The Dual Behavior of Quantum Fields and the Big Bang

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We modify the propagator of the quantum fields for the quarks and gluons. With that we have finite results (without ultraviolet divergence) in the perturbation theory. Then we search for $a^2 p^2 \to 0$ and $a^2 k^2 \to 0$ with fixing the Lagrangian parameters Z_i, therefore we can ignore our modification. We find the situation $a^2 p^2 \to 0$ and $a^2 k^2 \to 0$ associates with the free particles situation $g \to 0$ (g is the coupling constant) and the situation $a \neq 0$ associates with the perturbation breaking. We try to give the modification terms $a^2 p^2/(1 + a^2 p^2)$ and $a^2 k^2/(1 + a^2 k^2)$ physical aspects, for that we find the corresponding terms in the Lagrangian. To do that we find the role of those terms in the Feynman diagrams, in self energies, quarks gluons vertex, … We see we can relate the propagator modification to fields dual behavior, pairing particle-antiparticle appears as scalar particles with mass $1/a$. For the quarks we can interrupt these particles as pions with charges $(-1, 0, +1)$. If we used the propagator modification for deriving the quarks static potential $U(r)$ of exchanged gluons and pions we find $U(0) \sim 1/a$ if we compare this with the Coulomb potential we find the length a equivalent to the smallest distance between the interacting quarks. We use the static potential in quarks plasma study. We find the free and confinement quarks phases. We suggest a nuclear compression. We find there is a decrease in the global pressure due to the nuclear condensation. We use this decrease in the Friedman equations solutions, we find we can control the dark matter and dark energy, we can cancel them.

1 Quarks and gluons propagator modification

To remove the ultraviolet (UV) divergences in the quarks and gluons perturbed interaction, we modify the propagator like:

$$\overline{\Delta}_{\mu\nu}^{ab}(k^2) = \frac{g_{\mu\nu}\,\delta^{ab}}{k^2 - i\varepsilon}\left(1 - \frac{a^2 k^2}{1 + a^2 k^2}\right) \quad \text{for gluons} \qquad (1.1)$$

$$\overline{S}_{ij}(p) = \frac{-\not{p}\,\delta_{ij}}{p^2 - i\varepsilon}\left(1 - \frac{a^2 p^2}{1 + a^2 p^2}\right) \quad \text{for quarks} \qquad (1.2)$$

the indexes a and b are gluons indexes, i and j color indexes and a is critical length, $\hbar = c = 1$. We use this modification in calculating the quarks self-energy for the perturbation interaction with the gluons, then we renormalize the interaction and search for the condition $a^2 p^2 \to 0$ and $a^2 k^2 \to 0$. We have

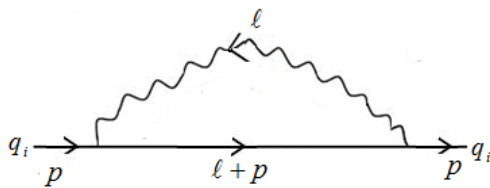Fig. 1: The quarks self energy in strong interaction.

$$i\Sigma_{ij}(\not{p}) = \int \frac{d^4\ell}{(2\pi)^4}\left[ig_s\gamma^\mu T_{ik}^a \frac{\overline{S}_{kl}(\not{p}+\ell)}{i} ig_s\gamma^\nu T_{lj}^b\right]\frac{\overline{\Delta}_{\mu\nu}^{ab}(\ell^2)}{i}$$

$$= g_s^2 T_{ik}^a T_{lj}^b \int \frac{d^4\ell}{(2\pi)^4}\left[\gamma^\mu \frac{(-\not{p}-\ell)\delta_{kl}}{(p+\ell)^2}\gamma^\nu\right]\frac{g_{\mu\nu}\delta^{ab}}{\ell^2}.$$

So

$$i\Sigma_{ij}(\not{p}) = g_s^2 T_{ik}^a T_{kj}^a \int \frac{d^4\ell}{(2\pi)^4}\left[\gamma^\mu \frac{(-\not{p}-\ell)}{(p+\ell)^2}\gamma^\nu\right]\frac{g_{\mu\nu}}{\ell^2}$$

$$= g_s^2 C(R)\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4}\left[\gamma^\mu \frac{(-\not{p}-\ell)}{(p+\ell)^2}\gamma_\mu\right]\frac{1}{\ell^2}$$

using $\gamma^\mu(-\not{p}-\ell)\gamma_\mu = 2(-\not{p}-\ell)$, it becomes

$$i\Sigma_{ij}(\not{p}) = 2g_s^2 C(R)\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4}\frac{(-\not{p}-\ell)}{(p+\ell)^2}\frac{1}{\ell^2}.$$

