac{\bar{b}^2 \bar{\mu}_0}{2} \int_{ct} \int_y \int_x \frac{\gamma^2}{(x - vt)^2 + \gamma^2 y^2} dx dy d(ct) \tag{138}$$

where ℓ is the length of the dislocation and as before, Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

Evaluating the integral over x [40],

$$\begin{aligned}
 \frac{W_{\perp}^S}{\ell} = & \frac{\bar{b}^2 \bar{\mu}_0}{2} \gamma^2 \int_{ct} \int_y dy d(ct) \\
 & \left[\frac{1}{\gamma y} \arctan \left(\frac{x - vt}{\gamma y} \right) \right]_{\sqrt{y^2 - b^2}}^{\sqrt{\Lambda^2 - y^2}}
 \end{aligned} \tag{139}$$

where the limits corresponding to the maximum cut-off parameter Λ and minimum cut-off parameter b are stated explicitly. Applying the limits of the integration, we obtain

$$\frac{W_{\perp}^S}{\ell} = \frac{\bar{b}^2 \bar{\mu}_0}{2} \gamma^2 \int_{ct} \int_y dy d(ct) \left\{ \frac{1}{\gamma y} \arctan \left(\frac{\sqrt{\Lambda^2 - y^2} - vt}{\gamma y} \right) - \frac{1}{\gamma y} \arctan \left(\frac{\sqrt{y^2 - b^2} - vt}{\gamma y} \right) \right\}. \tag{140}$$

This integration over y is not elementary and likely does not lead to a closed analytical form. If we consider the following simpler integral, the solution is given by

$$\int_y \frac{1}{\gamma y} \arctan \left(\frac{x - vt}{\gamma y} \right) dy = -\frac{i}{2} \left[\text{Li}_2 \left(-i \frac{x - vt}{\gamma y} \right) - \text{Li}_2 \left(i \frac{x - vt}{\gamma y} \right) \right] \tag{141}$$

where $\text{Li}_n(x)$ is the polylogarithm function. As pointed out in [44], “[t]he polylogarithm arises in Feynman diagram integrals (and, in particular, in the computation of quantum electrodynamics corrections to the electrons gyromagnetic ratio), and the special cases $n = 2$ and $n = 3$ are called the dilogarithm and the trilogarithm, respectively.” This is a further indication that the interaction of strain energies are the physical source of quantum interaction phenomena described by Feynman diagrams as will be seen in Section 7.

6.4.2 Edge dislocation longitudinal strain energy

The longitudinal strain energy of a moving edge dislocation is given by

$$W_{\parallel}^E = \int_V \mathcal{E}_{\parallel}^E dV \tag{142}$$

where the strain energy density \mathcal{E}_{\perp}^E is given by (86), viz.

$$\mathcal{E}_{\parallel}^E = \frac{1}{2} \bar{\kappa}_0 \bar{b}^2 \left(\frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\gamma_1 y}{(x - vt)^2 + \gamma_1^2 y^2} \right)^2 \tag{143}$$

and V is the 4-dimensional volume of the edge dislocation. The volume element dV in cartesian coordinates is given by $dx dy dz d(ct)$.

Substituting for $\mathcal{E}_{\parallel}^E$, (142) becomes

$$W_{\parallel}^E = \int_V \frac{1}{2} \bar{\kappa}_0 \bar{b}^2 \left(\frac{2\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{\gamma_1 y}{(x - vt)^2 + \gamma_1^2 y^2} \right)^2 dx dy dz d(ct). \tag{144}$$

As before, \bar{b} is taken out of the integral from (106), and the integral over z is handled by considering the strain energy per

unit length of the dislocation:

$$\frac{W_{\parallel}^E}{\ell} = 2 \bar{\kappa}_0 \bar{b}^2 \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \int_{ct} \int_y \int_x \frac{(\gamma_1 y)^2}{((x - vt)^2 + \gamma_1^2 y^2)^2} dx dy d(ct) \tag{145}$$

where ℓ is the length of the dislocation and as before, Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

