Schrödinger Equation for a Half Spin Electron in a Time Dependent Magnetic Field

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The behaviour of an electron with mass m_e and half spin when passing through a magnetic field with fixed strength B_0 is studied. The motion of the particle is restricted to a ring with radius R, thus assuming periodic boundary conditions. We also focused on magnetic field evolving adiabatically in time, the magnetic field is expressed as a function of angle ϕ and θ i.e only the direction of the magnetic field vectors change while the strength B_0 is kept fixed. Expression for eigenenergies were drawn for a fixed energy and sample values of α, ω, θ and $x = mR^2/\hbar^2$.

1 Introduction

An intriguing example emerging from asymmetric spin-interactions are skyrmion lattices. In 1989 Alexey Bogdanov predicted that for anisotropic chiral magnets there is a new magnetic order consisting of topologically stable spin whirls, named skyrmions after the English particle physicist Tony Skyrme, who showed that localized solutions to non-linear quantum field theories may be interpreted as elementary particles. Briefly speaking, skyrmions are topologically stable whirls in fields.

In 2009, a new magnetic order was observed in Manganese Silicide (MnSi) for specific temperatures and magnetic fields by Mühlbauer et al [1]. The physics of an electron moving through the magnetic field can be analyzed from two different points of view:

From the point of view of the electron, i.e. considering the problem in terms of emergent electric and magnetic fields, the change in spin orientation is equal to an effective Lorentz force acting on the electron, which is perpendicular to its motion [2]. As a result, the magnetic field induces a deflection of the electron, which can be measured by making use of the topological Hall-effect [3]. Because of the electron carrying an electric charge, a potential may be measured perpendicular to the direction of the current. Since the magnetic structure of the skyrmion lattice is very smooth, the adjustment of the spin of the electron to the magnetization of the skyrmion lattice can be considered an adiabatic process.

On the other hand, there must be a corresponding counterforce acting on the skyrmion. This force, arising from the transfer of angular momentum from the conduction electrons to the local magnetic structure (cf. [4]), can for example result in a drift of the domains of the lattice. A 1-D model of an electron passing over a static magnetic field has previously been investigated in the Bachelor's thesis of M. Baedorf [5]. Berry phase physics and spin-scattering in time-dependent magnetic fields has been studied by Sarah Maria Schroeter [6].

In this work, the behaviour of an electron with mass m_e, when passing through a magnetic field with a fixed strength B_0 is studied.

2 Formulation of the problem

The behaviour of a half spin particle, more specifically an electron, when passing through a magnetic field with a fixed strength B_0 is considered. The parameter ϕ sets the position where the particular magnetic field is measured. At every position ϕ on border of the circle, we attach an imaginary 3D-sphere which determines the direction of the field vector. In effect, the magnetic field is constituted by mere spherical coordinates. In addition, we allow variation of both angle ϕ and θ in time with frequency of ω_1 and ω_2 respectively:

$$B(r,t) = B_0\,\hat{n}(\phi,\theta,t) \tag{1}$$

$$B(r,t) = B_0 \begin{bmatrix} \sin(\theta-\omega_2 t)\cos(\phi-\omega_1 t) \\ \sin(\theta-\omega_2 t)\sin(\phi-\omega_1 t) \\ \cos(\theta-\omega_2 t) \end{bmatrix} \tag{2}$$

$$B(r,t) = B_0 \begin{bmatrix} \sin(\tilde{\theta})\cos(\tilde{\phi}) \\ \sin(\tilde{\theta})\sin(\tilde{\phi}) \\ \cos(\tilde{\theta}) \end{bmatrix} \tag{3}$$

where $\tilde{\phi} = \phi - \omega_1 t$ and $\tilde{\theta} = \theta - \omega_2 t$. The Hamiltonian is made up of a kinetic part and a part arising from the interaction of particle with the magnetic field:

