A Modern Interpretation of the Dirac-Electron Continuity Equation

William C. Daywitt

National Institute for Standards and Technology (retired), Boulder, Colorado. E-mail: wcdaywitt@me.com

This paper re-derives the Dirac continuity equation for the electron from the viewpoint of the Planck vacuum (PV) theory. Results show the equation to be a spacetime equation (whose line elements are cdt and dx^k) that equates the normalized ct-gradient of the probability density ($\psi^\dagger\psi$) to the normalized negative divergence of the quantity ($\psi^\dagger\boldsymbol{\alpha}\psi$).

1 Introduction

The Dirac equation that defines the free-electron spinor field $\psi = \psi(\mathbf{r}, t)$ [1, p.74]

$$ic\hbar\frac{\partial\psi}{c\partial t} = (c\boldsymbol{\alpha}\cdot\widehat{\mathbf{p}} + mc^2\beta)\psi \tag{1}$$

where $\widehat{\mathbf{p}}$ $(=-i\hbar\nabla)$ is the vector momentum operator, can be expressed as

$$ic\hbar\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha}\cdot\nabla\right)\psi = mc^2\beta\psi \tag{2}$$

where c is the speed of light, $\hbar$ is the reduced Planck constant, and m is the electron mass. The spinor field ψ is the 4×1 column vector

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}. \tag{3}$$

The two 4x4 matrices in (1) and (2) are defined by

$$\alpha_k = \begin{pmatrix}0 & \sigma_k\\ \sigma_k & 0\end{pmatrix} \quad\text{and}\quad \beta = \begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix} \tag{4}$$

where $k = (1, 2, 3)$ and I is the 2×2 unit matrix. The three 2×2 Pauli spin matrices σ_k are [1, p. 12]

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{5}$$

and the operator on the left side of (2) reduces to

$$\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha}\cdot\nabla\right) = \left(\frac{\partial}{c\partial t} + \sum_{k=1}^{3}\alpha_k\frac{\partial}{\partial x^k}\right). \tag{6}$$

In its rest frame the massive electron core $(-e_*, m)$, with its zero-point derived mass m [2], exerts the two-term coupling force [3, Sec. 7-8]

$$F(r) = \frac{e_*^2}{r^2} - \frac{mc^2}{r} = \frac{(-e_*)(-e_*)}{r^2} - \frac{mm_*G}{r_*r} \tag{7}$$

on the PV quasi-continuum, where e_* is the massless bare charge and $G\,(= e_*^2/m_*^2)$ is Newton's gravitational constant. The first $(-e_*)$ in (7) belongs to the electron and the second to the separate Planck particles making up the degenerate PV state. The two terms in (7) represent respectively the Coulomb repulsion between the electron charge and the separate PV charges, and the second their mutual gravitational attraction.

The particle/PV coupling force (7) vanishes at the electron Compton radius $r_c\,(= e_*^2/mc^2)$. In addition, the vanishing of $F(r_c)$ is a Lorentz invariant constant [4] that leads to the important Compton-(de Broglie) relations

$$r_c\cdot mc^2 = r_d\cdot cp = r_L\cdot E = r_*\cdot m_*c^2 = e_*^2 \quad (= c\hbar) \tag{8}$$

where $r_d = r_c/\beta_0\gamma_0$ and $r_L = r_c/\gamma_0$, and $r_*\,(= e_*^2/m_*c^2)$ and m_* are the Compton radius and mass of the Planck particles within the PV state. The ratio of the electron speed v to the speed of light c is β_0 and $\gamma_0 = 1/(1-\beta_0^2)^{1/2}$. The relativistic momentum and energy following from the invariance of $F(r_c) = 0$ are $p\,(= m\gamma_0 v)$ and $E\,(= m\gamma_0 c^2)$, from which $E = (m^2c^4 + c^2p^2)^{1/2}$ is the relativistically important energy-momentum relationship.

Using (8), (2) can be expressed as

$$ie_*^2\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha}\cdot\nabla\right)\psi = mc^2\beta\psi \tag{9}$$

or

$$ir_c\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha}\cdot\nabla\right)\psi = \beta\psi \tag{10}$$

where the partial derivatives within the parentheses are normalized by the Compton radius r_c. The spinor field that is the hermitian conjugate of ψ is the 1×4 row vector $\psi^\dagger = (\psi_1^\dagger, \psi_2^\dagger, \psi_3^\dagger, \psi_4^\dagger)$. Then, pre-multiplying (10) by $\psi^\dagger$ leads to

$$ir_c\psi^\dagger\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha}\cdot\nabla\right)\psi = \psi^\dagger\beta\psi\,. \tag{11}$$