The integrand has a functional form similar to that of (138), and a similar solution behaviour is expected. Evaluating the integral over x [40],

$$\frac{W_{\parallel}^E}{\ell} = 2 \bar{\kappa}_0 \bar{b}^2 \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \int_{ct} \int_y dy d(ct) \left[\frac{1}{2} \frac{x - vt}{(x - vt)^2 + (\gamma_1 y)^2} + \frac{1}{2\gamma_1 y} \arctan \left(\frac{x - vt}{\gamma_1 y} \right) \right] \frac{\sqrt{\Lambda^2 - y^2}}{\sqrt{y^2 - b^2}} \tag{146}$$

where the limits corresponding to the maximum cut-off parameter Λ and minimum cut-off parameter b are stated explicitly. Applying the limits of the integration, we obtain

$$\frac{W_{\parallel}^E}{\ell} = 2 \bar{\kappa}_0 \bar{b}^2 \frac{\bar{\mu}_0^2}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \int_{ct} \int_y dy d(ct) \left\{ \frac{1}{2} \frac{\sqrt{\Lambda^2 - y^2} - vt}{(\sqrt{\Lambda^2 - y^2} - vt)^2 + (\gamma_1 y)^2} - \frac{1}{2} \frac{\sqrt{y^2 - b^2} - vt}{(\sqrt{y^2 - b^2} - vt)^2 + (\gamma_1 y)^2} + \frac{1}{2\gamma_1 y} \arctan \left(\frac{\sqrt{\Lambda^2 - y^2} - vt}{\gamma_1 y} \right) - \frac{1}{2\gamma_1 y} \arctan \left(\frac{\sqrt{y^2 - b^2} - vt}{\gamma_1 y} \right) \right\}. \tag{147}$$

This integration over y is again found to be intractable, including that of (140), and likely does not lead to a closed analytical form. In the arctan Λ integral of (140) and (147), we can make the approximation $\sqrt{\Lambda^2 - y^2} \simeq \Lambda$ and evaluate this term as seen in (141):

$$\int_y \frac{1}{\gamma_1 y} \arctan \left(\frac{\Lambda - vt}{\gamma_1 y} \right) dy = -\frac{i}{2} \left[\text{Li}_2 \left(-i \frac{\Lambda - vt}{\gamma_1 y} \right) - \text{Li}_2 \left(i \frac{\Lambda - vt}{\gamma_1 y} \right) \right] \tag{148}$$

where $\text{Li}_n(x)$ is the polylogarithm function as seen previously.

6.4.3 Edge dislocation transverse strain energy

The transverse strain energy of a moving edge dislocation is given by

$$W_{\perp}^E = \int_V \mathcal{E}_{\perp}^E dV \quad (149)$$

where the strain energy density \mathcal{E}_{\perp}^E is given by (90), viz.

$$\begin{aligned} \mathcal{E}_{\perp}^E = 2\bar{\mu}_0 \bar{b}^2 \frac{c^4}{v^4} & \left\{ \frac{\alpha^4 (3 + \gamma^2)}{(x - vt)^2 + \gamma^2 y^2} - \right. \\ & - 2\alpha^2 \frac{\left(3 + \frac{1}{\gamma^2}\right) \gamma_l \gamma (x - vt)^2 + \left(2\gamma_l^2 - \frac{v^2}{c^2}\right) \gamma_l \gamma y^2}{\left((x - vt)^2 + \gamma_l^2 y^2\right) \left((x - vt)^2 + \gamma^2 y^2\right)} + \\ & \left. + \frac{(3 + \gamma^2) \gamma_l^2 (x - vt)^2 + 2\left(\alpha^2 + \gamma_l^4 - \frac{3}{8} \frac{v^4}{c_l^4}\right) \gamma_l^2 y^2}{\left((x - vt)^2 + \gamma_l^2 y^2\right)^2} \right\} \quad (150) \end{aligned}$$

and V is the 4-dimensional volume of the edge dislocation. The volume element dV in cartesian coordinates is given by $dx dy dz d(ct)$.

