A Model of Dust-like Spherically Symmetric Gravitational Collapse without Event Horizon Formation

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Some dynamical aspects of gravitational collapse are explored in this paper. A time-dependent spherically symmetric metric is proposed and the corresponding Einstein field equations are derived. An ultrarelativistic dust-like stress-momentum tensor is considered to obtain analytical solutions of these equations, with the perfect fluid consisting of two purely radial fluxes — the inwards flux of collapsing matter and the outwards flux of thermally emitted radiation. Thermal emission is calculated by means of a simplistic but illustrative model of uninteracting collapsing shells. Our results show an asymptotic approach to a maximal space-time deformation without the formation of event horizons. The size of the body is slightly larger than the Schwarzschild radius during most of its lifetime, so that there is no contradiction with either observations or previous theorems on black holes. The relation of the latter with our results is scrutinized in detail.

1 Introduction

The aim of this paper is to discuss several open problems of conceptual interest concerning black holes and, in particular, to elaborate a simple model of dust-like spherically symmetric gravitational collapse with account of both the inwards flux of the collapsing matter and the outwards flux of emitted thermal radiation. We illustrate how the latter may avoid the formation of event horizons. The metric considered in this work is time-dependent, unlike the Schwarzschild one. Spherical polar coordinates will be used and there will be no need for analytical extensions (such as the one given by the Kruskal-Szekeres chart) because the occurrence of an event horizon at the Schwarzschild radius will be avoided.

In Sec. 2 the main historical events concerning the development of the well-known concept of black hole are reviewed and its precise significance is shortly but precisely detailed. In Sec. 3 some open problems of the common black hole model are pointed out and their relationship with the corresponding historical findings is emphasized. Section 4 deals with the development of the metric of the present model: First of all, in subsec. 4.1 a time-dependent spherically symmetric metric in spherical polar coordinates is presented and the corresponding Einstein field equations are specified. Secondly, a dust-like energy momentum tensor for a purely radial motion with account of an ultrarelativistic collapsing matter and thermally emitted radiation is obtained in subsec. 4.2. Temporal evolution of the metric components is studied in subsec. 4.3, with the absence of emitted thermal radiation being detailed as a particular case. Fourthly, in subsec. 4.4 it is shown that there should exist a limit where the inwards flux of collapsing matter and the outwards flux of thermal radiation become *compensated*. It is also shown the asymptotic character of the approximation to this limit. Some additional considerations about the total mass and the edge of the collapsing body will be made in subsec. 4.5. Finally, our results are discussed in Sec. 5, paying a special attention to the plausibility of the different hypothesis and the implications of their alternatives.

2 Important historical results concerning black holes

Several historical results in General Relativity led to the concept of black hole. The following list includes some of the most important ones:

1. K. Schwarzschild found in 1916 an exact solution of the Einstein field equations describing the field created by a point particle [1]. (According to Birkhoff's theorem, this solution is also valid for any spherically symmetric body at a distance larger than its radius [2].)
2. J. R. Oppenheimer and G. M. Volkoff discovered in 1939 the existence of upper limit for the mass of neutron stars, above which gravitational collapse could not be avoided [3].
3. In 1967 J. Wheeler used the term "black hole" to name a "gravitationally completely collapsed star" [5].
4. S. Hawking and R. Penrose proved in 1970 that, under certain circumstances, singularities could not be avoided. This is known as the Hawking-Penrose theorem of singularity [6].

   All these results concerning black holes arise basically from Einstein's General Relativity. On the other hand, there exist two important features in the description of black holes which require from both Thermodynamics and Quantum Field Theory (QFT):

5. J. Bekenstein defined the entropy of black holes in 1972 and, based on thermodynamic grounds, deduced the need for black-hole radiation [7].
6. In 1974 S. Hawking justified Bekenstein's speculations about the existence of black-hole radiation from the

point of view of QFT. Hawking model implies the creation of particles of negative mass near the event horizon of black holes. The conservation of information is not clearly ensured by this model [8].

3 Some open problems in gravitational collapse

In this section we discuss if the previous historical results genuinely imply the actual existence of black holes as physical objects. It is widely believed that these findings prove the existence of black holes. The argument supporting black hole formation is the following:

1. There exist stars which are massive enough to exceed the Oppenheimer-Volkoff limit at the end of their "vital cycle". Those stars must finally enter collapse.
2. According to the Hawking-Penrose theorem of singularity, all the mass inside an event horizon must reach a single central point, that is, form a singularity.
3. The solution of the Einstein field equations for the metric of a "point mass" is the Schwarzschild metric, that describes a black hole.

Entering collapse, however, does not immediately lead to the formation of an event horizon and, while the event horizon is not formed, the Hawking-Penrose theorem of singularity is not properly applicable (notice that one of its conditions of application is equivalent either to the existence of an event horizon, or to an expanding Universe taken as a whole). Hence, *a priori* entering collapse must not necessarily lead to a complete collapse.

Certainly, the period of time involved in the process of collapse may be proven to be infinite from the point of view of any external observer (that is, from our perspective on Earth). On the other hand, a "free falling observer" would measure a finite period of time for the collapse, at least if nothing destroys it before reaching its goal [4, 10]. A well-known feature of General Relativity is that space and time are relative but events are absolute. Consequently, it is necessary to reconcile the observations from both reference frames.