Taking the hermitian conjugate of (10), post-multiplying by ψ, then yields [1, p. 76]

$$-ir_c\left(\frac{\partial}{c\partial t} + \boldsymbol{\alpha}\cdot\nabla\right)\psi^\dagger\psi = \psi^\dagger\beta\psi\,. \tag{12}$$

Subtracting (12) from (11) finally leads to the continuity equations [1, p. 76]

$$ir_c\left[\frac{\partial(\psi^\dagger\psi)}{c\partial t} + \nabla\cdot(\psi^\dagger\boldsymbol{\alpha}\psi)\right] = 0 \tag{13}$$

or

$$\frac{\partial(\psi^\dagger\psi)}{c\partial t/r_c} + \sum_{k=1}^{3} \frac{\partial(\psi^\dagger\alpha_k\psi)}{\partial x^k/r_c} = 0 \tag{14}$$

for the electron. From (8), the presence of r_c in these two equations connects the electron core dynamics to a wave traveling within the vacuum state [5].

2 Comments and Conclusions

Dividing (13) by ir_c yields the equation

$$\frac{\partial(\psi^\dagger\psi)}{\partial t} + \nabla\cdot(\psi^\dagger c\boldsymbol{\alpha}\psi) = 0 \tag{15}$$

where the 4×4 matrix $c\boldsymbol{\alpha}$ looks like a velocity operator because of the speed of light c. This observation then leads intuitively to the standard continuity equation [1, p. 76]

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0 \tag{16}$$

where $\rho = \psi^\dagger\psi$ is the probability density and $j^k = \psi^\dagger c\alpha_k\psi$ is the kth component of the probability current density. Integrating (16) over the volume V (assumed to contain the electron core $(-e_*, m)$), and using the divergence theorem, leads to [1, p. 77]

$$\frac{\partial}{\partial t}\int_V d\rho\, d^3x + \int_S \mathbf{j}\cdot d\vec{S} = 0\,. \tag{17}$$

where the surface S surrounds the volume V.

So far, so good. But there is a problem: treating $c\boldsymbol{\alpha}$ as a free-space matrix velocity leads to a tortured interpretation of that operator that cries out for a better explanation. From the PV perspective, that explanation is apparent from equation (14)

$$\frac{\partial(\psi^\dagger\psi)}{c\partial t/r_c} + \sum_{k=1}^{3} \frac{\partial(\psi^\dagger\alpha_k\psi)}{\partial x^k/r_c} = 0$$

where the Minkowski-like line elements, cdt and dx^k associated with the partial derivatives, are normalized by the electron Compton radius r_c. The form of this equation suggests that it is associated with a distorted spacetime [6, p. 27] (the distortion coming from the r_c and the α_k), rather than a free-space velocity dynamic. Furthermore, the absence of the dynamical electron parameters p and E from (8), and the fact that $c\boldsymbol{\alpha}$ is not a recognizable free-space operator, suggest that (14) refers to a PV substructure dynamic [7] (driven by the electron core dynamic), where the normalized ct-gradient of $(\psi^\dagger\psi)$ equals the normalized negative divergence of $(\psi^\dagger\boldsymbol{\alpha}\psi)$.

Finally, the assumption that the PV is a degenerate state implies that the Planck-particle energy eigenstates are full. So if there is a current wave propagating within the PV, it cannot involve a Planck particle current (because the Planck particles are not free to move macroscopically). Thus $c\boldsymbol{\alpha}$ must refer, in part, to a localized percussion-like spinor wave within that vacuum state, analogous to a wave traveling on the surface of a kettle drum.

Equations (13) and (14) and the previous two paragraphs represent the PV view of the Dirac-electron continuity equation.

Submitted on April 15, 2016 / Accepted on April 30, 2016

References

1. Gingrich D.M. Practical Quantum Electrodynamics. CRC, The Taylor & Francis Group, Boca Raton, London, New York, 2006.
2. Daywitt W.C. Why the Proton is Smaller and Heavier than the Electron. *Progress in Physics*, 2014, v. 10, 175.
3. Daywitt W.C. The Trouble with the Equations of Modern Fundamental Physics. *American Journal of Modern Physics, Special Issue: Physics Without Higgs and Without Supersymmetry*, 2016, v. 5, no. 1-1, 22.
4. Daywitt W.C. The de Broglie Relations Derived from the Electron and Proton Coupling to the Planck Vacuum State. *Progress in Physics*, 2015, v. 11(2), 189.
5. Daywitt W.C. The Dirac-Electron Vacuum Wave. *Progress in Physics*, 2016, v. 12(3), 222.
6. Leighton R.B. Principles of Modern Physics. McGraw-Hill Book Co., New York, Toranto, London, 1959.
7. Daywitt W.C. The Strong and Weak Forces and their Relationship to the Dirac Particles and the Vacuum State. *Progress in Physics*, 2015, v. 11(1), 18.