Substituting for \mathcal{E}_{\perp}^E as before, taking \bar{b} out of the integral from (106), and handling the integral over z by considering the strain energy per unit length of the dislocation, (149) becomes

$$\begin{aligned} \frac{W_{\perp}^E}{\ell} = 2\bar{\mu}_0 \bar{b}^2 \frac{c^4}{v^4} & \int_{ct} \int_y \int_x dx dy d(ct) \\ & \left. \left\{ \frac{\alpha^4 (3 + \gamma^2)}{(x - vt)^2 + \gamma^2 y^2} - \right. \right. \\ & - 2\alpha^2 \frac{\left(3 + \frac{1}{\gamma^2}\right) \gamma_l \gamma (x - vt)^2 + \left(2\gamma_l^2 - \frac{v^2}{c^2}\right) \gamma_l \gamma y^2}{\left((x - vt)^2 + \gamma_l^2 y^2\right) \left((x - vt)^2 + \gamma^2 y^2\right)} + \\ & \left. \left. + \frac{(3 + \gamma^2) \gamma_l^2 (x - vt)^2 + 2\left(\alpha^2 + \gamma_l^4 - \frac{3}{8} \frac{v^4}{c_l^4}\right) \gamma_l^2 y^2}{\left((x - vt)^2 + \gamma_l^2 y^2\right)^2} \right\} \right. \quad (151) \end{aligned}$$

where ℓ is the length of the dislocation and as before, Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

Again, the integrand has functional forms similar to that of (138) and (145). A similar, but more complex, solution behaviour is expected, due to the additional complexity of (151).

7 Dislocation interactions in quantum physics

As mentioned in Section 6, the basic Feynman diagrams can be seen to represent screw dislocations as photons, edge dislocations as particles, and their interactions. More specifically, the external legs of Feynman diagrams that are on mass-shell representing real particles correspond to dislocations, while the virtual off mass-shell particles are replaced by the interaction of the strain energy densities. The exchange of virtual

particles in QED interactions can be taken as the perturbation expansion representation of the forces resulting from the overlap of the strain energy density of the dislocations. The Feynman diagram propagators are replaced by the dislocation strain energy density interaction expressions.

The properties of Burgers vectors and dislocations [14, see pp.25-26] have rules similar to those of Feynman diagrams, but not equivalent as virtual particles are replaced by dislocation strain energy density interactions. A Burgers vector is invariant along a dislocation line. Two Burgers circuits are equivalent if one can be deformed into the other without crossing dislocation lines. The resultant Burgers vector within equivalent Burgers circuits is the same.

Dislocation nodes are points where multiple dislocations meet. If all the dislocation vectors ξ_i are taken to be positive away from a node, then

$$\sum_{i=1}^N \xi_i = 0 \quad (152)$$

for the N dislocations meeting at the node. Burgers vectors are conserved at dislocation nodes.

In this section, we consider the interactions of dislocations which are seen to result from the force resulting from the overlap of their strain energy density in the *STC* [14, see p. 112].

7.1 Parallel dislocation interactions

From Hirth [14, see pp. 117-118], the energy of interaction per unit length between parallel dislocations (including screw and edge dislocation components) is given by

$$\begin{aligned} \frac{W_{12}}{\ell} = & -\frac{\bar{\mu}_0}{2\pi} (\mathbf{b}_1 \cdot \boldsymbol{\xi}) (\mathbf{b}_2 \cdot \boldsymbol{\xi}) \log \frac{R}{R_{\Lambda}} - \\ & - \frac{\bar{\mu}_0}{\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} (\mathbf{b}_1 \times \boldsymbol{\xi}) \cdot (\mathbf{b}_2 \times \boldsymbol{\xi}) \log \frac{R}{R_{\Lambda}} - \\ & - \frac{\bar{\mu}_0}{\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \frac{[(\mathbf{b}_1 \times \boldsymbol{\xi}) \cdot \mathbf{R}][(\mathbf{b}_2 \times \boldsymbol{\xi}) \cdot \mathbf{R}]}{R^2} \quad (153) \end{aligned}$$

where $\boldsymbol{\xi}$ is parallel to the z axis, $(\mathbf{b}_i \cdot \boldsymbol{\xi})$ are the screw components, $(\mathbf{b}_i \times \boldsymbol{\xi})$ are the edge components, R is the separation between the dislocations, and R_{Λ} is the distance from which the dislocations are brought together, resulting in the decrease in energy of the “system”.