It is usually assumed that the free falling observer actually reaches the singularity in a finite time, and the infinite-lasting collapse measured by the external observer is justified in the following way: the free falling body has already reached the central singularity, but as the light emitted from the body inside the black hole never escapes from it, we cannot see it falling; furthermore, the light emitted near the event horizon of the black hole comes to us with a great delay, making us believe that it is still falling.

In fact, there are compelling reasons that make us doubt about the previous explanation: The Schwarzschild metric is symmetric under temporal inversion, which suggests that trajectories in the corresponding space-time should be also reversible, in contrast to the most common interpretation of black holes and their event horizon. Furthermore, General Relativity is not only intended to explain what an observer "sees" in a given reference frame, but what truly "occurs" in there. Additionally, S. Hawking defended the incompatibility of event horizons with Quantum Mechanics [9].

Solution of this apparent paradox requires a careful analysis of what an external observer would exactly see when looking at a body free falling towards a black hole. On the one hand, it would see the free-falling body approaching asymptotically to the event horizon of the black hole, without ever crossing it. On the other hand, according to Hawking's law of black hole radiation, the observer should also see the whole black hole evaporating in a very large, but finite period of time. The evaporation of the whole mass of the black hole must logically include that of the free-falling body as well. Were it not to be like this, that is, if the crossing of the event horizon had to be accomplished before the emission of thermal radiation, it would never emit thermal radiation and the laws of Thermodynamics would be infringed. As the temporal order of causally-related events is always the same for all reference frames, we must conclude that the free falling observer should also observe its own complete evaporation before having reached the event horizon. If it had reached the singularity in a finite period of time, its complete evaporation must have occurred in a finite and *lesser* period of time.

Not only should these considerations be valid for the free-falling body approaching a black hole, but also for the process of collapse itself [28]. Consequently, collapsing bodies should never becomes black holes. On the contrary, they should asymptotically tend to form an event horizon until the time at which they become completely emitted in the form of radiation. An equivalent thesis has already been defended by Mitra [14–18], Robertson and Leiter [19–21], Vachaspati *et al.* [11, 12], and by Piñol and López-Aylagas [13]. In addition, there exist some calculations in string theory which point towards the same direction [22].

Thus, the metric of a collapsing body shall never be in a strict sense Schwarzschild's one (as it never completely collapses) but a time-dependent metric. In the next section, we solve the Einstein field equations of a time-dependent spherically symmetric metric. Several simplifications are considered to make calculations plausible, but the essential Physics of the problem is respected.

4 Deduction of a metric for gravitational collapse

4.1 Einstein field equations

As we have already pointed out, our goal in this paper is to study the temporal evolution of a spherically symmetric gravitational collapse. Rotations and local inhomogeneities are beyond the scope of the present work. Therefore, the starting point shall be a time-dependent spherically symmetric metric, which in spherical polar coordinates is given by the expression

$$d\tau^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2, \tag{1}$$

where $\nu = \nu(r,t)$ and $\lambda = \lambda(r,t)$. Notice that geometrized units have been used ($G = 1, c = 1$). The corresponding Einstein field equations for such metric are the following [23]:

$$8\pi T_0^0 = -e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2}\,, \tag{2}$$

$$8\pi T_1^1 = -e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2}\,, \tag{3}$$

$$\begin{aligned} 8\pi T_2^2 = -\frac{1}{2}\, e^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} + \frac{\nu'\lambda'}{2}\right) + \\ + \frac{1}{2}\, e^{-\nu}\left(\ddot{\lambda}\,\frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\,\dot{\nu}}{2}\right), \end{aligned} \tag{4}$$

$$8\pi T_3^3 = 8\pi T_2^2\,, \tag{5}$$

$$8\pi T_0^1 = -e^{-\lambda}\frac{\dot{\lambda}}{r}\,. \tag{6}$$

Subtraction of 3 from 2 yields the identity

$$8\pi\left(T_0^0 - T_1^1\right) = \frac{e^{-\lambda}}{r}\left(\nu' + \lambda'\right). \tag{7}$$

It will be useful to define a function $\phi(r,t)$

$$-2\phi \equiv \nu + \lambda \tag{8}$$

so that

$$\nu = -\lambda - 2\phi\,, \qquad 8\pi\left(T_0^0 - T_1^1\right) = \frac{e^{-\lambda}}{r}\left(-2\phi'\right). \tag{9}$$

A mathematical structure for the stress-momentum tensor must be specified in order to solve the previous equations, which will be discussed in next subsection.

4.2 A dust-like stress-momentum tensor of ultrarelativistic particles

The stress-momentum tensor of a perfect fluid may be written in terms of the energy density ρ, the pressure p and the four-velocity u^α as:

$$T_\beta^\alpha = g_{\beta\delta}\,(\rho + p)\, u^\alpha u^\delta - \eta_\beta^\alpha\, p\,. \tag{10}$$

If the pressure appears to be very small compared to the energy density, in the limit $p \to 0$ one obtains the stress-momentum tensor of dust:

$$T_\beta^\alpha = g_{\beta\delta}\,\rho\, u^\alpha\, u^\delta. \tag{11}$$

In our model we deal with a dust-like stress-momentum tensor. For the sake of simplicity, we shall consider the perfect fluid splitting into two perfectly radial fluxes: a flux of ingoing collapsing matter and a second flux of outgoing thermal radiation. Both the ingoing collapsing matter and the outgoing thermal radiation are going to be dealt as ultrarelativistic particles. It has been already established that the matter in a process of gravitational collapse reaches celerities near the speed of light [24]. It is also a well-known fact that, despite photons being "massless", a photon gas may be assimilated to a gas of ultrarelativistic particles with an effective mass density [25].

