Type Ia Supernovae Progenitor Problem and the Variation of Fundamental Constants

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Cosmological observations strongly suggest our universe is the interior of an expanding black hole. If the constant mass of the universe is assumed then from the equation for Schwarzschild radius: $r_S = 2Gmc^{-2}$ it follows that proportionality constant Gc^{-2} depends linearly on the universe's radius R_u, identified with r_S, i.e. $Gc^{-2} \sim R_u$, $M_u = const$. Because the Chandrasekhar limit M_{Ch} relates to the speed of light and to the Newton's constant as $M_{Ch} \sim (c/G)^{3/2}$ so expansion involves gradual decrease of M_{Ch}. In result, a single white dwarf can alone become the Type Ia supernova progenitor, which provides a complementary solution to single-degenerate and double-degenerate models for SNe Ia. Both alternative scenarios: $G \sim R_u$ and $c \sim R_u^{-1/2}$ are analyzed in regard of their consistence with observations, and their consequences to cosmology.

1 Introduction

On account of the supposed uniformity of their absolute magnitude, the Type Ia supernovae (SNe Ia) play an important role of "standard candles" in cosmology. A tight correlation between the peak light output and the light-curve width (width-luminosity relation) results from the way SNe Ia originate from white dwarfs (WDs) — the final remnants for low and medium mass stars. According to the current understanding, the carbon-oxygen (CO) thermonuclear fusion triggering the supernova explosion takes place in compact binary systems in either of two principal progenitor channels. A single-degenerate (SD) model (Whelan & Iben [80]) predicts that CO WD accretes matter from the companion, usually the red giant or the main sequence star. Just before approaching the Chandrasekhar mass-limit $M_{Ch} \approx 1.44 M_{\odot}$ for which electron degeneracy pressure becomes insufficient to prevent the gravitational collapse, the WD's core reaches the ignition temperature for the runaway carbon and oxygen fusion into heavier elements. In a preceding time lasting usually $\sim 10^6$ yr WD processes the transferred matter falling onto its surface through the accretion disc. In this phase, called "nuclear-burning white dwarf" (NBWD) the hydrogen-helium fusion releases energy in a form of copious X-radiation, observed as "super-soft X-ray source" (Di Stefano [17]).

Instead, the double-degenerate (DD) model (Webbink [79], Iben & Tutukov [37]) predicts that two WDs of the combined mass $\geqslant M_{Ch}$ form a compact binary system and subsequently spiral towards each other in a common envelope. Eventually, they collide and merge and, after exceeding the Chandrasekhar limit, explode as SN Ia. Unlike in accrete scenario the merging WDs are not expected to be the source of X-radiation until a short time preceding the supernova explosion. The X-ray signatures of SD and DD channels differ significantly, which makes them easy to distinguish. The DD model admits a broader range of progenitor mass and SNe Ia luminosity; thus is thought to be responsible for the non-standard SNe Ia explosions.

These two basic models (hereinafter collectively referred to as "SNe Ia binary paradigm") do not however provide a fair explanation to the diversity in the observed characteristics of SNe Ia and the paucity of their potential progenitors. The relevant SNe Ia progenitor problem amounts to the following two items. First is the problem of SNe Ia rate: the total number of potential progenitors seems to be inadequate to the number of observed SNe Ia events. Second is the problem of SNe Ia properties: the observed light-curves and remnants spectra do not match satisfactorily the detailed predictions of SD and DD models.

Our goal here is to provide a solution to the progenitor problem based on assumption of the varying Chandrasekhar mass, a consequence of varying constant Gc^{-2}. It's not been a century yet since one realized our universe has a turbulent history behind and some kind of final fate ahead. Compared with the prior model of eternal and basically invariable universe, this forms quite different ground for thinking about physical fundamental constants. One cannot ascribe logical necessity to any of fundamental constants (class C "universal" constants, according to Uzan's nomenclature(Uzan [74]) as e.g. in the case of mathematical constant π or the Euler's number e. Likewise, one cannot obtain them by pure deduction in a way similar to that Eddington tried (ineffectively) to do with the fine structure constant alpha. For the time being, they work as "free parameters". Hence, still valid is Dirac's opinion: "It is usually assumed that the laws of nature have always been the same as they are now. There is no justification for this. The laws may be changing, and in particular quantities which are considered to be constants of nature may be changing with cosmological time" (Dirac [16]). Let us complement this opinion with another one: "Ignoring the possibility of varying constants could lead to a distorted view of our universe and if such a variation is established corrections would have to be applied" (Uzan [74]).

2 The SNe Ia progenitor problem: a brief overview

The question of identity of Type Ia supernovae progenitors is widely considered as the "major unsolved problem in astrophysics" (Maoz & Mannucci [47]). The main problem is the discrepancy between the observed SNe Ia rate and the number of potential progenitors. Taking into account the estimated rate of SNe Ia ($\sim (10^{-3} - 10^{-2})\text{yr}^{-1}$ events in a typical spiral or elliptical galaxy) and the mean/median delay time for the SNe Ia progenitors ($\sim$ (0.5 – 1) Gyr for DD channel and $\sim$ (2 – 3) Gyr for SD channel), X-ray sources should manifest in thousands in any such galaxy including the Milky Way. Meanwhile, the X-ray flux from the sample of six neighboring spiral galaxies obtained from Chandra X-ray Observatory is a factor of 30-50 times fainter than expected (Gilfanov & Bogdan [29]). In some of SNe Ia previously thought to originate in SD channel no remnants of red giant has been observed (Schaeffer & Pagnotta [70], Li et al. [42], Nugent et al. [56]). Generally, in most cases red giants have been excluded as possible ex-companions in binaries. The discrepancy between the observed amount of X-ray sources and the assessed numbers of SNe Ia led to conclusion that accrete scenario is not a primary route to supernovae, giving priority to the merger scenario. Gonzalez Hernandez et al. [30] estimate that fewer than 20% of SNe Ia is produced in SD channel. Gilfanov & Bogdan [29] opt for even more stringent limit $\leqslant$ 5% of total population. Di Stefano [17] indicates the lack of 90% – 99% of the required number of X-ray sources. She argues (Di Stefano [18]) that companion stars forming the double degenerates do not age at the same rate and thus do not become WDs at the same time; for that reason the common envelope phase should be preceded by a symbiotic pre-double-degenerate phase with the hydrogen-helium fusion similar to NBWD. Thus, merger channel should also produce X-ray flux comparable to the accrete channel prior to the common envelope phase, which puts into doubt DD model as an effective explanation.

A vital problem is the paucity of the observed white dwarfs mergers. According to Gilfanov [28] "…too few double-white-dwarf systems appeared to exist". One expects the ESO Supernovae Type Ia Progenitor Survey (SPY) (Napiwotzki et al. [54, 55]) and the ongoing Sloan White dwArf Radial velocity data Mining Survey (SWARMS) (Badenes et al. [3], Mullally et al. [53]) to provide evidences for the merger channel (DD) as the main route to SNe Ia. Badenes & Maoz [4] using Doppler techniques isolated 15 WD binaries from a sample of $\approx 4,000$ WDs brought by Sloan Digital Sky Survey (SDSS). They compared the rate of WD binaries with the rate of SNe Ia in the Milky Way-like Sbc galaxies and found a "remarkable agreement" between them. However, a majority of these WD binaries appeared to be sub-Chandrasekhar, although usually with total mass relatively close to M_{Ch} $(1.1 - 1.2M_{\odot})$.