Now we use the gluon modified propagator

$$\overline{\Delta}_{\mu\nu}^{ab}(k^2) = \frac{g_{\mu\nu}\delta^{ab}}{k^2 - i\varepsilon}\left(1 - \frac{a^2 k^2}{1 + a^2 k^2}\right)$$

we get

$$i\Sigma_{ij}(\not{p}) = 2g_s^2 C(R)\delta_{ij}$$

$$\int \frac{d^4\ell}{(2\pi)^4}\frac{(-\not{p}-\ell)}{(p+\ell)^2}\frac{1}{\ell^2}\left(1 - \frac{a^2 \ell^2}{1 + a^2 \ell^2}\right) \qquad (1.3)$$

$$= 2g_s^2 C(R)\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4}\frac{(-\not{p}-\ell)}{(p+\ell)^2}\frac{1}{\ell^2}\frac{1}{1 + a^2\ell^2}. \qquad (1.4)$$

For massive quarks, the self-energy becomes:

$$i\Sigma_{ij}(\not{p}) = g_s^2 C(R)\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4}\frac{N}{(p+\ell)^2 + m_q^2}\frac{1}{\ell^2 + m_\gamma^2}\frac{1}{1 + a^2\ell^2}$$

with $N = \gamma^\mu(-\not{p} - \not{\ell} + m)\gamma_\mu$, using the Feynman formula:

$$\frac{1}{((p+\ell)^2 + m^2)\cdot(\ell^2 + m_\gamma^2)\cdot(1/a^2 + \ell^2)}$$
$$= \int dF_3 \frac{1}{\left[((p+\ell)^2 + m^2)x_1 + (\ell^2 + m_\gamma^2)x_2 + (1/a^2 + \ell^2)x_3\right]^3}$$

with $\int dF_3 = 2\int_0^1 dx_1 dx_2 dx_3\, \delta(x_1 + x_2 + x_3 - 1)$ and setting the transformation $q = \ell + x_1 p$ with changing the integral to be over q and making transformation to Euclidean space, the self-energy becomes [2]

$$i\Sigma_{ij}(\not{p}) = g_s^2 C(R)\delta_{ij} i \int \frac{d^4\bar{q}}{(2\pi)^4}\frac{1}{a^2}\int dF_3 \frac{N}{[\bar{q}^2 + D]^3}$$

with $D = -x_1^2 p^2 + x_1 p^2 + x_1 m^2 + x_2 + m_\gamma^2 + (1 - x_1 - x_2)1/a^2$. The linear term in q integrates to zero, using $q = \ell + x_1 p$, N is replaced with [2]

$$N \to -2(1 - x_1)\not{p} - 4m.$$

Using the relation

$$\int \frac{d^d\bar{q}}{(2\pi)^d}\frac{(\bar{q}^2)^a}{(\bar{q}^2 + D)^b} = \frac{\Gamma(b - a - \frac{d}{2})\Gamma(a + \frac{d}{2})}{(4\pi)^{\frac{d}{2}}\Gamma(b)\Gamma(\frac{d}{2})} D^{-(b-a-\frac{d}{2})},$$

the integral over q in Euclidean space becomes:

$$\Sigma_{ij}(\not{p}) = g_s^2 C(R)\delta_{ij}\frac{1}{a^2}\int dF_3 N \frac{\Gamma(3-2)\Gamma(2)}{(4\pi)^2\Gamma(3)\Gamma(2)} D^{-(3-2)}$$
$$= g_s^2 C(R)\delta_{ij}\frac{1}{a^2}\int dF_3 \frac{N}{16\pi^2 \times 2} D^{-1}.$$