It could be expected that the relation between pressure and mass-energy density should be given by the identity $p = \frac{\rho}{3}$ due to the particles being ultrarelativitic. A closer insight into this points out that the above identity would only be properly applicable to an *isotropic* gas and not to the higly *directed* movement considered in the present work. The consideration of two purely "radial" fluxes shall simplify calculations and it is in this sense that a "dust-like" stress-momentum tensor may be used. A similar approach has been already adopted by Borkar and Dhongle [26].

With account of the metric 1 the coefficients of the dust energy-momentum tensor 11 become

$$T_0^0 = e^{-2\phi}\, e^{-\lambda}\,\rho\left(u^0\right)^2, \tag{12}$$

$$T_1^1 = -\,e^{\lambda}\,\rho\,\left(u^1\right)^2, \tag{13}$$

$$T_0^1 = e^{-2\phi}\, e^{-\lambda}\,\rho\, u^0\, u^1. \tag{14}$$

For a purely radial movement (characterized by $d\Omega = 0$) Eq. 1 leads to the relation

$$d\tau^2 = e^{-2\phi}e^{-\lambda}dt^2 - e^{\lambda}dr^2 \tag{15}$$

which, with account of the identities $\frac{dt}{d\tau} \equiv u^0$ and $\frac{dr}{d\tau} \equiv u^1$, becomes

$$1 = e^{-2\phi}e^{-\lambda}\left(u^0\right)^2 - e^{\lambda}\left(u^1\right)^2. \tag{16}$$

Isolating $\left|u^1\right| = \sqrt{(u^1)^2}$, we obtain

$$\left|u^1\right| = e^{-\phi}e^{-\lambda}u^0\left[1 - \frac{e^{2\phi}e^{\lambda}}{(u^0)^2}\right]^{\frac{1}{2}}. \tag{17}$$

In the ultrarelativistic limit $u^0 \to \infty$ ($u^0 \gg e^{2\phi}e^{\lambda}$) the component u^1 of the four-velocity becomes

$$\left|u^1\right| = e^{-\phi}e^{-\lambda}u^0. \tag{18}$$

Notice that this same relation could have been obtained by imposing the identity $d\tau \sim 0$ in Eq. 15.

Concerning the sign of u^1, it is clear that $u^1 < 0$ for ingoing matter and $u^1 < 0$ for outgoing thermal radiation, i.e.

$$u^1_{in} = -e^{-\phi}\, e^{-\lambda}\, u^0, \tag{19}$$

$$u^1_{out} = e^{-\phi}\, e^{-\lambda}\, u^0. \tag{20}$$

4.2.1 Stress-momentum tensor of the ingoing matter

If we denote the energy density of the infalling matter by ρ_{in}, according to Eqs. 12, 13, 14 and 19 we have

$$T^0_{0,in} = e^{-2\phi}\, e^{-\lambda}\, \rho_{in}\, \left(u^0\right)^2, \tag{21}$$

$$T^1_{1,in} = -e^{-2\phi}\, e^{-\lambda}\, \rho_{in}\, \left(u^0\right)^2 = -T^0_{0,in}, \tag{22}$$

$$T^1_{0,in} = -e^{-3\phi}\, e^{-2\lambda}\, \rho_{in}\, \left(u^0\right)^2 = -e^{-\phi}\, e^{-\lambda}\, T^0_{0,in}. \tag{23}$$

4.2.2 Stress-momentum tensor of the outgoing thermal radiation

Denoting the energy density of the outgoing thermal radiation by ρ_{out}, according to Eqs. 12, 13, 14 and 20 we obtain

$$T^0_{0,out} = e^{-2\phi}\, e^{-\lambda}\, \rho_{out}\, \left(u^0\right)^2, \tag{24}$$

$$T^1_{1,out} = -e^{-2\phi}\, e^{-\lambda}\, \rho_{out}\, \left(u^0\right)^2 = -T^0_{0,out}, \tag{25}$$

$$T^1_{0,out} = e^{-3\phi}\, e^{-2\lambda}\, \rho_{out}\, \left(u^0\right)^2 = e^{-\phi}\, e^{-\lambda}\, T^0_{0,out}. \tag{26}$$

4.2.3 Total stress-momentum tensor of the collapsing body

Addition of the stress-momentum tensors of both the infalling matter and the outgoing thermal radiation leads to the total stress-momentum tensor of the collapsing body, which is given by the expressions

$$T^0_0 = e^{-2\phi}\, e^{-\lambda}\, (\rho_{in} + \rho_{out})\, \left(u^0\right)^2, \tag{27}$$

$$T^1_1 = -e^{-2\phi}\, e^{-\lambda}\, (\rho_{in} + \rho_{out})\, \left(u^0\right)^2 = -T^0_0, \tag{28}$$

$$T^1_0 = -e^{-3\phi}\, e^{-2\lambda}\, (\rho_{in} - \rho_{out})\, \left(u^0\right)^2 = -e^{-\phi}\, e^{-\lambda} \left(\frac{\rho_{in} - \rho_{out}}{\rho_{in} + \rho_{out}}\right) T^0_0. \tag{29}$$

Once the mathematical structure of the stress-momentum tensor of the collapsing body is established, we are able to study the temporal evolution of the collapse by solving the Einstein field equations 2-6.