The components of the interaction force per unit length between the parallel dislocations are obtained by differentiation:

$$\begin{aligned} \frac{F_R}{\ell} &= -\frac{\partial(W_{12}/\ell)}{\partial R} \\ \frac{F_{\theta}}{\ell} &= -\frac{1}{R} \frac{\partial(W_{12}/\ell)}{\partial \theta}. \quad (154) \end{aligned}$$

Substituting from (153), (154) becomes

$$\begin{aligned} \frac{F_R}{\ell} &= \frac{\bar{\mu}_0}{2\pi R} (\mathbf{b}_1 \cdot \boldsymbol{\xi}) (\mathbf{b}_2 \cdot \boldsymbol{\xi}) + \\ &+ \frac{\bar{\mu}_0}{\pi R} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} (\mathbf{b}_1 \times \boldsymbol{\xi}) \cdot (\mathbf{b}_2 \times \boldsymbol{\xi}) \\ \frac{F_\theta}{\ell} &= \frac{\bar{\mu}_0}{\pi R^3} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} [(\mathbf{b}_1 \cdot \mathbf{R}) [(\mathbf{b}_2 \times \mathbf{R}) \cdot \boldsymbol{\xi}] + \\ &+ (\mathbf{b}_2 \cdot \mathbf{R}) [(\mathbf{b}_1 \times \mathbf{R}) \cdot \boldsymbol{\xi}]]. \end{aligned} \quad (155)$$

7.2 Curved dislocation interactions

In this section, we extend the investigation of curved dislocations initiated in Section 5, to the interaction energy and interaction force between curved dislocations [14, see pp. 106-110]. The derivation considers the interaction between two dislocation loops, but has much more extensive applications, being extendable to the interaction energy between two arbitrarily positioned segments of dislocation lines.

If a dislocation loop 1 is brought in the vicinity of another dislocation loop 2, the stresses originating from loop 2 do work $-W_{12}$ on loop 1 where W_{12} is the interaction energy between the two dislocation loops. The work done on loop 1 represents a decrease in the strain energy of the total system. In that case, if W_{12} is negative, the energy of the system decreases and an attractive force exists between the loops [14, see p. 106].

The interaction energy between the two dislocation loops is given by [14, see p. 108]

$$\begin{aligned} W_{12} &= -\frac{\bar{\mu}_0}{2\pi} \oint_{C_1} \oint_{C_2} \frac{(\mathbf{b}_1 \times \mathbf{b}_2) \cdot (d\mathbf{l}_1 \times d\mathbf{l}_2)}{R} + \\ &+ \frac{\bar{\mu}_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{(\mathbf{b}_1 \cdot d\mathbf{l}_1) (\mathbf{b}_2 \cdot d\mathbf{l}_2)}{R} + \\ &+ \frac{\bar{\mu}_0}{2\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \oint_{C_1} \oint_{C_2} \frac{(\mathbf{b}_1 \times d\mathbf{l}_1) \cdot \mathbf{T} \cdot (\mathbf{b}_2 \times d\mathbf{l}_2)}{R} \end{aligned} \quad (156)$$

where \mathbf{T} is given by

$$T_{ij} = \frac{\partial^2 R}{\partial x_i \partial x_j}. \quad (157)$$

The force produced by an external stress acting on a dislocation loop is given by [14, see p. 109]

$$d\mathbf{F} = (\mathbf{b} \cdot \boldsymbol{\sigma}) \times d\mathbf{l} \quad (158)$$

where $\boldsymbol{\sigma}$ is the stress tensor in the medium, \mathbf{b} is the Burgers vector, and $d\mathbf{l}$ is the dislocation element. This equation can be used with (104) to determine the interaction force between dislocation segments.