$$H_0(r,t) = \frac{\hat{p}^2}{2m_e} + B(r,t)\frac{g_s|\mu_B|}{h}\,S \tag{4}$$

where S is the electron spin, g_s is the spin g-factor and μ_B is the Bohr magneton

$$|\mu_B| = \frac{|e|\hbar}{2m_e}.$$

We confine ourselves to the xy-plane, with the real space parameter $\theta = \pi/2$ and radius R kept fixed. The nabla-operator is simplified as:

$$\nabla = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_{\tilde{\theta}} \frac{1}{R} \frac{\partial}{\partial \tilde{\theta}} + \hat{e}_{\tilde{\phi}} \frac{1}{R \sin\tilde{\theta}} \frac{\partial}{\partial \tilde{\phi}} \tag{5}$$

which becomes

$$\nabla^2 = \left(\frac{1}{R}\frac{\partial}{\partial\tilde{\phi}}\right)^2. \tag{6}$$

Thus, we can now rewrite the Hamiltonian H_0 as

$$H_0 = -\frac{\hbar^2}{2mR^2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 + |\mu_B|\, B_0(r,t)\,\sigma \tag{7}$$

where σ is a vector of Pauli matrices and for any unit vector $\hat{n}$, we find a rotation matrix $\Re$ such that $\Re\hat{\tilde{\phi}} = \hat{n}$ so that (7) can be rewritten as

$$H_0 = \frac{\hbar^2}{mR^2}\left(-\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 + \frac{|\mu_B|B_0}{\hbar^2/mR^2}\,\hat{n}\,\sigma\right) \tag{8}$$

$$H_0 = \frac{\hbar^2}{mR^2}\left(-\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 + \alpha\hat{n}\sigma\right) = \frac{\hbar^2}{mR^2}\,\tilde{H}_0 \tag{9}$$

where

$$\alpha = \frac{|\mu_B|B_0}{\hbar^2/mR^2}\,; S = \frac{\hbar}{2}\,\sigma\,; g_s = 2.$$

Combining the operators generating the translation and rotation gives

$$g = -i\hbar\frac{\partial}{\partial S}\,1 + \frac{\hbar}{2R}\,\sigma_z = -\frac{i\hbar}{R}\frac{\partial}{\partial\tilde{\phi}}\,1 + \frac{\hbar}{2R}\,\sigma_z \tag{10}$$

$$\tilde{g} = -i\frac{\partial}{\partial\tilde{\phi}}1 + \frac{\sigma_z}{2} \tag{11}$$

where $\tilde{g}$ is a rescaled version of g. By careful construction of g, $\tilde{H}_0$ and $\tilde{g}$ commute, consequently H_0 and g indeed commute.

$$[\tilde{H}_0, \tilde{g}] = \left[-\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 + \alpha\hat{n}\sigma, -i\frac{\partial}{\partial\tilde{\phi}}1 + \frac{\sigma_z}{2}\right] \tag{12}$$

$$\begin{aligned}[\tilde{H}_0, \tilde{g}] = &\left[-\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2, -i\frac{\partial}{\partial\tilde{\phi}}\right] + \left[\alpha\hat{n}\sigma, -i\frac{\partial}{\partial\tilde{\phi}}\right] + \\ &+ \left[-\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2, \frac{\sigma_z}{2}\right] + \left[\alpha\hat{n}\sigma, \frac{\sigma_z}{2}\right]\end{aligned} \tag{13}$$

$$\begin{aligned}[\tilde{H}_0, \tilde{g}] = &\, i\alpha\left(\frac{\partial}{\partial\tilde{\phi}}, \hat{n}\sigma\right) + \\ &+ \frac{\hat{n}\sigma}{2}\Big([\sigma_x,\sigma_z], [\sigma_y,\sigma_z], [\sigma_z,\sigma_z]\Big)\end{aligned} \tag{14}$$