Some of researches (Hachisu et al. [34], Van Kerkwijk et al. [40], Zhu et al. [82], Maoz & Mannucci [47]) claim that the requirement as to the total mass of merging CO WDs (i.e $1.4M_{\odot}$) is too restrictive. This would match observations of super-Chandra WD progenitor stars with the combined mass reaching $2.4 - 2.8M_{\odot}$. According to the respective models, the observed number of SNe Ia can be explained provided the wider range of combined mass: smaller than M_{Ch} (sub-M_{Ch} merger channel) or bigger than M_{Ch} (super-M_{Ch} merger channel), dependently on detailed conditions such as rotation, magnetic fields, metallicity and the host galaxy population. This would account for better agreement with observations, both as to the rate of SNe Ia and to the differences in their properties. The controversial point of these models is that they require special fine-tuning to be effective. Maoz & Mannucci [48] attribute some of discrepancies as caused by "deadly sins", i.e. incorrect or inadequate methods in measuring and analyzing the SNe Ia rates. They admit however the "detailed models still falls short of the observed number (of SNe Ia) by at least factor of a few".

Di Stefano [18] suggests that, possibly, only a small fraction of accreting WDs can be detected and identified as X-ray sources. This may occur by two reasons: either the winds from a companion giant reprocess the supersoft X-ray radiation into the radiation of longer wavelengths, or the duty cycle of nuclear burning is to low to be detected. However, neither of these solutions has been properly recognized and confirmed as yet. Another proposal (Di Stefano et al. [19]) links the mass of progenitor with the angular momentum gained from the donor star together with matter. The angular momentum prevents the super-M_{Ch} WD from collapse, which widens the potential range of SNe Ia progenitors. The relevant "spin-up/spin-down" models predict the existence of numerous WD "ticking bombs" waiting to explode until their rotation slows down to a proper level.

There is a broad agreement (e.g. Totani et al. [73], Maoz et al. [48], Mennekens et al. [50], Hachisu et al. [33]) as to the key role of "delay time distribution" (DTD) — the number of SNe Ia events in unit time as a function of time elapsed since starburst, in predicting the SNe Ia rates. It seems that DTD (indicated as t^{-1} power law) favors the DD scenario. Hachisu et al. [33] found a good agreement of DTD with SD model either, provided the donor stars are both red giants and the main-sequence stars. Undoubtedly, DTD introduces an indispensible methodological order to the SNe Ia progenitor problem. In general however, regarding DTD did not bring a decisive breakthrough so far in the question of identity of SNe Ia progenitors.

It has gradually become evident that SNe Ia are not "standard candles" in the originally attributed sense. Their intrinsic luminosity is neither considered nor demanded to be exactly uniform, which gives priority to the more "capacious" merger channel. Instead of standard candles, SNe Ia are currently interpreted as "standarizable candles", which means that utilizing them as the correct distance indicators requires due calibration. This in turn demands better recognizing of their origin and nature. The study by Linden, Virey & Tilquin [43] revealed a likely positive correlation between the SNe Ia absolute brightness and distance, which may put in question the actually determined cosmological parameters. The observed relationship between the intrinsic color and ejecta velocity may help in reducing systematic biases in the estimates of distance (Foley et al. [25]). Instead, Sullivan et al. [72] point to the relationship between the luminosity of SNe Ia and metallicity of their hosts, while metallicity is supposed to depend on redshift. Gallagher et al. [26] comparing the spectra of a sample of 29 early elliptical galaxies of the age exceeding 5 Gyr with the general sample from SDSS including younger galaxies, find a strong correlation between the absolute magnitude of SNe Ia and the age of host galaxies while, most likely, "... the observed trend with metallicity is merely an artifact brought about the evolutionary entanglement of age and metallicity". These findings may help in recognizing the properties of SNe Ia, which is particularly important for the question of dark energy and the relevant accelerating expansion of the universe (Riess et al. [66], Perlmutter et al. [60]). The supposed correlation between the absolute magnitude and distance suggests the presence of a time dependent factor in the effective SNe Ia progenitor model.

3 Varying Chandrasekhar limit as the postulated main route to SNe Ia

The mass-limit formula for white dwarfs based on the equation of state for ideal Fermi gas (Chandrasekhar [11]) reads

$$M_{Ch} = 4\pi \left(\frac{K_2}{\pi G}\right)^{3/2} \omega_3^0 \,, \tag{1}$$

where ω_3^0 is the numerical constant equal to 2.018, derived from the explicit solution of the Lane-Emden equation for the polytropic index $n = 3$. The constant K in the general case connects pressure and density: $P = K\rho^{(n+1)/n}$ while in the case including white dwarfs (i.e. for $n = 3$) becomes specified as $P = K_2\rho^{4/3}$. Since K_2 is defined as

$$K_2 = \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} \frac{hc}{(\mu_e m_H)^{4/3}} \tag{2}$$

(μ_e-mean molecular mass per electron, m_H-mass of hydrogen atom), so substituting gives

$$M_{Ch} = 4\pi \left[\frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} \frac{hc}{(\mu_e m_H)^{4/3}\pi G}\right]^{3/2} \omega_3^0 \,. \tag{3}$$

Collecting the pure numbers, and considering that $\hbar = h/2\pi$, one gets

$$M_{Ch} \approx 1.11065 \times 10^{54} \mu_e^{-2} m_P^3 \,, \tag{4}$$

where $m_P = (\hbar c/G)^{1/2}$ is the Planck mass. Since CO WDs are mainly composed of carbon-12 and oxygen-16, and because in both cases atomic number equal to half the atomic weight so one has $\mu_e = 2$, leading to $M_{Ch} \approx 1.44 M_{\odot}$. It is important that Chandrasekhar mass it is proportional to the cube of Planck mass:

$$M_{Ch} \sim m_P^3 \,. \tag{5}$$

Assuming $\hbar = const$ it relates to the speed of light and to the Newton's gravitational constant as

$$M_{Ch} \sim (c/G)^{3/2}. \tag{6}$$

(We use tilde for linear dependence in the cases when the variability of a reference quantity [here: c and G] is hypothetical. Instead, the symbol of proportionality [exact or approximate] $\propto$ is used when variation of a reference quantity is obvious or certain, e.g. cosmic time t or radius of universe R_u).

From this relationship it follows that any cosmological model postulating varying G or/and c (except the case they change accordingly) implies the postulate of varying M_{Ch}. This fact has not been properly explored so far. What we propose here is the "varying Chandrasekhar mass-limit" model (VCM) in which M_{Ch} decreases in cosmic time. VCM postulates that the currently known value of Chandrasekhar limit refers solely to the present epoch while in general:

$$M_{Ch}(\text{past}) > 1.44 M_{\odot} > M_{Ch}(\text{future}). \tag{7}$$

This determines a scenario for the single WD progenitors of SNe Ia, which can be outlined as follows. Once an individual WD is formed, it keeps its mass approximately constant during the cooling process while the Chandrasekhar limit gradually decreases in time. Eventually, it equates or approaches a given WD's mass triggering the SN Ia explosion. From a logical point of view, an effect of SN Ia caused by decreasing M_{Ch} reminds bringing water to a boil by reducing the atmospheric pressure without supplying heat. Hence, single WDs are, along with binary WDs, the potential progenitors of SNe Ia.