4.3 Temporal evolution of collapse

Substitution of T^1_0 by Eq. 27 in Eq. 6 leads to the following equation:

$$-e^{-\phi}\, e^{-\lambda} \left(\frac{\rho_{in} - \rho_{out}}{\rho_{in} + \rho_{out}}\right) 8\pi T^0_0 = -e^{-\lambda}\, \frac{\dot{\lambda}}{r}. \tag{30}$$

From this an expression for the temporal evolution of λ may be isolated:

$$\dot{\lambda} = e^{-\phi} \left(\frac{\rho_{in} - \rho_{out}}{\rho_{in} + \rho_{out}}\right) 8\pi r T^0_0. \tag{31}$$

Initially it is expected that $\rho_{in} \gg \rho_{out}$, as the amount of energy emitted in the form of thermal radiation should reasonably correspond to a very small proportion of the total energy of the collapsing body. In that case, $\left(\frac{\rho_{in}-\rho_{out}}{\rho_{in}+\rho_{out}}\right) \sim 1$ and $\dot{\lambda} \sim e^{-\phi}\left(8\pi r T^0_0\right)$, so that λ shall be a strictly increasing function with time and it is expected to acquire considerably large values. In any case, for $\lambda \gg 1$ we have the asymptotic expression

$$8\pi T^0_0 = \frac{1}{r^2} + O(e^{-\lambda}), \tag{32}$$

and therefore,

$$\dot{\lambda} = e^{-\phi} \left(\frac{\rho_{in} - \rho_{out}}{\rho_{in} + \rho_{out}}\right) \frac{1}{r} + O(e^{-\lambda}). \tag{33}$$

On the other hand, we need to estimate as well the value of ϕ. From Eqs. 9 and 28 we obtain

$$\phi' = -\frac{1}{2}\, e^{\lambda}\, 8\pi r\, \left(T^0_0 - T^1_1\right) = -e^{\lambda}\, \left(8\pi r T^0_0\right), \tag{34}$$

which combined with Eq. 32 yields

$$\phi' = -\frac{e^{\lambda}}{r} + \left(\frac{1}{r} - \lambda'\right) \sim -\frac{e^{\lambda}}{r}. \tag{35}$$

According to Birkhoff's theorem, outside the radius R of the collapsing body the space-time geometry will be exactly Schwarzschild-like, so that $\phi = 0$ for $r > R$. Inside the collapsing body $T^0_0 > 0$ and consequently $\phi' < 0$. This yields $\phi > 0$ for $r < R$ and $\phi(R, t) = 0$ because of the analytic character of this function.

Equations 33 and 35 are not trivial to resolve analytically. For any time t, however, Eq. 33 and the fact that $\phi > 0$ for any $r < R$ lead to the following inequality:

$$\lambda(t, r) < \lambda(0, r) + \frac{t}{r}. \tag{36}$$

4.4 Asymptotic approach to a pseudo-stability phase

According to the results obtained in the previous section, for any given time t the function $\lambda(r, t)$ is analytic on the domain $r > 0$. Nonetheless, as Eq. 36 is an inequality, no specific values for this function have been provided.

It has been discussed that the ingoing flux of infalling matter is initially expected to be much larger than the outgoing flux of thermal radiation. Despite this, as λ becomes larger, according to Eq. 35 $|\phi'|$ must also increase. On the other hand, as $\phi \geqslant 0$ the ingoing flux must decrease according to Eq. 23.

As the values of $T^1_{0,in}$ may become as small as wanted, if λ and ϕ were not upper bounded it would not be unreasonable to think that the ingoing flux of infalling matter may eventually become *compensated* by the outgoing flux of thermal radiation. It could be discussed as well that, according to Eq. 26, the flux of outgoing thermal radiation may also become arbitrarily small, but we proceed first to analyse the details concerning the compesation of fluxes and the consequences of this hypothesis.

The condition for the compensation of both fluxes is naturally given by the equation

$$T^1_{0,in,s} + T^1_{0,out,s} = 0\,. \tag{37}$$

It must not be misunderstood as a transgression of Oppenheimer-Volkoff's theorem. The star *is not* in equilibrium. It is actually collapsing, as nothing prevents the infalling matter of keeping in collapse. There would simply be an additional flux (arguable in the basis of thermodynamic grounds, and justifiable by the conversion of a portion of the collapsing matter into thermal radiation due to the interaction of their respective fields) that would compensate the energy interchange across a given surface of r−radius.

In that hypothetical state of "stability", from Eqs. 23 and 26 a relation between the energy densities ρ_{in} and ρ_{out} can be derived

$$\rho_{in,s} = \rho_{out,s} = \frac{1}{2}\rho_s\,, \tag{38}$$

where the subindex s stands for "stability" (notice that the aforementioned relations are specific of that hypothetical phase). Several considerations concerning the emission of thermal radiation due to collapsing bodies must be made in order to proceed further with the theoretical development.

4.4.1 A model of Hawking-like radiation

According to Hawking [8], the temperature of a black hole is proportional to the inverse of its Schwarzschild radius (R_S) and the thermal radiation emission rate is proportional to the inverse of the square of R_S:

$$\dot{M}_H = -\frac{k}{R_S^2}\,. \tag{39}$$

We have denoted the thermal emission by $\dot{M}_H$ as it implies a loss in the total mass of the black hole.