with

$$[\sigma_i, \sigma_j] = 2i \in_{ijk} \sigma_k.$$

$$\begin{aligned}[\tilde{H}_0, \tilde{g}] = &\, i\alpha\bigg[\frac{\partial}{\partial\tilde{\phi}}(\sin\tilde{\theta}\cos\tilde{\phi}\sigma_x + \sin\tilde{\theta}\sin\tilde{\phi}\sigma_y + \\ &+ \cos\tilde{\theta}\sigma_z)\bigg] + i\alpha\hat{n}(-\sigma_y, \sigma_x, 0)\end{aligned} \tag{15}$$

$$\begin{aligned}[\tilde{H}_0, \tilde{g}] = &\, i\alpha\Big(-\sin\tilde{\theta}\sin\tilde{\phi}\sigma_x + \sin\tilde{\theta}\cos\tilde{\phi}\sigma_y\Big) + \\ &+ i\alpha\hat{n}(-\sigma_y, \sigma_x, 0)\end{aligned} \tag{16}$$

$$[\tilde{H}_0, \tilde{g}] = 0. \tag{17}$$

We have shown that $\tilde{H}_0$ and $\tilde{g}$ possess the same system of eigenfunctions, with that, we regard $\tilde{g}$ as a generalized momentum operator.

2.1 Solution to momentum operator

We now establish the eigenfunctions of $\tilde{g}$ solving the eigensystem

$$\left(-i\frac{\partial}{\partial\tilde{\phi}}1 + \frac{\sigma_z}{2}\right)|\psi\rangle = K|\psi\rangle \tag{18}$$

$$-i\frac{\partial}{\partial\tilde{\phi}}1\,|\psi\rangle = \left(K - \frac{\sigma_z}{2}\right)|\psi\rangle = \begin{pmatrix}\left(K-\frac{1}{2}\right) & 0 \\ 0 & \left(K+\frac{1}{2}\right)\end{pmatrix}|\psi\rangle \tag{19}$$

with eigenvalues

$$\lambda_{\frac{1}{2}} = \left(K \mp \frac{1}{2}\right) \tag{20}$$

and the respective eigenfunctions

$$|\psi_1\rangle = \begin{pmatrix}1\\0\end{pmatrix} e^{i\left(K-\frac{1}{2}\right)\phi} = \begin{pmatrix}\psi_1\\0\end{pmatrix} \tag{21}$$

$$|\psi_2\rangle = \begin{pmatrix}0\\1\end{pmatrix} e^{i\left(K+\frac{1}{2}\right)\phi} = \begin{pmatrix}0\\\psi_2\end{pmatrix}. \tag{22}$$

As we study the motion of a particle on a ring, we require $|\psi(\phi)\rangle$ to fill periodic boundary condition

$$|\psi(\phi)\rangle = |\psi(\phi+2\pi)\rangle e^{i\left(K\mp\frac{1}{2}\right)2\pi} = 1K = n + \frac{1}{2}; n \in Z \tag{23}$$

This means that the momentum is quantized. The general solution to (18) is linear combination of both eigenfunctions

$$|\psi(\phi)\rangle = C_1(t)|\psi_1(\phi)\rangle + C_2(t)|\psi_2(\phi)\rangle = \begin{pmatrix}C_1(t)|\psi_1(\phi)\rangle\\ C_2(t)|\psi_2(\phi)\rangle\end{pmatrix} \tag{24}$$

where $C_1(t)$ and $C_2(t)$ do not depend on ϕ.

2.2 Solution to the time-dependent Hamiltonian

Ultimately, we are interested in computing the time-dependent coefficients $C_1(t)$ and $C_2(t)$ in order to receive full solution of the Schrödinger equation when solving the time-dependent Schrödinger equation, we employ the solution to the momentum operator in order to simplify the eigensystem associated with $\tilde{g}$ as follows:

$$i\hbar\partial_t|\psi\rangle = H_0|\psi\rangle = \frac{\hbar^2}{mR^2}\left(-\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 + \alpha\hat{n}\sigma\right)|\psi\rangle. \qquad (25)$$

See the last page for intermediate equations (26) and (27)

$$i\hbar\partial_t|\psi\rangle = H_{0,K,\tilde{\phi}}(t)|\psi\rangle \qquad (28)$$

where $H_{0,K,\tilde{\phi}}(t)$ is defined by equation (27).