The self-energy becomes

$$\Sigma_{ij}(\not{p}) = g_s^2 C(R)\delta_{ij}\frac{1}{a^2}\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{N}{16\pi^2}\frac{1}{D}$$

$$= \frac{g_s^2 C(R)\delta_{ij}}{16\pi^2}\int_0^1 dx_1 \int_0^{1-x_1} dx_2$$
$$\frac{-2(1-x_1)\not{p} - 4m}{a^2\left[-x_1^2 p^2 + x_1 p^2 + x_1 m^2 + x_2 m_\gamma^2 + (1 - x_1 - x_2)/a^2\right]}.$$

We write

$$\Sigma_{ij}(\not{p}) = C(R)\delta_{ij}\frac{g_s^2}{8\pi^2}\int_0^1 dx_1 \int_0^{1-x_1} dx_2$$
$$\frac{-(1-x_1)\not{p} - 2m}{[a^2 f + (1 - x_1 - x_2)]} \quad (1.5)$$

with $f = -x_1^2 p^2 + x_1 p^2 + x_1 m^2 + x_2 m_\gamma^2$ this is a finite result (without divergences).

Now we renormalize the fermions propagator to give the real states and let $a \to 0$. The interacting quarks propagator becomes [2]:

$$\overline{S}(\not{p})^{-1} = \not{p} + m - \Sigma(\not{p}). \quad (1.6)$$

To renormalize the interacting field, we write it as

$$\overline{S}(\not{p})^{-1} = \not{p} + m - \Sigma(\not{p}) = Z_2\not{p} + Z_m m. \quad (1.7)$$

The parameters Z_2 and Z_m are the renormalization parameters, later we try to make them constants. For the interacting field ψ we have:

$$\langle 0|\, \psi(\not{p})\bar\psi(-\not{p})\,|0\rangle = \frac{1}{i}\frac{1}{\not{p} + m - \Sigma(\not{p})} = \frac{1}{i}\frac{1}{Z_2\not{p} + Z_m m}$$
$$= \frac{1}{iZ_2}\frac{1}{\not{p} + Z_2^{-1}Z_m m}.$$

We can rewrite as

$$\langle 0|\, \sqrt{Z_2}\psi(\not{p})\sqrt{Z_2}\bar\psi(-\not{p})\,|0\rangle = \frac{1}{i}\frac{1}{\not{p} + Z_2^{-1}Z_m m}$$

and make $m_0 = Z_2^{-1}Z_m m$ and $\psi_0 = \sqrt{Z_2}\psi$ with that we have bare fields ψ_0 that are like the free fields and like the classical fields, so we can make them independent of the interaction, so $\partial\psi_0/\partial p^2 = \partial m_0/\partial p^2 = 0$ for $a \to 0$ and by that we renormalize the interaction. We make ψ the interacting field with mass m the physical mass, but we have to make $\Re[\Sigma(-m)] = 0$ in (1.6) but with $m_\gamma^2 < 0$. From (1.5) and (1.7) we have

$$Z_2 = 1 + C(R)\frac{g_s^2}{8\pi^2}\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1 - x_1}{[a^2 f + (1 - x_1 - x_2)]}$$

$$Z_m = 1 + C(R)\frac{g_s^2}{8\pi^2}\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{2}{[a^2 f + (1 - x_1 - x_2)]}$$

and $f = -x_1^2 p^2 + x_1 p^2 + x_1 m_q^2 + x_2 m_\gamma^2$.

By that we remove the self-energy of the interacting quark and make the mass variable. For easiness we ignore m_q and m_γ so

$$Z_2 = 1 + C(R)\frac{\alpha_s}{2\pi}\int_0^1 (1-x)\ln\left(1 + \frac{1}{a^2 p^2 x}\right)dx$$
$$= 1 + \frac{C(R)\alpha_s}{4\pi(a^2 p^2)^2}\left[(a^2 p^2)^2 \ln\left(1 + \frac{1}{a^2 p^2}\right) -\right.$$
$$\left. - a^2 p^2 + (2a^2 p^2 + 1)\ln(a^2 p^2 + 1)\right].$$

Now we fix $Z_2 = constant$ and search for the situations $-a^2 p^2 \to 0$ for timelike and $a^2 p^2 \to 0$ for spacelike, we have

$$\frac{\alpha_s}{(a^2 p^2)^2}\left[(a^2 p^2)^2 \ln\left(1 + \frac{1}{a^2 p^2}\right) -\right.$$
$$\left. - a^2 p^2 + (2a^2 p^2 + 1)\ln\left(a^2 p^2 + 1\right)\right] = c.$$