4 Varying constants and the black-hole cosmology

Varying Chandrasekhar limit, as a hypothesis based on assumption of varying constants c or/and G is closely related to the black-hole cosmology. A constitutive observation of the respective models is the coincidence between the radius of observable universe and the Schwarzschild radius, supposed to be valid over the whole course of the universe's history. According to a hypothesis advanced by Pathria [58] and Good [31], the universe is the interior of a black hole existing, among many others, within a larger structure called multiverse.

The recent multiverse model by Popławski [64, 65] uses the Einstein-Cartan-Sciama-Kibble theory removing from General Relativity the constraint of symmetry in the affine connection, and regarding the antisymmetric variable torsion tensor in the Friedmann equations. The relevant cosmological scenario takes an advantage of the fact that most stars have a non-zero angular momentum. When a massive rotating star collapses to a (Kerr) black hole, the torsion of extremely dense matter inside the horizon prevents from the point singularity (replaced by the ring singularity). As a result, the black hole becomes a wormhole to another universe thought to originate in "big bounce". As far as our own universe is the interior of a black hole existing in another universe, any black hole in our universe is thought to contain (produce) a separate universe. The new universe is interpreted as a "white hole" — a time reversal black hole whose expansion, e.g. such as observed in our universe is driven by the torsion, identified with dark energy. This model predicts the presence of traces of primordial torsion in a form of slight anisotropies in both cosmic and nanoscopic scales. Some reported evidences of the preferred handedness of spiral galaxies (dipole asymmetry of the value 0.0408 ± 0.011 based on SDSS data sample containing 15,158 spiral galaxies with the redshift < 0.085) seem to support the idea of cosmic parity violation (Longo [44]). However, the area covered by this sample is still too small to derive unambiguous conclusions. According to Neta Bahcall "The directional spin of spiral galaxies may be impacted by other local gravitational effects".

Besides, even if the filaments forming the cosmic web are uniformly distributed, anisotropy connected with rotation will break the homogeneity in a deeper sense. In the isotropic cosmic space, the "center" is a purely relative concept connected with the notion of observable universe. But it is no longer relative in the anisotropic space with the fixed axis of rotation. The spinning universe implies, besides anisotropy, the presence of preferred points. We may think about analogies between directional spin of spiral galaxies and the Coriolis effects on the Earth, e.g. manifesting itself in different spin of hurricanes in north and south hemispheres. Anyway, the question of spinning universe is, in the end, a matter of (further) observations.

The model here proposed (VCM) bases on formal resemblance of our universe with a black hole (and thus we shall use the Schwarzschild equation for radius) yet does not settle whether the universe is a black hole in the literal sense. It seems instead that crucial property of the universe conceived as the interior of a black hole is that its total energy amounts to zero. In this regard, the black-hole cosmologies are close to the "zero-energy universe" theories.

The legitimacy for interpreting the universe in terms of a black hole depends on its parameters, in particular size, density and mass. Recent estimations concerning the radius of observable universe point to the value $\geqslant 14$ Gpc (4.3×10^{26} m) or 28 Gpc in diameter. Cornish et al. [12] analyzing the WMAP data in search of the matched back-to-back circles predicted by various nontrivial topologies, settled the low bound of diameter of the last scattering surface of fundamental domain for 24 Gpc. Bielewicz & Banday [6], using similar methods extended this value to 27.9 Gpc. This admittedly does not prejudge the question of size, yet, provided the multi-connected space of universe, constraints the topology scale from below. An additional (though partly linked) difficulty comes out from the potential difference between the notions of entire and observable universe. In principle, entire universe may significantly surpass the observable universe (as inflationary theory predicts), but it can be as well slightly smaller due to nontrivial topology. The respective ratio may also change in time. Presumably, the black hole parameters describe the entire universe, and not just the universe currently observed. However, this distinction becomes important only insofar as "entire", by virtue of convention, denotes the biggest physically connected object defined according to the horizon problem of the early universe. Assuming the approximately linear rate of expansion after the end of inflationary epoch (or from the beginning), the parameters of the so defined "entirety" should not significantly differ from the "observable" parameters. Bearing in mind the obvious uncertainties, we shall use in calculations the value $10^{27} m$ for the universe's radius.

The critical density for a flat universe derived from Friedmann equation for the Hubble constant obtained from Planck telescope: $H_0 = 67.15\,\mathrm{kms^{-1}Mpc^{-1}}$ is $\rho_c = 3H^2/8\pi G \approx 0.85 \times 10^{-26}\,\mathrm{kgm^{-3}}$. The resultant total mass for $R_u = 10^{27}$ m amounts to $M_u \approx 1.44 \times 10^{54}$kg (we shall use 10^{54}kg in calculations). Considering the approximated values of gravitational constant: $G \approx 6.7 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$ and the speed of light: $c \approx 3 \times 10^8\,\mathrm{ms^{-1}}$ ($c^2 \approx 10^{17}\,\mathrm{m^2s^{-2}}$) one obtains the numerical relationship connecting radius and mass:

$$10^{27} = 10^{-10}10^{54}10^{-17}\,(\mathrm{m}), \tag{8}$$

which means that equation for the Schwarzschild radius:

$$r_S = 2Gmc^{-2} \tag{9}$$

apparently applies to the universe as

$$R_u \approx GM_u c^{-2}. \tag{10}$$

We postulate that universe constantly fulfills the "black-hole condition" (BHC), which means that it is always fulfilled:

$$R_u \equiv r_S\,. \tag{11}$$

Together with assumption $M_u = const$, and the general assumption of isotropy of cosmic space, BHC implies

$$Gc^{-2} \propto R_u\,. \tag{12}$$

5 Models with varying constants

In the intensive discussion on the variability of fundamental constants, variation of c is probably the leading topic. A majority of the "variable speed of light" (VSL) models conceived as a challenge to inflation restricts the variation of c

to the early superluminary universe (Moffat [51], Albrecht & Magueijo [2], Magueijo & Smolin [45]). These models do not match BHC since, after restoring the local Lorentz invariance, the light is thought to travel at the presently measured speed. Likewise, assuming the change of c refers totally to the time preceding the structure formation, they would not imply the variability of M_{Ch}.

In some VSL models, the change in c value has been considered as a continuous process spread over the whole lifespan of the universe. Dicke's theory of gravity (Dicke [13]), developing the earlier considerations by Einstein [22, 23] explains the cosmological redshift as a result of c decreasing with time, which somehow corresponds with the steady state theory. However, this model does not predict the change of c to be a measurable effect since it assumes the units of length and time to change accordingly.

In turn, variability of G has been proposed in some scalar-tensor models modifying the Einstein's General Relativity, in particular the Brans-Dicke theory [9] inspired by Mach's principle, with the time and space dependent scalar field ϕ modifying the Newton's constant. A similar as to the general structure and conclusions model by Hoyle & Narlikar [36] originates from considerations concerning the action on distance. Petit [61, 62] advanced a model with joint variation of G, c and h decoding the Hubble's law in a static universe. One of the first models postulating varying G, and likely the most influential one, is the Dirac's "large number hypothesis" (Dirac [15]). From the supposed coincidence between two ratios: radius of universe (expressed as ct) vs. radius of electron, and electrostatic force vs. gravitational force between proton and electron (both of them yielding $\approx 10^{40}$), Dirac derived a conclusion that G changes as the inverse of cosmic time: $G \propto t^{-1}$, while the mass of universe increases as $M_u \propto t^2$. Provided the approximately linear relationship between time and radius ($R_u \propto t$), LNH satisfies BHC. However, LNH also implies $M_{Ch} \propto t^{3/2}$, which compared with the standard assumption of constant G makes the SNe Ia progenitor problem even more puzzling. A model proposed by the present author (Rybicki [69]) has postulated $G \propto R_u$, $M_u = const$, yet then with no reference to BHC and the SNe Ia progenitor problem.