2.2.1 Setting up the Schrödinger equation for the time-dependent coefficients

To set up the Schrödinger equation for the time-dependent coefficients $C_1(t)$ and $C_2(t)$ is by transforming the Schrödinger equation for $|\psi\rangle$:

$$\begin{aligned} i\hbar\partial_t|\psi\rangle\begin{pmatrix} C_1(t)\psi_1 \\ C_2(t)\psi_2 \end{pmatrix} &= H_0(t)\begin{pmatrix} C_1(t)\psi_1 \\ C_2(t)\psi_2 \end{pmatrix} \\ i\hbar\partial_t|\psi\rangle\begin{pmatrix} C_1(t) \\ C_2(t)\frac{\psi_2}{\psi_1} \end{pmatrix} &= H_0(t)\begin{pmatrix} C_1(t) \\ C_2(t)\frac{\psi_2}{\psi_1} \end{pmatrix}. \end{aligned} \qquad (29)$$

Employing the equation (27) computed solution to the momentum operator, we know that $\frac{\psi_2}{\psi_1} = e^{i\phi}$ and may write (see the last page for intermediate equations (30) and (31)):

$$i\hbar\partial_t\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} = H_{0,K,\omega}\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} \qquad (32)$$

where $H_{0,K,\omega}$ is defined by equation (31).

2.3 Moving into a rotating coordinate system

To solve the eigensystem, we transform $H_{0,K,\omega}(t)$ by changing into a coordinate system rotating clockwise with a frequency $\omega = \omega_1$

$$\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix} = e^{-\frac{i}{\hbar}S_z\omega t}\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} = e^{-\frac{i}{2}\sigma_z\omega t}\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix}. \qquad (33)$$

In another way, (33) becomes

$$\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} = e^{\frac{i}{2}\sigma_z\omega t}\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix} \qquad (34)$$

where

$$\begin{aligned} e^{\frac{i}{2}\sigma_z\omega t} &= \Sigma_n\frac{\left(\frac{i}{2}\sigma_z\omega t\right)^n}{n!} = \Sigma_n\frac{\left(\frac{i}{2}\omega t\right)^n}{n!}\begin{pmatrix} 1^n & 0 \\ 0 & (-1)^n \end{pmatrix} \\ &= \begin{pmatrix} e^{\frac{i}{2}\omega t} & 0 \\ 0 & e^{-\frac{i}{2}\omega t} \end{pmatrix}. \end{aligned}$$

Substituting of (32) in (34) gives:

$$\begin{aligned} & i\hbar\partial_t\begin{pmatrix} e^{\frac{i}{2}\omega t} & 0 \\ 0 & e^{-\frac{i}{2}\omega t} \end{pmatrix}\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix} \\ &= H_{0,K,\omega}\begin{pmatrix} e^{\frac{i}{2}\omega t} & 0 \\ 0 & e^{-\frac{i}{2}\omega t} \end{pmatrix}\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix}. \end{aligned} \qquad (35)$$

Multiplying L.H.S of (35) by $e^{\frac{i}{2}\sigma_z\omega t}$, we obtain

$$\begin{aligned} & \begin{pmatrix} e^{-\frac{i}{2}\omega t} & 0 \\ 0 & e^{\frac{i}{2}\omega t} \end{pmatrix}\begin{pmatrix} e^{\frac{i}{2}\omega t}\left(\frac{-\hbar\omega}{2} + i\hbar\partial_t\right) & 0 \\ 0 & e^{-\frac{i}{2}\omega t}\left(\frac{\hbar\omega}{2} + i\hbar\partial_t\right) \end{pmatrix} \\ & \begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix} = \begin{pmatrix} -\frac{\hbar\omega}{2} & 0 \\ 0 & -\frac{\hbar\omega}{2} \end{pmatrix}\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix} + i\hbar\partial_t\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix}. \end{aligned}$$