In what follows, both the approach and the nomenclature adopted in the study of the mass and its mathematical relation with the components of the stress-momentum tensor and with the functions $\nu(r,t)$ and $\lambda(r,t)$ of the metric 1 are the ones given in Ref. [23]. The total mass of a spherically symmetric body of radius R is given by the following expression:

$$M = \int_0^R 4\pi r^2 T^0_0(r,t)dr. \tag{40}$$

Analogously, the mass contained inside a surface of radius r (concentric to the spherically symmetric body of interest) is given by

$$m(r,t) = \int_0^r 4\pi\tilde{r}^2 T^0_0(\tilde{r},t)\,d\tilde{r}. \tag{41}$$

Comparing Eqs. 2 and 41, the following relation can be set between $m(r,t)$ and $\lambda(r,t)$:

$$e^{-\lambda(r,t)} = 1 - \frac{2m(r,t)}{r}\,, \tag{42}$$

and therefore we have $-e^{-\lambda}\dot{\lambda} = -\frac{2\dot{m}}{r}$ or, equivalently,

$$\dot{\lambda} = \frac{2\dot{m}}{r}\,e^{\lambda}. \tag{43}$$

Despite the fact that there is solely "one" function $\lambda(r,t)$, it is useful to split $\dot{\lambda}$ into the sum of $\dot{\lambda}_{in}$ (due to the ingoing flux $\dot{m}_{in}$ of collapsing matter) and $\dot{\lambda}_{out}$ (due to the outgoing flux $\dot{m}_{out}$ of thermal radiation). In so doing we obtain

$$\dot{\lambda} = \dot{\lambda}_{in} + \dot{\lambda}_{out} \tag{44}$$

with

$$\dot{\lambda}_{in} = \frac{2\dot{m}_{in}}{r}\,e^{\lambda}, \qquad \dot{\lambda}_{out} = \frac{2\dot{m}_{out}}{r}\,e^{\lambda}. \tag{45}$$

As pointed out before, the thermal emission of black holes $\dot{m}_H$ is given by Eq. 39. On the other hand, Vachaspati *et al.* showed that the thermal emission of a collapsing shell approaching the Schwarzschild's radius of a black hole would follow a law of the same style [11]: according to their calculations, the temperature of the collapsing shell turns out to be proportional to the Hawking's one ($T_V \sim 2.4T_H$, where T_V stands for Vachaspati's temperature and T_H for Hawking's temperature).

With account of Eq. 42 the metric 1 becomes

$$\begin{aligned} d\tau^2 = &\left(1 - \frac{2m(r,t)}{r}\right)e^{-2\phi(r,t)}\,dt^2 - \\ &- \left(1 - \frac{2m(r,t)}{r}\right)^{-1} dr^2 \;-\; r^2\,d\Omega^2, \end{aligned} \tag{46}$$

where the resemblance with Schwarzschild's metric results evident. Certainly, there exist two main differences between Eq. 46 and the Schwarzschild's metric: 1) the mass is not a constant, but a function of the radius. 2) there is an additional factor $e^{-2\phi(r,t)}$ in the coefficient g_{00}.

However, if we 1) deal with motions whose variation in the r-coordinate is small enough and 2) assume a temporal proximity to the hypothetical stationary case that we postulated (i.e., $\dot{m}(r,t) \sim 0$ and $\dot{\phi}(r,t) \sim 0$), then the metric 46 may be *locally* transformed into the Scwarzschild's one.

In fact, in the *vicinity* of a given radius R_a, where $m(r,t) \sim M_a$ and $\phi(r,t) \sim \Phi_a$, we have

$$d\tau^2 \sim \left(1 - \frac{2M_a}{r}\right)d\tilde{t}^2 - \left(1 - \frac{2M_a}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{47}$$

with

$$d\tilde{t} \equiv e^{-\Phi_a}dt. \tag{48}$$

At this point it is time to introduce our Hawking-like radiation model. We will conceptually split the collapsing body into a sequence of concentric spherical shells, each of which asymptotically approaches its corresponding radius $r = 2M_a$ in the coordinate system given by the metric 47. We assume that these collapsing shells do not interact with each other. Along the lines of Ref. [12] it can be deduced that the radiation law obtained for a spherical shell asymptotically approaching in time t the event horizon of a black hole is also valid for any of the concentric shells asymptotically approaching in time $\tilde{t}$ its corresponding $r = 2M_a$ radius in our model. Consequently,

$$\frac{dm_{out}}{d\tilde{t}} = -\frac{k}{r^2} \tag{49}$$

and so

$$\dot{m}_{out} \equiv \frac{dm_{out}}{dt} = \frac{d\tilde{t}}{dt}\frac{dm_{out}}{d\tilde{t}} = -e^{-\phi}\frac{k}{r^2}\,. \tag{50}$$

From this, we straightforwardly obtain the identity

$$\dot{\lambda}_{out} = \frac{2e^{\lambda}}{r}\left(\frac{-e^{-\phi}k}{r^2}\right) = -e^{-\phi}\,\frac{2k\,e^{\lambda}}{r^3}\,. \tag{51}$$

On the other hand, according to Eqs. 6 and 23 an equivalent expression for $\dot{\lambda}_{in}$ is given by

$$\dot{\lambda}_{in} = e^{-\phi}\left(8\pi r T^0_{0,in}\right)\,. \tag{52}$$

From Eqs. 12, 21 and 32 we conclude that, asymptotically,

$$8\pi T^0_{0,in} = \frac{\rho_{in}}{\rho_{in} + \rho_{out}}\,\frac{1}{r^2} + O(e^{-\lambda}) \tag{53}$$

and therefore, with account of Eq. 38, we obtain

$$\dot{\lambda}_{in} = e^{-\phi}\left(\frac{\rho_{in}}{\rho_{in} + \rho_{out}}\right)\frac{1}{r} \simeq e^{-\phi}\frac{1}{2r}\,. \tag{54}$$