A question underlying the varying constants models is whether the postulated changes in dimensional constants are physically meaningful. A long-lasting controversy over this subject has not been concluded so far. Some physicists (e.g. Barrow [5], Duff [20]) claim that only the (potential) change in dimensionless constants matters, e.g. the coupling constants of fundamental forces such as fine structure constant α, gravitational coupling constant α_G , or the masses of elementary particles related to Planck mass contributing to standard model. Instead, dimensional constants such as $\hbar$, c, G, e, or k may change in value dependently on the (arbitral) choice of units, thus being merely the "human constructs" or "conversion factors". Others (Okun [57], Veneziano [76]) consider as indispensable in shaping the fundamental theories respectively three (G, c and $\hbar$) and two (c and string length λ_s) dimensional constants.

From the "dimensionless" point of view as applied to BHC, no matter whichever of dimensional constants is thought to vary; only what counts is the change of $\alpha_G = Gm_e^2/\hbar c$. Since we discriminate here between the change of G and c treated as different solutions of BHC, so this question demands a clarifying comment. Let's start with two remarks: 1) There is no doubt that $Gc^{-2} \propto R_u$ implies the variability of α_G; 2) The fact that dimensional constant changes its numerical value together with the change of unit is trivial, and as such contributes nothing to discussion.

Let the increase of α_G be observed, correlated with the increase of R_u. Assuming $m_e = const$, $\hbar = const$, we conclude that it is either $G \propto R_u$ or $c \propto R_u^{-1}$ which, according to the "dimensionless" paradigm, we treat as fully equivalent (i.e. physically indistinguishable) interpretations of $\alpha_G \propto R_u$. However, from $Gc^{-2} \propto R_u$ it follows: $G \propto R_u \Rightarrow \alpha_G \propto R_u$, and $c \propto R_u^{-1/2} \Rightarrow \alpha_G \propto R_u^{1/2}$, which obviously differs from $\alpha_G \propto R_u$. Thus, G and c cannot be considered as "conversion factors" within BHC.

As we show in next sections, the Planck units of length and time react differently depending on whether G or c is postulated to vary. Besides, each of respective solutions affects entropy in a different way. We thus agree with the anonymous referee cited in Duff's paper: "It is true that if the fundamental "constants" $\hbar$, c, G, k ... are truly constant, then they do indeed only act as conversion factors and can e.g. be set equal to unity. However, when they are postulated (or discovered experimentally to vary) in time, then we have to take into account that varying one or the other of these constants can have significant consequences for physics" (Duff [20]).

6 Basics of the VCM hypothesis

Expressed in the here proposed nomenclature, our main idea consists in postulating VCM as being the consequence of BHC. Any model satisfying BHC makes the Planck units variable, and thus determines new parameters of the Planck era.

Identifying the mass in the equation for Schwarzschild radius with Planck mass: $m \equiv m_P$ gives

$$r_S = Gm_P c^{-2} = G(\hbar c G^{-1})^{1/2} c^{-2} = (\hbar G c^{-3})^{1/2} = \ell_P\,. \quad (13)$$

Accordingly, the black hole becomes the Planck particle. Implementing the Planck mass to the reduced Compton wavelength $\lambda/2\pi = \hbar m^{-1} c^{-1}$ makes the Planck particle the only one black hole whose Schwarzschild radius equals the Compton wavelength

$$\lambda/2\pi = \hbar(G\hbar^{-1}c^{-1})^{1/2}c^{-1} \equiv (\hbar G c^{-3})^{1/2} = \ell_P\,. \quad (14)$$

Rewriting the Schwarzschild equation for the Planck particle:

$\ell_p \approx Gm_p c^{-2}$ gives the identity

$$(\hbar G c^{-3})^{1/2} \equiv G(\hbar c G^{-1})^{1/2} c^{-2}, \tag{15}$$

which means that Planck particle's property of being a black hole is insensible to the change of G or/and c.

From $Gc^{-2} \propto R_u$ it follows $m_P \propto R_u^{-1/2}$; hence for $R_u \to 0$ the Planck mass tends to infinity. However, to avoid singularities (and also taking into account that Planck mass should have "realistic" reference), we assume that in the newly defined Planck era (denoted P_0) the Planck mass coincidences with the mass of universe:

$$m_{P_0} \equiv M_u\,. \tag{16}$$

Thus, the initial value of Schwarzschild radius becomes

$$r_{S_0} \approx GM_u c^{-2}. \tag{17}$$

This can be also obtained by expressing the Newton's constant in the equation $R_u = GM_u c^{-2}$ in terms of Planck units, namely: $G = \ell_P m_P^{-1} c^2$. Then

$$R_u = \ell_P M_u m_P^{-1} \tag{18}$$

and so

$$R_u \ell_P^{-1} = M_u m_P^{-1} \tag{19}$$

meaning that identity $R_{u_0} \equiv \ell_{P_0}$ becomes a consequence of the conjecture $M_u m_{P_0}^{-1} = 1$.We have thus arrived at conclusion that the universe at its initial stage (here called "primordial Planck era" — PPE) had the form of a quantum mechanical black hole identified with a single one "primordial Planck particle" (PPP), described by equation:

$$\ell_{P_0} = G_0 m_{P_0} c_0^{-2}\,. \tag{20}$$

Accordingly, the notion of PPP becomes coherent with the concept of the universe emerging from "nothing" due to the Heisenberg uncertainty.

From $M_u \approx 10^{54} kg$, provided $m_{P_0} \equiv M_u$, it follows

$$m_{P_0} m_P^{-1} \approx 10^{62} \tag{21}$$

a factor hereinafter denoted by δ.

Because $M_{Ch} \sim m_P^3$ so

$$M_{Ch_0} M_{Ch}^{-1} = \delta^3. \tag{22}$$

Obviously, M_{Ch_0} as related to the early universe, is a formal entity only. To be a physically meaningful concept, Chandrasekhar limit demands a proper physical "enviroment" (atoms, elements, stars). It belongs then to the epoch of structure formation starting from Population III stars. Provided the universe expanded in a roughly uniform rate, BHC can be expressed as the approximate function of cosmic time: $Gc^{-2} \propto t$. From the whole range of possible BHC scenarios, the two deserve special attention, namely: 1) $G \propto R_u$ i.e. $G \propto t$, $c = const$, and 2) $c \propto R_u^{-1/2}$, $G = const$, both analyzed in the next sections.