Also multiplying R.H.S of (35) by $e^{\frac{i}{2}\sigma_z\omega t}$ we have:

$$\begin{aligned} & \begin{pmatrix} e^{-\frac{i}{2}\omega t} & 0 \\ 0 & e^{\frac{i}{2}\omega t} \end{pmatrix}H_K\begin{pmatrix} e^{\frac{i}{2}\omega t} & 0 \\ 0 & e^{-\frac{i}{2}\omega t} \end{pmatrix}\begin{pmatrix} \tilde{C_1(t)} \\ \tilde{C_2(t)} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}\left(K - \frac{1}{2}\right)^2 + \alpha\cos\tilde{\theta} & \alpha\sin\tilde{\theta} \\ \alpha\sin\tilde{\theta} & \frac{1}{2}\left(K + \frac{1}{2}\right)^2 - \alpha\cos\tilde{\theta} \end{pmatrix}\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix}. \end{aligned}$$

As a consequence, (35) yields (see the last page for intermediate equation (36))

$$i\hbar\partial_t\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix} = C\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix}. \qquad (37)$$

Comparing (37) with the corresponding static Schrödinger equation for time-independent coefficients, one observes that C is the Hamiltonian one receives when considering static magnetic field (cf. [5]) combined with an additional matrix

$$\begin{pmatrix} \frac{\omega mR^2}{2\hbar} & 0 \\ 0 & \frac{-\omega mR^2}{2\hbar} \end{pmatrix}.$$

We now deal with time-independent $\tilde{\theta}$ and time-dependent $\tilde{\phi}$, so that $\tilde{\theta} = \theta = constant$. As eigenvalues of the operator C we get (see the last page for equation (38)), which correspond to the energies of the lower and upper band. E_- corresponds to a magnetic moment which is parallel to the magnetic field.

2.4 Determining the rotated time-dependent coefficients

To determine the solution to (36) i.e find a representation of the rotated time-dependent coefficients $\tilde{C(t)}_1$ and $\tilde{C(t)}_2$, an equation of the form

$$i\hbar\partial_t\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix} = C\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix}$$

can immediately be found to have the solution

$$\begin{pmatrix} \tilde{C}_{1+}(t) \\ \tilde{C}_{2+}(t) \end{pmatrix} = e^{-iE_+t}X_+ \qquad (39)$$

$$\begin{pmatrix} \tilde{C}_{1-}(t) \\ \tilde{C}_{2-}(t) \end{pmatrix} = e^{-iE_-t}X_+ \qquad (40)$$

where E_+, E_- and X_+, X_- are the eigenvalues and corresponding normalized eigenvectors of the matrix C respectively.

More precisely, the later are found to be given by equation (41) and normalization factor (42) given on the last page.

2.5 Establishing the solution to the initial Schrödinger equation

Combining (39) and (40) with already computed static parts of the wave function (21) and (22) as well as multiplying the respective components with the e factor which sets the wave function back into a non-rotating coordinate system (see (34)), we receive the exact solutions to the initial Schrödinger equation (27)

$$|\psi\rangle_{K,+} = e^{-iE_+t}\begin{pmatrix} x_{1,+}e^{i\left(K-\frac{1}{2}\right)\phi}e^{i\frac{\omega}{2}t} \\ x_{2,+}e^{i\left(K+\frac{1}{2}\right)\phi}e^{-i\frac{\omega}{2}t} \end{pmatrix}; \quad {}_{+,K}\langle\psi|\psi\rangle_{K,+} = 1 \tag{43}$$

$$|\psi\rangle_{K,-} = e^{-iE_+t}\begin{pmatrix} x_{1,-}e^{i\left(K-\frac{1}{2}\right)\phi}e^{i\frac{\omega}{2}t} \\ x_{2,-}e^{i\left(K+\frac{1}{2}\right)\phi}e^{-i\frac{\omega}{2}t} \end{pmatrix}; \quad {}_{-,K}\langle\psi|\psi\rangle_{K,-} = 1 \tag{44}$$

The solutions (43) and (44) specific to energies E_- and E_+ (and respective bands + and -) corresponding to the solution to one K, hence the indices.