The stability phase is naturally defined by the condition

$$\dot{\lambda}_s = 0 \tag{55}$$

and therefore, from Eqs. 44, 51, 54 and 55 we obtain the relation

$$-e^{-\phi_s}\frac{2k\,e^{\lambda_s}}{r^3} + e^{-\phi_s}\frac{1}{2r} = 0. \tag{56}$$

Equivalently,

$$e^{\lambda_s} = \frac{1}{4k}\,r^2, \tag{57}$$

from which a functional dependence of λ on r is obtained for the stability phase

$$\lambda_s(r) = -\ln(4k) + \ln\left(r^2\right). \tag{58}$$

Taking into account Eq. 35, from the previous equation we easily obtain an expression for ϕ_s:

$$\phi'_s = -\frac{e^{\lambda_s}}{r} = -\frac{r}{4k}. \tag{59}$$

Integration over r with account of the contour condition $\phi(R,t) = 0\ \forall t$ discussed in the previous section yields the identity

$$\phi_s(r) = \int_R^r \phi'_s(\tilde{r})\,d\tilde{r} = \frac{1}{8k}\left(R^2 - r^2\right), \tag{60}$$

and thus

$$e^{-\phi_s(r)} = e^{\frac{-1}{8k}\left(R^2 - r^2\right)}. \tag{61}$$

It must be noticed that the existence of the postulated stability phase is self-consistent and that it may be clearly derived from equations 45: both $\left|\dot{\lambda}_{in}\right|$ and $\left|\dot{\lambda}_{out}\right|$ decrease as $\phi(r,t)$ increases by a factor $e^{-\phi(r,t)}$, but only $\left|\dot{\lambda}_{out}\right|$ increases as $\lambda(r,t)$ increases (by a factor $e^{\lambda(r,t)}$). Consequently, even when initially $\left|\dot{\lambda}_{out}\right| \ll \left|\dot{\lambda}_{in}\right|$ at large enough times both quantities should become of the same magnitude.

Nonetheless, a significant issue concerning the behaviour of $\lambda(r,t)$ for small values of r must be remarked. We are going to deal it with detail in the following subsection.

4.4.2 Corrections to the equation of λ_s for small radii

From Eq. 42, as $m(r,t) \geqslant 0\ \forall r,t$, it becomes evident that also $\lambda(r,t)$ must be $\geqslant 0\ \forall r,t$. However, in Eq. 58, it can be checked that it yields $\lambda_s = 0$ at $r = 2\sqrt{k}$ and $\lambda_s < 0$ for $r < 2\sqrt{k}$. Consequently, the mentioned expression cannot be valid for small radii.

As it has been clearly established in subsec. 4.3, if no outwards flux of thermal radiation is taken into account the values of $\lambda(r,t)$ would grow in an unlimited way. Thus, at large times, it would become great enough to imply the T^0_0 component of the stress-momentum tensor to approach the asymptotic expression given in Eq. 32. By contrast, in the previous subsection we have actually taken into account the emission of thermal radiation, and it has been performed with the Hawking-like law specified in Eq. 39, which entails a most prominent emission rate for inner shells. As a consequence, λ_s values decrease at small radii (or, what is the same, it results to be a strictly increasing function with r).

For radii $r \gg 2\sqrt{k}$, all the calculations which have been deduced after Eq. 32 are completely justified. Fortunately, that corresponds to most values of r, since $k \ll 1$ (certainly, the thermal evaporation process takes place at a considerably slow rhythm).

Thus, the steps which we have followed in order to determine $\lambda_s(r)$ must be reviewed in order to obtain a valid expression for small radii. A suitable analytical solution to the problem is far from being straightforward, but we are going to analyse it a bit more of care in the following lines.

Firstly, the complete identity of T^0_0 in Eq. 2 must be used instead of Eq. 32. Therefore, the expression for λ_{in}, instead of the one specified in Eq. 54, according to 52 will be

$$\dot{\lambda}_{in} = e^{-\phi}\frac{1}{2r}\left(1 - e^{-\lambda}\left(1 - r\lambda'\right)\right). \tag{62}$$

From this, keeping the same radiation law of Eq. 39 and the expression for λ_{out} of Eq. 51, it is not hard to follow that the stability condition in Eq. 55 entails

$$e^{\lambda_s} = \frac{r^2}{4k}\left(1 - e^{-\lambda_s}\left(1 - r\lambda_s'\right)\right). \quad (63)$$

When $\lambda_s \gg 1$, Eq. 57 is recovered. As we had already signalled, its resolution in the regions where the mentioned limit ceases to be valid is far from being trivial. Nonetheless, a possibility could consist in the application of an iterative method. Instead of making $e^{\lambda_s} \to 0$, in the right side of the equation we may use as a first approximation (well, actually as a *second* approximation) the expression for λ_s obtained in Eq. 58 (being its derivative $\lambda_s' = 2/r$):

$$e^{*\lambda_s} \sim \frac{r^2}{4k}\left(1 - \frac{4k}{r^2}\left(1 - r\frac{2}{r}\right)\right) = \frac{r^2}{4k} + 1, \quad (64)$$

where the asterisk (*) stands for "iterated".