As each element $d\mathbf{l}$ of a dislocation loop is acted upon by the forces caused by the stress of the other elements of the

dislocation loop, the work done against these corresponds to the self-energy of the dislocation loop. The self-energy of a dislocation loop can be calculated from (156) to give [14, see p. 110]

$$\begin{aligned} W_{self} &= \frac{\bar{\mu}_0}{8\pi} \oint_{C_1=C} \oint_{C_2=C} \frac{(\mathbf{b} \cdot d\mathbf{l}_1) (\mathbf{b} \cdot d\mathbf{l}_2)}{R} + \\ &+ \frac{\bar{\mu}_0}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \oint_{C_1=C} \oint_{C_2=C} \frac{(\mathbf{b} \times d\mathbf{l}_1) \cdot \mathbf{T} \cdot (\mathbf{b} \times d\mathbf{l}_2)}{R} \end{aligned} \quad (159)$$

where \mathbf{T} is as defined in (157).

More complicated expressions can be obtained for interactions between two non-parallel straight dislocations [14, see pp. 121-123] and between a straight segment of a dislocation and a differential element of another dislocation [14, see pp. 124-131]. This latter derivation can be used for more arbitrary dislocation interactions.

7.3 Physical application of dislocation interactions

In Quantum Electrodynamics, these correspond to particle-particle and particle-photon interactions, which are taken to be mediated by virtual particles. This is in keeping with the QED picture, but as shown above, particle-particle and particle-photon interactions physically result from the overlap of their strain energy density which results in an interaction force. Again, this improved understanding of the physical nature of dislocation interactions demonstrates that the interactions do not need to be represented by virtual particle exchange as discussed in Section 6.

This theory provides a straightforward physical explanation of particle-particle and particle-photon interactions that is not based on perturbation theory, but rather on a direct evaluation of the interactions.

7.4 Photons and screw dislocation interactions

Screw dislocations interact via the force resulting from the overlap of the strain energy density of the dislocations in the *STC* [14, see p. 112].

As seen in Section 6.2, screw dislocations in the spacetime continuum are identified with the massless, transverse deformations, photons. As pointed out in [45], it has been known since the 1960s that photons can interact with each other in atomic media much like massive particles do. A review of collective effects in photon-photon interactions is given in [46].

In QED, photon-photon interactions are known as photon-photon scattering, which is thought to be mediated by virtual particles. This is in keeping with the QED picture, but as shown in this work, photon-photon interactions physically result from the overlap of their strain energy density. This improved understanding of the physical nature of photon-photon interactions demonstrates that the interaction does not need to

be represented by virtual particle exchanges, in that the nature of the physical processes involved is now understood.

From (153), the energy of interaction per unit length between parallel screw dislocations (photons) is given by

$$\frac{W_{12}^{ss}}{\ell} = -\frac{\bar{\mu}_0}{2\pi} (\mathbf{b}_1 \cdot \boldsymbol{\xi}) (\mathbf{b}_2 \cdot \boldsymbol{\xi}) \log \frac{R}{R_\Lambda} \quad (160)$$

where $\boldsymbol{\xi}$ is parallel to the z axis, $(\mathbf{b}_i \cdot \boldsymbol{\xi})$ are the screw components, R is the separation between the dislocations, and R_Λ is the distance from which the dislocations are brought together, resulting in the reduction in the energy of the 2-photon “system”.

From (155), the components of the interaction force per unit length between the parallel screw dislocations are given by:

$$\frac{F_R^{ss}}{\ell} = \frac{\bar{\mu}_0}{2\pi R} (\mathbf{b}_1 \cdot \boldsymbol{\xi}) (\mathbf{b}_2 \cdot \boldsymbol{\xi}) \quad (161)$$

$$\frac{F_\theta^{ss}}{\ell} = 0.$$

The interaction force is radial in nature, independent of the angle θ , as expected.

8 Physical explanations of QED phenomena

As we have seen in previous sections, spacetime continuum dislocations have fundamental properties that reflect those of phenomena at the quantum level. In particular, the improved understanding of the physical nature of interactions mediated by the strain energy density of the dislocations. The role played by virtual particles in Quantum Electrodynamics is replaced by the work done by the forces resulting from the dislocation stresses, and the resulting interaction of the strain energy density of the dislocations. In this section, we examine the physical explanation of QED phenomena provided by this theory, including self-energy and mass renormalization.