3 Numerical solution to the eigenenergies

First, let us turn back to the exact eigenenergies we computed in section 2.2, equation (38). We consider an incoming wave function with a fixed energy $\in$ (given on the last page). For a fixed energy $\in_n = \in_o + n\,\omega$ there are maximal four real solutions for $K(n, \sigma, \delta)$, which correspond to the propagation directions $\delta = l, r$ and the two possible eigenenergies of the respective wave functions, i.e. the alignment of the spin $\sigma = +, -$ with respect to the magnetic field, (see Fig. 1).

4 Discussion

The Schrödinger equation for a half spin particle in a time dependent magnetic field is presented. Depending on the energy, there are up to four real solutions for K. The energy function $E_+(K)$ lies below the function $E_-(K)$ for all specific K, (see Fig. 1). For a fixed energy below the minimum of E_- there are no real solutions. For a fixed energy between both minima there are two real solutions which correspond to a spin aligned in the direction of the magnetic field and waves propagating towards the left or the right. For an energy above two minima there are four real solutions. In this case, both directions of propagation and both spin orientations occur.

5 Conclusion

In this paper, the exact wave function of a particle moving through a non-colinear time-dependent magnetic field is computed. Also, it is confirmed that the motion of a half spin of an electron through the chosen magnetic field is an adiabatic problem evolving with time. We found that for a time-dependence of the position of the electron, there are no emergent electric fields since the undisturbed Hamiltonian can be mapped onto a time- independent one by unitary transformations.

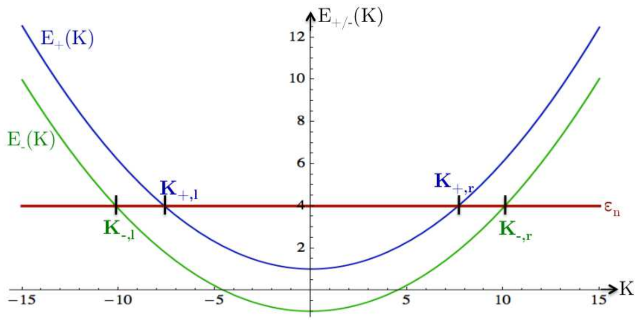

Fig. 1: Eigenenergies $E_\pm(K)$ plotted versus the momentum eigenvalue K for sample values of α, ω, θ and $x = mR^2/\hbar^2$. The points of intersection K_i with a fixed energy $\in$ determine the propagation direction and the spin alignment of the wave function. We set $\alpha = 10$, $\omega = 0.1$, $\theta = \pi$ and $x = mR^2/\hbar^2 = 10$.

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$$i\hbar\partial_t|\psi\rangle = \frac{\hbar^2}{mR^2}\begin{pmatrix} -\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 + \alpha\cos\tilde{\theta} & \alpha\sin\tilde{\theta}e^{-i\tilde{\phi}} \\ \alpha\sin\tilde{\theta}e^{i\tilde{\phi}} & -\frac{1}{2}\left(\frac{\partial}{\partial\tilde{\phi}}\right)^2 - \alpha\cos\tilde{\theta} \end{pmatrix}|\psi\rangle \tag{26}$$