Thus,

$$\lambda_s * \sim \ln\left(\frac{r^2}{4k} + 1\right), \quad (65)$$

which can be assimilated to Eq. 58 for large values, but that has the advantage of accomplishing the necessary condition $\lambda(r,t) \geqslant 0 \; \forall r, t$.

In the next subsection, we are not going to have longer into account the corrections for small radii, but we will focus onto temporal variations of $\lambda(r,t)$ when approaching the stability phase described by 58 (only valid for $r > 2\sqrt{k}$).

4.4.3 Small variations of $\lambda_{(r,t)}$ before the stability phase

According to Eqs. 44, 51 and 54 we have

$$\dot{\lambda}(r,t) = e^{-\phi}\left(\frac{1}{2r} - \frac{2ke^{\lambda}}{r^3}\right). \quad (66)$$

In the stability phase, defined by Eq. 55, the functional dependence of λ is given by Eq. 57. Now we proceed to study small variations of $\lambda(r,t)$ before it acquires the stability value, that is,

$$\lambda(r,t) = \lambda_s(r) - \lambda_\Delta(r,t). \quad (67)$$

Notice that, by definition, $\dot{\lambda}_s(r) = 0$. This fact implies

$$\dot{\lambda}(r,t) = -\dot{\lambda}_\Delta(r,t). \quad (68)$$

Furthermore, because of the inequality $\lambda_\Delta \ll \lambda$, we will consider $\phi \simeq \phi_s$. Therefore, from Eqs. 57, 66, 67 and 68 we obtain the expression

$$\dot{\lambda}_\Delta = -\frac{e^{-\phi_s}}{2r}\left(1 - e^{-\lambda_\Delta}\right). \quad (69)$$

In the limit $\lambda_\Delta \ll 1$ we can approximate $1 - e^{-\lambda_\Delta} \sim \lambda_\Delta$, so that

$$\dot{\lambda}_\Delta = -\frac{e^{-\phi_s}}{2r}\lambda_\Delta + O\left(\lambda_\Delta^2\right), \quad (70)$$

whose integration over t leads to the following solution

$$\lambda_\Delta = A(r)\exp\left(-\frac{e^{-\phi_s}}{2r}t\right) = A(r)\exp\left(-e^{\frac{-1}{8k}\left(R^2 - r^2\right)}\frac{t}{2r}\right), \quad (71)$$

where $A(r)$ is an arbitrary positive defined function depending on the initial conditions of the problem.

Therefore, according to the hypothesis of the model, $\lambda_{(r,t)}$ asymptotically approaches its stability value:

$$\lambda(r,t) = -\ln(4k) + \ln\left(r^2\right) - A(r)\exp\left(-e^{\frac{-1}{8k}\left(R^2 - r^2\right)}\frac{t}{2r}\right). \quad (72)$$

4.5 Some considerations about the mass and the edge of the collapsing body

From Eq. 19 the infalling velocity $\dot{r}_{in}$ of any collapsing shell in the present model is given by

$$\dot{r}_{in} \equiv \frac{dr}{dt} = \frac{dr}{d\tau}\frac{d\tau}{dt} = \frac{u^1_{in}}{u^0} = -e^{-\phi}e^{-\lambda}. \quad (73)$$

According to Eqs. 42 and 73 and with account of the contour condition $\phi(R,t) = 0 \; \forall t$, the motion of the edge R of a collapsing body of mass M must be given by the expression

$$\dot{R} = -\left(1 - \frac{2M}{R}\right), \quad (74)$$

whose solution for large enough times is

$$R = 2M + \Delta R_0 e^{\frac{-t}{2M}} \quad (75)$$

with ΔR_0 being a constant depending on the initial conditions of the collapse.

An important detail must be pointed out. In the previous equations we have dealt with the total mass M of the collapsing body as if it was a constant. It may be actually considered constant in practice for long periods of time but, in fact, it slowly diminishes due to the emission of thermal radiation, unless the surrounding background presents a greater CMB temperature or news amounts of infalling mass are provided. Thus, having into account that $R_S = 2M$, from Eq. 39,

$$\dot{M} = \frac{-k}{R_S^2} = \frac{-k}{4M^2}. \quad (76)$$

Therefore,

$$M(t) = \left(M_0^3 - \frac{3kt}{4}\right)^{\frac{1}{3}}, \quad (77)$$

from which the evaporation time t_v may be isolated:

$$t_v = \frac{4M_0^3}{3k}. \quad (78)$$

5 Discussion

The model of gravitational collapse presented in this paper contains an important number of simplifications which have allowed us to find analytical solutions of the coefficients of the metric all over the space at any given time (for small radius values, we have seen that some special considerations must be taken into account, but no essential contradiction is risen). The results obtained are self-consistent and do not lead to the formation of an event horizon, what would provide a simpler interpretation of the information loss problem: if no event horizon is formed, thermal radiation should be directly emitted by the collapsing body. Hence, there is no need for postulating a special mechanism of radiation such the one that S. Hawking proposed *ad hoc* for black holes. Let us now analyse more carefully the hypothesis that we have made, their implications and the consequences that would have been derived from making slightly different considerations.

Our starting point has been a time-dependent spherically symmetric metric. It is a well-known fact that spherical symmetry is an almost universal *approximate* characteristic of any celestial body. Two kind of phenomena certainly prevents it from being *perfect*: the first one is rotation (which implies the modification from spherical surfaces to ellipsoidal ones), while the second one consists of the local inhomogeneities of any *real* system.