8.1 Dislocation self-energy and QED self energies

Dislocation self energies are found to be similar in structure to Quantum Electrodynamics self energies. They are also divergent if integrated over all of spacetime, with the divergence being logarithmic in nature. However, contrary to QED, dislocation self energies are bounded by the density of dislocations present in the spacetime continuum, which results in an upperbound to the integral of half the average distance between dislocations. As mentioned by Hirth [14], this has little impact on the accuracy of the results due to the logarithmic dependence.

The dislocation self-energy is related to the dislocation self-force. The dislocation self-force arises from the force on an element in a dislocation caused by other segments of the *same* dislocation line. This process provides an explanation for the QED self-energies without the need to resort to

the emission/absorption of virtual particles. It can be understood, and is particular to, dislocation dynamics as dislocations are defects that extend in the spacetime continuum [14, see p. 131]. Self-energy of a straight-dislocation segment of length L is given by [14, see p. 161]:

$$W_{self} = \frac{\bar{\mu}_0}{4\pi} \left((\mathbf{b} \cdot \boldsymbol{\xi})^2 + \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} |(\mathbf{b} \times \boldsymbol{\xi})|^2 \right) L \left(\log \frac{L}{b} - 1 \right), \quad (162)$$

where there is no interaction between two elements of the segment when they are within $\pm b$, or equivalently

$$W_{self} = \frac{\bar{\mu}_0}{4\pi} \left((\mathbf{b} \cdot \boldsymbol{\xi})^2 + \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} |(\mathbf{b} \times \boldsymbol{\xi})|^2 \right) L \log \frac{L}{eb}, \quad (163)$$

where $e = 2.71828\dots$. These equations provide analytic expressions for the non-perturbative calculation of quantum self energies and interaction energies, and eliminate the need for the virtual particle interpretation.

In particular, the pure screw (photon) self-energy

$$W_{self}^S = \frac{\bar{\mu}_0}{4\pi} (\mathbf{b} \cdot \boldsymbol{\xi})^2 L \log \frac{L}{eb} \quad (164)$$

and the pure edge (particle) self-energy

$$W_{self}^E = \frac{\bar{\mu}_0}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} |(\mathbf{b} \times \boldsymbol{\xi})|^2 L \log \frac{L}{eb} \quad (165)$$

are obtained from (163), while (163) is also the appropriate equation to use for the dual wave-particle “system”.

8.2 Dislocation strain energy and QED mass renormalization

This approach also resolves and eliminates the mass renormalization problem. This problem arises in QED due to the incomplete description of particle energies at the quantum level. This paper shows that the strain energy density of an edge dislocation, which corresponds to a particle, consists of a longitudinal dilatation mass density term and a transverse distortion energy density term, as shown in (49), (50), and (51).

QED, in its formulation, only uses the transverse distortion strain energy density in its calculations. This is referred to as the bare mass m_0 . However, there is no dilatation mass density term used in QED, and hence no possibility of properly deriving the mass. The bare mass m_0 is thus renormalized by replacing it with the actual experimental mass m . Using the longitudinal dilatation mass density term as in this paper will provide the correct mass m and eliminate the need for mass renormalization.

9 Discussion and conclusion

This paper provides a framework for the physical description of physical processes at the quantum level based on dislocations in the spacetime continuum within the theory of the Elastodynamics of the Spacetime Continuum (*STCED*).

We postulate that the spacetime continuum has a granularity characterized by a length b_0 corresponding to the smallest elementary Burgers dislocation-displacement vector possible. One inference that comes out of this paper is that the basic structure of spacetime consists of a lattice of cells of size b_0 , rather than the “quantum foam” currently preferred in the literature. The “quantum foam” view may well be a representation of the disturbances and fragmentation of the b_0 lattice due to dislocations and other defects in the spacetime continuum.