$$i\hbar\partial_t|\psi\rangle = \frac{\hbar^2}{mR^2}\begin{pmatrix} \frac{1}{2}\left(K-\frac{1}{2}\right)^2 + \alpha\cos\tilde{\theta} & \alpha\sin\tilde{\theta}e^{-i\tilde{\phi}} \\ \alpha\sin\tilde{\theta}e^{i\tilde{\phi}} & \frac{1}{2}\left(K+\frac{1}{2}\right)^2 - \alpha\cos\tilde{\theta} \end{pmatrix}|\psi\rangle \tag{27}$$

$$i\hbar\partial_t\begin{pmatrix} C_1(t) \\ C_2(t)\frac{\psi_2}{\psi_1} \end{pmatrix} = \frac{\hbar^2}{mR^2}\begin{pmatrix} \left(\frac{1}{2}\left(K-\frac{1}{2}\right)^2 + \alpha\cos\tilde{\theta}\right)C_1(t) + \alpha\sin\tilde{\theta}e^{-i\tilde{\phi}}C_2(t)e^{i\phi} \\ \alpha\sin\tilde{\theta}e^{i\tilde{\phi}}C_1(t) + \left(\frac{1}{2}\left(K-\frac{1}{2}\right)^2 - \alpha\cos\tilde{\theta}\right)C_2(t)e^{i\phi} \end{pmatrix} \tag{30}$$

$$i\hbar\partial_t\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(K-\frac{1}{2}\right)^2 + \alpha\cos\tilde{\theta} & \alpha\sin\tilde{\theta}e^{i\omega_1 t} \\ \alpha\sin\tilde{\theta}e^{-i\omega_1 t} & \frac{1}{2}\left(K+\frac{1}{2}\right)^2 - \alpha\cos\tilde{\theta} \end{pmatrix}\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} \tag{31}$$

$$i\hbar\partial_t\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix} = \frac{h^2}{mR^2}\begin{pmatrix} \frac{1}{2}\left(K-\frac{1}{2}\right)^2 + \alpha\cos\tilde{\theta} + \frac{\omega mR^2}{2\hbar} & \alpha\sin\tilde{\theta} \\ \alpha\sin\tilde{\theta} & \frac{1}{2}\left(K+\frac{1}{2}\right)^2 - \alpha\cos\tilde{\theta} - \frac{\omega mR^2}{2\hbar} \end{pmatrix}\begin{pmatrix} \tilde{C}_1(t) \\ \tilde{C}_2(t) \end{pmatrix} \tag{36}$$

$$E_\pm = \frac{\hbar^2}{mR^2}\left(\frac{K^2+\frac{1}{4}}{2} \pm \sqrt{\frac{\left(K-\frac{\omega mR^2}{2\hbar}\right)^2}{4} - \alpha\left(K-\frac{\omega mR^2}{2\hbar}\right)^2\cos\theta + \alpha^2}\right) \tag{38}$$

$$X_\pm = \begin{pmatrix} x_{1,\pm} \\ x_{2,\pm} \end{pmatrix} = \frac{1}{N_\pm}\begin{pmatrix} \frac{\hbar^2}{mR^2}\left(-\frac{1}{2}\left(K+\frac{1}{2}\right)^2 + \alpha\cos\theta\right) + \frac{\hbar\omega}{2} + E_\pm \\ \frac{\hbar^2}{mR^2}\alpha\sin\theta \end{pmatrix} \tag{41}$$

$$N_\pm^2 = \left(\frac{\hbar^2}{mR^2}\left(-\frac{1}{2}\left(K+\frac{1}{2}\right)^2 + \alpha\cos\theta\right) + \frac{\hbar\omega}{2} + E_\pm\right)^2 + (\alpha\sin\theta)^2 \tag{42}$$

$$E_\pm = \frac{\hbar^2}{mR^2}\left(\frac{K^2+\frac{1}{4}}{2} \pm \sqrt{\frac{\left(K-\frac{\omega mR^2}{2\hbar}\right)^2}{4} - \alpha\left(K-\frac{\omega mR^2}{2\hbar}\right)^2\cos\theta + \alpha^2}\right) = const = \epsilon_o \tag{45}$$