7 Assumption $c \propto R_u^{-1/2}$, $G = const$: collision with the second law of thermodynamics

The initial value of speed of light derived from $m_{P_0} = (\hbar c_0/G)^{-1/2}$ and $m_{P_0} \equiv M_u \approx 10^{54}$ kg becomes $c_0 = 10^{132}$ ms^{-1}, yielding $c_0/c \approx 10^{124} = \delta^2$. The respective Planck length is (hereinafter, SI units always when omitted)

$$\ell_{P_0} = (\hbar G/c_0^3)^{-1/2} \approx 10^{-220} \tag{23}$$

a value equal to the Schwarzschild radius

$$r_S = Gm_{P_0}/c_0^2 \approx 10^{-220} \tag{24}$$

and to the Compton wavelength

$$\lambda_0 = \hbar M_u^{-1} c_0^{-1} \approx 10^{-220}. \tag{25}$$

The initial Planck time would amount to

$$t_{P_0} = (\hbar G/c^5)^{-1/2} \approx 10^{-352}. \tag{26}$$

From $E = m_{P_0} c_0^2$ it follows

$$\hbar = Et_{P_0}(10^{-34} = 10^{318} 10^{-352}). \tag{27}$$

As derived from $c \propto t^{-1/2}$, with the age of universe $\approx 13.8 \times 10^9$ yr the current rate of decrease in the speed of light becomes

$$\dot{c}/c \approx -2.7 \times 10^{-11} \text{yr}^{-1}. \tag{28}$$

Let us compare this prediction with the results obtained from observations of gas clouds spectra intersecting the distant quasars, the Oklo natural uranium fission reactor, and atomic clocks. In agreement with the VSL paradigm, the supposed change of α is usually interpreted as the change of c. For the approximate emission time connected with the observational data samples concerning quasars: $t_{EM} \approx 0.25\, t_0/0.85\, t_0$ covering ≈ 8.3 Gyr (here t_0 stands for the present moment), the reported values suggesting the change are: $\Delta c(t)/c = (-0.57 \pm 0.10) \times 10^{-5}$ (Webb et al. [77]), and $\Delta c(t)/c = (-1.09 \pm 0.17) \times 10^{-5}$ (Webb et al. [78]). At the same time, other groups (e.g. Chand et al. [10]) reported no detectable change in α value over the last 10-12 billion years. In the case of Oklo, for the respective operating time $t_{prev}/t_0 \approx 0.87$, Petrov et al. [63] obtained $\dot{\alpha}/\alpha = (-4+3) \times 10^{-17}$yr^{-1}, in fact signifying no detectable change. In turn, Lamoreaux & Torgerson [41] reported a decrease in alpha at the level -4.5×10^{-8} over the last 2 billion years, which consequently should be interpreted as the increase of the speed of light. Observations based on atomic clocks give a direct insight to the possible current rate of change. Peik et al. [59], using cesium atomic clock set the limit of annual change of the present variation of alpha for $\dot{\alpha}/\alpha = (-1.2 \pm 4.4) \times 10^{-15}$yr^{-1}. In turn, Rosenband et al. [67], based on the frequency ratio of Al^+ and Hg^+ in a single ion atomic clocks obtained a bound: $\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17}$yr^{-1}.

Except the data provided by Webb et al. suggesting decrease of c at the level $10^{-15}\mathrm{yr}^{-1}$, and the opposite one (as to general conclusion) provided by Lamoreaux & Torgerson, all other results seem to point the zero change. This suggests the failure of assumption $c \propto R_u^{-1/2}$. Besides, the question of entropy provides us with an additional argument against declining c. As is known, entropy is proportional to the horizon surface area, which normally (i.e. by assumption $G = const$, $c = const$) implies linear dependence on the squared mass. Let us apply the Bekenstein-Hawking formula for the entropy of black hole:

$$S_{BH} = Akc^3(4G\hbar)^{-1} \tag{29}$$

or, written in terms of Planck length,

$$S_{BH} = Ak(4\ell_P^2)^{-1}, \tag{30}$$

where A is the surface area for event horizon, and k the Boltzmann constant. For the spherically symmetrical black hole, the surface area is $A = m^2 8\pi G^2 c^{-2}$ so entropy becomes $S_{BH} = m^2 2\pi Gkc\hbar^{-1}$. Thus, despite increasing surface area $A = m^2 8\pi G^2 c^{-2}$, at the assumption $m \equiv M = const$, $G = const$ and $c \propto R_u^{-1/2}$, the entropy decreases according to $S_{BH} = m^2 2\pi Gkc\hbar^{-1}$ being dependent on the decreasing speed of light: $S_{BH} \sim c$. One obtains therefore

$$S_{BH(present)}/S_{BH(primordial)} = \delta^{-2}, \tag{31}$$

which violates the second law of thermodynamics applied to the universe as a whole. This does not exclude VSL models in general; in particular, does not exclude VSL applied to the very early universe. However, BHC is not agreeable with VSL conceived as a continuous process. Therefore, in the further considerations, we shall specify BHC as a model defined by the assumption $G \propto R_u$, $c = const$. We shall also treat this model as a right basis for the VCM hypothesis and the respective quantitative predictions.

8 Assumption $G \propto R_u$, $c = const$: parameters of the universe at Planck era

Provided $m_{P_0} \equiv M_u \approx 10^{54}$ kg , the initial value of Newton's constant derived from $m_{P_0} = (\hbar c/G_0)^{-1/2}$ is $G_0 \approx 10^{-134}$, yielding $G/G_0 = \delta^2$. The initial Planck length becomes

$$\ell_{P_0} = (\hbar G_0/c^3)^{-1/2} \approx 10^{-97} \tag{32}$$

equal to the Schwarzschild radius:

$$r_S = G_0 M_u c^{-2} \approx 10^{-97} \tag{33}$$

and to the (constant) value of Compton wavelength for the universe:

$$\lambda_0 = \hbar M_u^{-1} c^{-1} \approx 10^{-97}. \tag{34}$$

All three quantities apply to the initial size of universe R_{u_0}:

$$R_{u_0} \equiv \ell_{P_0} \equiv r_S \equiv \lambda_0\,. \tag{35}$$

The initial Planck time is

$$t_{P_0} = (\hbar G_0 c^{-5})^{1/2} \approx 10^{-105}. \tag{36}$$

Hence,

$$\hbar = E_{P_0} t_{P_0} \approx 10^{-34}, \tag{37}$$

where $E_{P_0} = M_u c^2 \approx 10^{71}$. The invariability of Planck constant is a consequence of the fact that, although individually Planck energy and Planck time change in time, their product remains constant:

$$E_{P(variable)} \times t_{P(variable)} = \hbar_{(\mathrm{constant})}\,. \tag{38}$$

In general, initial values of the base Planck units relate to their present equivalents as

$$m_{P_0}/m_P = \ell_P/\ell_{P_0} = t_P/t_{P_0} = \delta\,. \tag{39}$$

The horizon problem in PPE is solved so to speak by definition, since

$$ct_{P_0} = \ell_{P_0}\,, \tag{40}$$

which means that the whole primordial universe fits in a light cone.

The density in the primordial Planck era is

$$\rho_{(PPE)} = M_u \ell_{P_0}^{-3} \approx 10^{344} \tag{41}$$

equal to initial Planck density:

$$\rho_{P_0} = c^5 \hbar^{-1} G^{-2} \approx 10^{344}. \tag{42}$$

Let us compare this with the critical density derived from the Friedmann equation: $\rho_c = 3H^2(8\pi G)^{-1}$, as calculated for PPE. The current value of Hubble constant ($\approx 70\,\mathrm{kms}^{-1}/\mathrm{Mpc}$) expressed in SI units amounts to

$$H_{(now)} \approx 2.27 \times 10^{-18}\ \mathrm{s}^{-1} \tag{43}$$

yielding the respective value of the Hubble constant in PPE:

$$H_{(PPE)} = H_{(now)} \times \delta^2 \approx 10^{106}\ \mathrm{s}^{-1}. \tag{44}$$

Approximating $8\pi G_0 \approx 10^{-133}$, one obtains the PPE critical density:

$$\rho_{c(PPE)} \approx 10^{212} 10^{133} \approx 10^{345}. \tag{45}$$

Hence, it is likely that also in PPE

$$\rho_{(PPE)} \equiv \rho_c \tag{46}$$

which solves the flatness problem.