There are two types of dislocations: Edge dislocations correspond to dilatations (longitudinal displacements) which have an associated rest-mass energy, and are identified with particles. Screw dislocations correspond to distortions (transverse displacements) which are massless and are identified with photons when not associated with an edge dislocation. Arbitrary mixed dislocations can be decomposed into a screw component and an edge component, giving rise to wave-particle duality.

We consider both stationary and moving dislocations, and find that stationary dislocations are simpler to work with due to their cylindrical polar symmetry, but are of limited applicability. Moving screw dislocations are found to be Lorentz invariant. Moving edge dislocations involve both the speed of light corresponding to transverse displacements and the speed of longitudinal displacements c_l . However, the speed of light c upper limit also applies to edge dislocations, as the shear stress becomes infinite everywhere at $v = c$, even though the speed of longitudinal deformations c_l is greater than that of transverse deformations c .

We calculate the strain energy density of both stationary and moving screw and edge dislocations. The strain energy density of the screw dislocation is given by the transverse distortion energy density, and does not have a mass component. On the other hand, the dilatation strain energy density of the edge dislocation is given by the (massive) longitudinal dilatation energy density, and the distortion (massless) strain energy density of the edge dislocation is given by the transverse distortion energy density. This provides a solution to the mass renormalization problem in QED. Quantum Electrodynamics only uses the equivalent of the transverse distortion strain energy density in its calculations, and hence has no possibility of properly deriving the mass, which is in the longitudinal dilatation massive strain energy density term that is not used in QED.

The theory provides an alternative model for Quantum Electrodynamics processes, without the mathematical formalism of QED. In this framework, self-energies and interac-

tions are mediated by the strain energy density of the dislocations. The role played by virtual particles in Quantum Electrodynamics is replaced by the interaction of the strain energy densities of the dislocations. This theory is not perturbative as in QED, but rather calculated from analytical expressions. The analytical equations can become very complicated, and in some cases, perturbative techniques will need to be used to simplify the calculations, but the availability of analytical expressions permits a better understanding of the fundamental physical processes involved.

We provide examples of dislocation-dislocation interactions, applicable to photon-photon, photon-particle, and particle-particle interactions, and of dislocation self-energy calculations, applicable to photons and particles. These equations provide analytical expressions for the non-perturbative calculation of quantum self energies and interaction energies, and provides a physical process replacement for the virtual particle interpretation used in QED.

The theory proposed in this paper is formulated in a formalism based on Continuum Mechanics and General Relativity. This formalism is different from that used in Quantum Mechanics and Quantum Electrodynamics, and is currently absent of quantum states and uncertainties as is commonplace in quantum physics. Both formalisms are believed to be equivalent representations of the same physical phenomena. It may well be that as the theory is developed further, the formalism of orthonormal basis function sets in Hilbert spaces will be introduced to facilitate the solution of problems.

As shown in [47], it is a characteristic of Quantum Mechanics that conjugate variables are Fourier transform pairs of variables. The Heisenberg Uncertainty Principle thus arises because the momentum p of a particle is proportional to its de Broglie wave number k . Consequently, we need to differentiate between the measurement limitations that arise from the properties of Fourier transform pairs of conjugate variables, and any inherent limitations that may or may not exist at the quantum level, independently of the measurement process. Quantum theory currently assumes that the inherent limitations are the same as the measurement limitations. As shown in [47], quantum measurement limitations affect our perception of the quantum environment only, and are not inherent limitations of the quantum level, i.e. there exists a physical world, independently of an observer or a measurement, as seen here. See also the comments in [48, pp. 3–15].

This framework lays the foundation to develop a theory of the physical description of physical processes at the quantum level, based on dislocations in the spacetime continuum, within the theory of the Elastodynamics of the Spacetime Continuum. The basis of this framework is given in this initial paper. This framework allows the theory to be fleshed out in subsequent investigations. Disclinations in the spacetime continuum are expected to introduce new physical processes at the quantum level, to be worked out in future investigations. Additional spacetime continuum fundamental

processes based on ongoing physical defect theory investigations will emerge as they are applied to *STCED*, and will lead to further explanation of current quantum physics challenges.

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