Concerning rotation, it constitutes *per se* a very interesting but mathematically complex problem. To deal properly with a rotating process of gravitational collapse, a kind of modified time dependent Kerr metric should be formulated (in the same way that in this paper a kind of "time-dependent Schwarzschild metric" has been proposed). From an intuitive point of view, however, one would expect that rotation should lead to a genuinely *slower* collapsing process (due to the "centrifugal" effect of angular momentum). Concerning local inhomogeneities, a detailed study of the effect of small perturbations on the metric could constitute another *per se* attractive problem, but *a priori* it is not unreasonable to assume that the emission of gravitational waves should tend to diminish these effects with time. This is a consequence of the "no hair" theorem for black holes (even when we have found no black hole in the mathematical development of this article).

About the temporal dependence of the metric coefficients, it appears to be a strict logical requirement of the problem. The displacement of the infalling matter along the collapsing process must necessarily imply a temporal change in the metric coefficients. In this sense, Schwarzschild metric -a good solution for the stationary "punctual mass" problem- is not the best choice for the question of collapse itself. In words of J. A. Wheeler, "matter tells spacetime how to curve, and curved spacetime tells matter how to move". With our choice of time-dependent metric, Kruskal-Szekeres coordinates are not needed because the ordinary polar spherical coordinates cover the entire spacetime manifold and the functions $\lambda(r, t)$ and $\nu(r, t)$ are analytic all over the space.

With respect to the choice of stress-momentum tensor, its dust-like nature has been greatly aimed for the sake of simplicity. As it has been already emphasized in the pertinent section, it seems paradoxal to consider simultaneously the features of "dust-like" and "ultrarelativistic" because the relation between pressure and energy density in an ultrarelativistic *gas* turns out to be $p = \frac{1}{3}\rho$. Nonetheless, two subtle points should be raised here: First of all, the concept of "ultrarelativistic dust" is not as strange as it appears to be, since a privileged direction of motion has been considered (the ultrarelativistic motion is highly "directed" towards purely radial lines). Secondly, even if a relation of proportionality between p and ρ would have been chosen, that would not have changed the fact that all the other stress-momentum tensor components could be expressed as a product of certain factors and T_0^0. It is straightforward to check that changing the aforementioned factors would not alter drastically the subsequent mathematical development. As a matter of fact, the "linearity" between T_0^1 and T_0^0 has allowed us to set a temporal dependence for λ. In fact, as $\dot{\lambda}$ turns out to be proportional to T_0^0, the function λ would only diverge if T_0^0 became infinite too. Nevertheless, when λ increases T_0^0 does not diverge but tends to $\frac{1}{8\pi r^2}$. In a similar way, it may be proved that ν, or $\phi = -\frac{1}{2}(\nu + \lambda)$, is also a well-behaved function despite [reasonable] modifications in the stress-momentum tensor.

Thus, whether we consider thermal radiation or not, the study of the temporal evolution of a spherically symmetric gravitational collapse in spherical polar coordinates does not lead to incoherences, but constitutes a sensible alternative to the usual black hole model. In addition, when thermal radiation is considered, very high (but finite) values of λ are obtained at any given r. Definitely, the radiation law proposed in this paper has been deduced in a rather "heuristic" way by assuming the extensibility of the calculations detailed in Ref. [12] to a model of *scarcely interacting* collapsing shells. Certainly, in the original paper by Vachaspati *et al.* the emission of radiation was calculated from a spherical Nambu-Goto domain wall using the functional Schrödinger formalism, with vacuum close to the wall. Therefore, our *analytical* extension of their results to "inner shells" may be cautiously considered, but it is a reasonable hypothesis, specially having into account Birkhoff theorem (according to which, in a system with spherical symmetry, the gravity in a surface is basically determined by the mass of the matter contained in the inner, not outer, shells). As a matter of fact, it is a much more consistent assumption than some of those that may be found in the published works, as the use of a *strictu sensu* Hawking radiation in a *process* of gravitational collapse (as, for instance, in Ref. [28]), as Hawking radiation implies (essentialy, not just formally) a transition from vacuum, and in truth a collapsing star is not void.

On the other hand, even if the genuine radiation law appeared to be completely different, it would still be true that an asymptotic approach to a "stationary" phase (where the value of λ would stop increasing) should happen. In fact, this phase should be always reached just by assuming the reasonable hypothesis that the outgoing flux of thermal radiation should not diminish with time (the temperature of the collapsing body should be expected to rise with the progression of collapse), while the ingoing flux of collapsing matter should become smaller as the spacetime deformation becomes larger.

In summary, even when several of the assumptions of the model of gravitational collapse proposed in this paper may be considered excessively "idealistic", it provides an illustrative description of how a time-dependent metric should be the most logical choice for the study of gravitational collapse and that the polar spherical coordinates of an asymptotic observer (a scientific on the Earth, not an astronaut falling into a black hole) are sufficient to cover the whole collapsing process. The supposed completion of the collapsing process in a finite proper time for a co-mobile observer would never be truly accomplished due to the invariance of causal order for any relativistic system (in a finite and lesser proper time, the co-mobile observer would be fully evaporated by the emission of thermal radiation). The astronomic objects already *identified* as "black holes" could equally correspond to "asymptotically collapsing bodies". Empirically, few differences would be expected. From a theoretical point of view, the latter ones may be obtained in a very natural way from the Einstein field equations and avoid many of the paradoxes and illogical aspects of the former ones. Thus, according to Occam's razor, asymptotic collapse should be preferred to black holes.

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