In contrast to the previously considered assumption $c \propto R_u^{-1/2}$, $G = const$, the thermodynamic arrow of time becomes well defined. Considering $G/G_0 = \delta^2$, from $S_{BH} = m^2 2\pi Gkc\hbar^{-1}$ it follows

$$S_{BH(present)}/S_{BH(primodial)} = \delta^2. \tag{47}$$

In the cosmological scenario based on assumption $G \propto R_u$, $c = const$, the expansion is linear, or roughly linear, including the early epoch. This means that $G \propto R_u$ is tantamount to $G \propto t$; in particular, $G_0(10^{-134})$ coincides with $t_{P_0}(10^{-105})$. At some additional assumptions, this scenario could be modified so as to regard nonlinear expansion during early epochs. However, considering that basic motives for invoking inflation (horizon problem and flatness problem) are absent in BHC scenario, inflation appears to be basically redundant.

9 Assumption $G \propto R_u$, $c = const$: question of consistence with observational tests of G variability

Provided the approximately uniform rate of Hubble flow, the derived from $G \propto R_u$ current rate of increase of G becomes a simple inverse of the age of universe. In fact, the Hubble time does not significantly differ from estimations of the age of universe derived from Friedman equation equipped with definite values of k and Λ. Whereas these estimations range from ≈ 13.798 Gyr (Lambda-CDM concordance model based on data from Planck satellite and WMAP) to $\approx$ 13.82 Gyr (Planck mission), the Hubble time ranges between ≈ 13.7 Gyr and ≈ 14.26 Gyr according to the current extreme estimates of the Hubble constant: ≈ 72 and and $\approx 67\,\mathrm{kms^{-1}Mpc^{-1}}$ respectively. Thus, on the average, the Hubble time only slightly exceeds the supposed age of universe. Interpreting $G \propto R_u$ as $G \propto t$ and estimating the age of the universe for $\approx 13.8\times10^9$yr gives the current rate of change:

$$\dot{G}/G \approx 7.25 \times 10^{-11}\mathrm{yr}^{-1}. \tag{48}$$

Let us compare this prediction with the constraints put upon G variation, derived from different sources (paleontology and geophysics, celestial mechanics, stellar physics, cosmology). A handful of representative results covering the whole range are:

- — paleontological data connected with Earth temperature: $|\dot{G}/G| < 2.0\times10^{-11}\mathrm{yr}^{-1}$ (Eichendorf & Reinhardt [21]);
- — increase of Earth radius: $\dot{G}/G = (-0.5 \pm 2) \times 10^{-11}\mathrm{yr}^{-1}$ (Blake [8]);
- — stability of the radii of Earth, Moon and Mars: $-\dot{G}/G \leqslant 8 \times 10^{-12}\mathrm{yr}^{-1}$ (McElhiny et al. [49]);
- — stability of the orbit of Mars (Mariner 9 and Mars orbiter data): $\dot{G}/G = (-2\pm10)\times10^{-12}\mathrm{yr}^{-1}$ (Shapiro [71]);
- — systematic deviations from the Keplerian orbital periods of Moon: $\dot{G}/G \leqslant (3.2 \pm 1.1)\times 10^{-11}\mathrm{yr}^{-1}$ (Van Flandern [75]);
- — lunar laser ranging (LLR): $|\dot{G}/G| < 6 \times 10^{-12}\mathrm{yr}^{-1}$ (Dickey et al. [14]); LLR: $\dot{G}/G \leqslant (4 \pm 9) \times 10^{-13}\mathrm{yr}^{-1}$ (Williams et al. [81]);
- — spin-down of pulsar JP1953: $-\dot{G}/G < 5.8 \pm 1 \times 10^{-11}$ yr^{-1} (Mansfield [46]);
- — pulsar timing PSR B1913+16: $\dot{G}/G \leqslant (4 \pm 5) \times 10^{-12}$ yr^{-1} (Kaspi et al. [38]);
- — luminosity function of white dwarfs (cooling age): $-\dot{G}/G \leqslant 3^{+1}_{-3} \times 10^{-11}\mathrm{yr}^{-1}$ (Garcia-Berro et al. [27]);
- — pulsating white dwarf data G117-B15A: $|\dot{G}/G| \leqslant 4.10\times 10^{-10}\mathrm{yr}^{-1}$ (Biesiada & Malec [7]);
- — SNe Ia luminosity vs. redshift: $\dot{G}/G = (-3, +7.3) \times 10^{-11}\mathrm{yr}^{-1}$ (Mould & Uddin [52]);
- — helioseismology: $|\dot{G}/G| \leqslant 1.6\times10^{-12}\mathrm{yr}^{-1}$ (Guenther et al. [32]);
- — big bang nucleosynthesis (BBN): $|\dot{G}/G| \leqslant 9. \times 10^{-13}$ yr^{-1} (Accetta et al. [1]); BBN: $|\dot{G}/G| \leqslant 1.7\times10^{-13}$ yr^{-1} (Rothman & Matzner [68]).

One can easily notice that BHC prediction hardly matches the minority of the above bounds. However, a closer insight into methodology reveals various circumstances hidden behind the digits. We shall discuss them now, one by one.

9.1 Accuracy of the constraints on G variation and accuracy in measurements of the value of G

Unlike in the case of other fundamental constants, the increasing precision of measurements of G value is accompanied by increasing discrepancy of the obtained results. This led the CODATA to widen the uncertainty range from 0.013% to 0.15%. We ask whether this uncertainty may impinge on the G variability tests. This question does not seem groundless taking into account the ratio between typical bound put on the annual rate of change of G ($\sim$ 10^{-11}) and the uncertainty range of G value (1.5×10^{-3}), roughly ten-billionth! To better realize the scale, imagine we test the Wegener's continental drift theory (btw unaccepted for a long time) by settling a constraint on the annual rate of relative motion between two continents, say, America and Europe. Assume we determine two points (measuring devices) placed on each of these continents, and estimate the distance between them for 5 thousand kilometers. However, due to hypothetic imperfection of measuring techniques, this distance is only known with the relative uncertainty 0.15%, which translates into 7.5*km*. Assume next that, undeterred by this immense inaccuracy, we derive the constraint for the drift rate for $10^{-11}\mathrm{yr}^{-1}$, i.e. 0.05 mm/year, while the drift rate estimated by the theory amounts to $7.25 \times 10^{-11}\mathrm{yr}^{-1}$, i.e. 0.36 mm/year (in fact, Wegener estimated the speed of drift for 2.5 m/year, while the currently observed rate amounts to about 2.5 cm/year).

Obviously, measuring a given value and measuring a change in this value are, basically, two different things; yet the mentioned discrepancy is too significant to be ignored. This in particular happens when a constraint depends on assumptions that are themselves encumbered by sizeable uncertainty (see subsection 9.3). In the above fictional example, before drawing ultimate conclusions as to the correctness of Wegener's idea, one should certainly aim at eliminating the distance uncertainty or try to find its hidden sources. Otherwise, any ultimate conclusions as to the change of distance could not be considered reliable. There is no reason to assume the question of variability of Newton's constant should subject to different rules.

9.2 Differences in notation and the question of autonomy of particular constraints

There is no unique notation for the constraints on G variation; for different reasons, particular constraints (or their groups) are expressed in different mathematical forms. Revealing their meaning provides us with a better insight into the question of autonomy. The (here called) canonical form, $\dot{G}/G \leqslant (a \pm b) \times 10^{-c}\text{yr}^{-1}$ (a positive/negative, b, c positive) reads: "An annual rate of increase/decrease of G, not greater than $a \times 10^{-c}$ has been observed, with the uncertainty range equal to $\pm b \times 10^{-c}$". If $a = 0$, it means that no change has been observed, although b still describes the range of uncertainty of that finding. Expression $-\dot{G}/G$, instead of $\dot{G}/G$, means that given constraint concerns solely (is design to detect) the decrease rate of G. This takes place when a theory predicting the decrease of G (e.g. Dirac's LNH) is tested, and thus respective assumptions are the base of derivation. In turn, the form $|\dot{G}/G|$ reads: "The possibility of G variation (including increase and decrease in equal degree) fits in the range..." However, $|\dot{G}/G|$ is sometimes used as equivalent to $-\dot{G}/G$, in particular when aimed at testing Dirac's hypothesis (e.g. Eichendorf & Reinhardt [21]). This form implies a to be indistinguishable from b, i.e. treats expressions "(rate) not greater than" and "with the uncertainty range" as tantamount to each other. Another way to identify the range of possible change with the range of uncertainty is the form $\dot{G}/G = (-b_1, +b_2) \times 10^{-c}\,\text{yr}^{-1}$, $b_1 \neq b_2$. Although apparently similar to $|\dot{G}/G|$, this form indicates the observed tendency (i.e. increase or decrease) and thus seems to be basically equivalent to the canonical form; e.g. the term $(-2, +4)$ could be expressed as (1 ± 3). An alternative use of the relation symbols $<$, $\leqslant$ and $=$ in each of the above forms can be interpreted (dependently on the context) as a gradable expression of conviction as to the observed tendency. In particular, symbols $<$ and $\leqslant$, when used in the canonical form, play the role of additional proviso (apart of b term) due to general uncertainty; for example, if $|a|$ is greater than b then using $=$ unambiguously points to the observed change of G. Instead, using $<$ or $\leqslant$ weakens this statement, suggesting the change to be only probable.

Let us assume that, generally, all observations meet the criteria of scientific rigor. Apart of proper methodology and precision, this would also mean the unbiased standpoint as to the principal question, i.e. whether the Newton's constant is a true constant. Provided that, the postulate of autonomy says that each constraint should be interpreted in accordance with the sense of its notation and with regard to the underlying assumptions (usually not reflected in notation). In particular, weaker constraints should not be treated as "worse" than the stronger ones but, for the most part, as speaking in favor of variability.

9.3 Dependence on the employed theory and assumptions

Many factors involved in determination of the bounds put on G variation are theory or assumption dependent. For example, stringent constraints derived from BBN (Accetta et al. [1], Rothman & Matzner [68]) are valid only for Brans-Dicke theory; likewise, the constraint derived by Guenthner et al. [32] bases on the Brans-Dicke type theory with varying G. Most of constraints, even when not visibly shown in their notation, base on observations testing Dirac's LNH, i.e. are focused on the possible decrease of G. This in particular concerns the results derived from geophysical and paleontological data: impact of the Earth surface temperature on ancient organisms, expansion of Earth and the relevant difference in paleolatitudes between two sites of known separation (allowing to deduce the paleoradius), spin-down of the Earth due to its expansion, recession of the Moon and its impact on tides reflected in fossils. The respective data depend on too many conditions to repose excessive trust in their precision, and thus to consider them as fully reliable assumptions. In his extensive review study, Uzan [74] pays attention on these other sources of uncertainty connected with particular constraints.

9.4 Variation of Newton's constant and the age of universe

Assuming that increase of G extends the age of universe, the rate of G variation would be smaller than the here quoted value $7.25 \times 10^{-11}\text{yr}^{-1}$ thus better fitting observations. However, according to the Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}\,, \qquad (49)$$

variation of G has a negligible impact on the age of universe. For $k = 0$ (flat universe) and $\Lambda = 0$, density becomes critical ($\rho_c = 3H^2(8\pi G)^{-1}$, and thus Friedmann equation reduces itself to identity $H^2 \equiv H^2$ becoming insensible to the change of G. In such a case, the age of universe simply equals the inverse of Hubble's constant ($t = H^{-1}$). However, for $\Lambda \neq 0$, currently estimated for $\Lambda_{(const)} \approx 10^{-52}$ m, dark energy (in a form of cosmological constant) predominates from a certain moment, so that t and H^{-1} more and more diverge. In an accelerating universe driven by dark energy, the rate of increase of G determined by $G \propto R_u$ also accelerates, which means that its declining in the unit time gradually slows down. Hence, in the far future, $G \propto R_u$ will translate to $G \propto H^{-1}$ rather than to $G \propto t$.

9.5 Equivalence of gravity and inertia

As is known, there are (currently) four notions of mass: 1) active gravitational mass — measure of ability to create gravitational field or curvature, 2) passive gravitational mass — measure of "sensing" the gravitational field by a body (in the Newtonian depiction, respectively: measure of the force exerted by a body and measure of the force experienced by a body), 3) inertial mass — measure of resistance against the force accelerating a body (including the force of gravity), and 4) mass as a measure of energy according to $E = mc^2$. Numerous experiments performed over a long time up to present days have shown with increasing precision that inertial mass and passive gravitational mass are proportional to each other: $m_{(inert)} \sim m_{(pass)}$ (week equivalence principle). In turn, since active and passive gravitational masses are interchangeable according to the Newton's third law, so also active gravitational mass and inertial mass are proportional to each other:

$$m_{(act)} \sim m_{(inert)} \,. \tag{50}$$

The active gravitational mass is proportional to the Newton's constant: $m_{(act)} \sim G$. In fact $m_{(act)}$ is inseparable from G, which means that any change in the active mass should be interpreted as the change in the Newton's constant. Consequently, inertial mass is thought to follow the putative variation of G:

$$\Delta G \rightarrow \Delta m_{(inert)} \,. \tag{51}$$

This would make the so-called "inertial reaction force" always (i.e. also in the time-slice experiments) equivalent to the gravitational force. At the same time, variation of the Newton's constant would not affect the mass interpreted as the source of positive energy. Accordingly, the tests on G variation derived from celestial mechanics (e.g. LLR) would be basically ineffective, while the other ones (e.g. based on stellar physics) would still remain valid.

10 Quantitative predictions of the varying Chandrasekhar limit hypothesis, based on $G \propto R_u$, $c = const$

We shall now consider the VCM hypothesis in the form related to the BHC specified as $G \propto R_u$ i.e. $G \propto t$. On the assumption that the rate of Hubble expansion is approximately uniform, the Chandrasekhar limit depends on cosmic time as $M_{Ch} \propto t^{-3/2}$. This determines characteristic "delay time" for a single white dwarf, defined as the time needed to reach the WD's mass by the decreasing M_{Ch}. It makes thereby a basis for the quantitative predictions of VCM as to the rate of supernovae events, interpreted as a function of cosmic time. While, in general, the anticipated by VCM ability of a single WD to become the supernova meets the problem of the paucity of SNe Ia progenitors, the detailed predictions obviously demand more circumstantial investigation. One has to regard: 1) the number of single WDs within a given area (in particular, the number of their representative sample); 2) the mean/median mass of this sample; 3) the respective "delay time" for the median mass, determined by $M_{Ch} \propto t^{-3/2}$. Besides, in predicting the rate of distant SNe Ia one should also regard the related to distance intrinsic time of the observed events, and a corresponding value of Chandrasekhar limit. Once a distance is well defined, the respective limit should be treated as constant, considering the negligible (compared with the assumed rate of change in M_{Ch}) time devoted to observation. Instead, for the nearby SNe Ia one may fairly assume $M_{Ch} \approx 1.4 M_\odot$.

Let us apply the above to our Galaxy. For the sake of simplicity (an also taking into account the uncertainty in all data), we shall not regard the contribution of SNe Ia originated in binaries. We aim to estimate the present rate of SNe Ia, deriving it from accessible data, according to the above quoted three points. As is known, the Galaxy contains roughly 100-400 billion stars, above 97% of them supposed to end as white dwarfs, which however includes both actual WDs and the potential ones. According to the estimations based on SPY project, the space density of WDs within the radius of 20 pc is $(4.8 \pm 0.5) \times 10^{-3}$ pc^{-3} while the corresponding mass density amounts to $(3.2 \pm 0.3) \times 10^{-3} M_\odot$ pc^{-3}, which gives the overall mean mass $(M)_{WD} \approx 0.665 M_\odot$ (Holberg et al. [35]). Instead Kepler et al. [39], basing on catalog elaborated by Eisenstein et al. [24] from the SDSS Data Release 4, found significant difference in the WD's mean mass between DA and DB stars (hydrogen and helium layers, respectively); namely $(M)_{DA} \approx 0.593 M_\odot$ and $(M)_{DB} \approx 0.711 M_\odot$. Considering the number of DA and DB in the sample (7167 and 507, respectively), one gets the $(M)_{WD} \approx 0.6 M_\odot$. We shall us this value in the further calculations.

In order to estimate the total number of white dwarfs in the Milky Way, we have to multiply the WD's space density by the Galaxy volume. Certainly, such an extrapolation is encumbered by significant uncertainty, as it is doubtful whether the sample obtained from the relatively close neighborhood (thin disc, in general) is typical for the whole Galaxy including thick disc, halo and the galactic bulge. Different parts of Galaxy vary in age, so WD's population is likely inhomogeneous in age and density. Evaluating the radius for 15,000 pc and the mean thickness for 5,000 pc and multiplying this by WDs' local density, one obtains: $(3.5 \times 10^{12}$ pc$^3) \times (5 \times 10^{-3}) \approx 1.7 \times 10^{10}$. This gives an insight into the actual number of WDs, consistent with a list brought by the Research Consortium on Nearby Stars (RECONS). According to the latter, 8 of the nearest 100 stars are the white dwarfs, which, provided this to be the representative ratio, gives the total number between 0.8×10^{10} to 3.2×10^{10} , dependently on the assumed total number of stars (100-400 billion).

The next step is to derive the "mean delay time" $(T)_{del}$ for the WD's mean mass $(M)_{WD}$. The respective algorithm reads

$$(T)_{del} = \left(\frac{t}{T_u} + 1 \right)^{2/3} \times T_u - T_u \tag{52}$$

T_u-age of universe, t – an auxiliary delay time not regarding the power index, yielding

$$t = \frac{T_u}{(M_{Ch}/\Delta M_{WD}) - 1}\,, \tag{53}$$

where $\Delta M_{WD} = M_{Ch} - (M)_{WD}$. After conversion, one has

$$(T)_{del} = \left(\frac{M_{Ch}}{(M)_{WD}}\right)^{2/3} \times T_u - T_u\,. \tag{54}$$

Inserting $M_{Ch} = 1.4M_\odot$, $(M)_{WD} = 0.6M_\odot$ and $T_u = 13.8$ Gyr, one obtains $(T)_{del} \approx 10$ Gyr. Dividing the number of white dwarfs in Galaxy by that time gives the rate of roughly 1-3 events per year, a frequency exceeding the observed rate by a factor $> 10^2$. However, this prediction does not concern the present rate but a hypothetic rate averaged over the above calculated $(T)_{del}$. One should not identify (or confuse) "averaged" with "uniform" mainly because WD's masses subject, in general, to the Gaussian distribution:

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\sigma)^2/2\sigma^2} \tag{55}$$

(σ-standard deviation, μ-mean of the distribution) and the respective probability function:

$$Prob[a \leqslant x \leqslant b] = \int_a^b f(x)\,dx\,. \tag{56}$$

The observed standard deviation is significantly smaller than one ($\sigma^2 \ll 1$) yielding substantial peak around the median mass $0.6M_\odot$. Obviously, only WDs of the mass close to $1.4M_\odot$, corresponding with the relatively short mean delay time, contribute to the present rate of SNe Ia. We assume that any single white dwarf of the mass close to M_{Ch} is, dependently on specific conditions (rotation, chemical composition), a potential SN Ia at any moment during the slated delay time. Admittedly, the most massive known WD only slightly exceeds $1.3M_\odot$; this however should be associated with the fact that less than one-millionth of the whole population of WDs in Galaxy are identified so far. A similar difficulty concerns specifying the expression "close to $1.4M_\odot$". Bearing in mind an inevitable uncertainty, let us determine the respective range for $[b-a] \approx 0.1M_\odot$, assuming that, dependently on detailed conditions, any WD of the mass between $1.3 - 1.4M_\odot$ may become the SNe-Ia. For that mass range, the unit normal distribution yields less than 0.1% of the entire population, say, $\approx 10^7$. The mean mass of this "representative sample" is $1.35M_\odot$. It follows:

$$T_{del} \approx (1.4/1.35)^{2/3} \times 13.8 - 13.8 \approx 0.34\,(\text{Gyr})\,. \tag{57}$$

The respective rate is then

$$\frac{10^7}{3\times 10^8} = 3\times 10^{-2}\,(\text{yr}^{-1}). \tag{58}$$

This still slightly exceeds the observed rate, provided the latter is $\leqslant 1$ events per 100 years. However, considering the mentioned above reservations, it would not be reasonable to attach excessive importance to this or that particular number. The real number of single WDs from the representative sample may prove to be much smaller than 10^7 . The mass-range of potential progenitors may appear slightly narrower or wider. In general, more accurate data may support or falsify our hypothesis.

11 Conclusion

We have considered the SNe Ia progenitor problem in the context of general problem of the constancy of fundamental constants. Basing on arguments derived from the black-hole cosmology, we have singled out the Newton's constant as the most probable candidate for "inconstant constant". Since the increase of G involves the decrease in the value of Chandrasekhar limit M_{Ch}, both questions meet together yielding a hypothesis according to which a single white dwarf can alone become the progenitor of SN Ia.

Admittedly, the ongoing progress in observational techniques together with an improvement in stellar physics may bring solution to the progenitor problem dispensed with violating the constancy of Chandrasekhar limit. A tacit heuristic strategy connected with searching for the SNe Ia progenitors consists in attempts of making the SD and DD models flexible enough to eliminate the observed discrepancies. For the time being however the problem still exists, which makes solutions going beyond the binary paradigm justifiable and noteworthy.

The unbiased estimations seem to support the main thesis of this article, i.e. that M_{Ch} decreasing according to $G \propto R_u$ may explain the paucity of SNe Ia progenitors. It is to be noted that, predicted by $G \propto R_u$ immense growth of the Newton's constant from the initial to present value ($G/G_0 = \delta^2 \approx 10^{124}$) almost completely applies to the very early and early universe, preceding structure formation. Since the oldest SNe Ia detected so far: SN UDS1 0Wil (Wilson) and SN 1997ff reach about 11 Gyr the part of increase of the Newton's constant shaping the Chandrasekhar limit does not exceed the one order of magnitude, being much smaller in the case of overwhelming majority of the observed events.

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