For spacelike $p^2 > 0$, we have Fig. 2. According to this figure, we have $a^2 p^2 = \exp(-c/\alpha_s) \to 0$ when $\alpha_s \to 0$ this is the
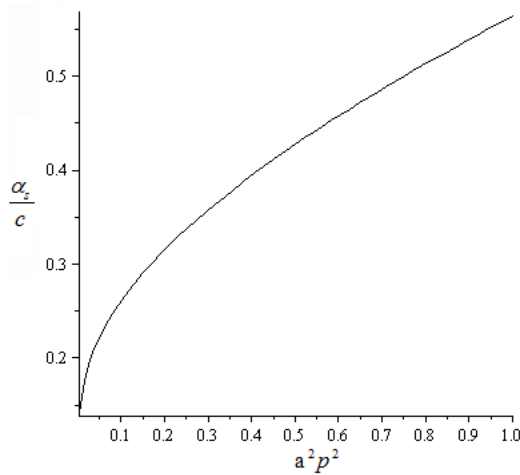
Fig. 2: The behavior of the length a with fixing Z_2.

decoupling; $p^2 \gg \Lambda_{QCD}^2$. It is the free quarks and gluons situation; $\alpha_s \to 0$ occurs at high energy for the free quarks phase. Because $ap \to 0$ so $p \ll 1/a$ this gives $r \gg a \to 0$, therefore the propagator modification is ignored. So the behavior of the length a is like the behavior of the coupling constant α_s and the modification terms are removed $ap \ll 1$ at high energy (free quarks phase).

For the limited low energy we fix $\alpha_s/a^2 = constant \times \sigma$, σ is string tension that appears in the low energy static potential $U(r)$ as we will see, for $a \to 0$ we have

$$Z_2 = 1 + C(R)\frac{\alpha_s}{4\pi}\left(\frac{3}{2} - \ln\left(p^2 a^2\right) + O(p^2 a^2)\right) \to 1, \text{ when } a \to 0$$

$$Z_m = 1 + C(R)\frac{\alpha_s}{\pi}\left(1 - \ln\left(p^2 a^2\right) + O(p^2 a^2)\right) \to 1, \text{ when } a \to 0$$

We know the strong interaction coupling constant α_s increases extremely at low limited energy, therefore, according to the figure, we can't let $a \to 0$, so we assume when the perturbation breaks down the length a could not be removed and takes non-zero value, let it be a_0, so the propagator modification takes place.

1.1 The confinement situation

According to Fig. 2 it is possible to have $ap > 1$ (the coupling constant α_s increases extremely at low energy), therefore $p > 1/a \to r < a$ which is the quarks confinement phase at low energy.

To study the quarks confinement, we use the modified gluons propagator in deriving the static potential of the quark-quark gluons exchange. We define this potential in momentum space using M matrix elements for quark-quark (gluons exchange) interaction, with $\omega_0 = k_0 = 0$ (like the Born approximation to the scattering amplitude in non-relativistic quantum mechanics [1])

$$iM = -i\tilde{V}(k)J^\mu(p_2', p_2)J_\mu(p_1', p_1)$$

with the transferred current $J^\mu(p', p) = \bar{u}(p')\gamma^\mu u(p)$ where spinor states $u(p)$ include the helicity states.

We find M matrix elements using the Feynman diagrams for quark-quark gluons exchange using color representation for one quark like

$$u(p)_{color \otimes spinor} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(p)_{spinor}.$$

For distinguishable quarks (only one diagram), we have

$$iM = \bar{u}^i(p_2')ig_s\gamma^\mu(T^a)_i^j u_j(p_2)\frac{\Delta_{\mu\nu}^{ab}(k^2)}{i}\bar{u}^k(p_1')ig_s\gamma^\nu(T^b)_k^\ell u_\ell(p_1)$$

with $k = p_2' - p_2 = p_1 - p_1'$.

Using Gell-Mann matrices, we consider the matrices $T^a = \lambda^a; \lambda_1, \dots, \lambda_8$ as $SU(3)$ generators, and using the modified gluons propagator we have

$$iM = \sum_{ijk\ell} ig_s^2 \bar{u}^i(p_2')\gamma^\mu(T^a)_i^j u_j(p_2)\frac{g_{\mu\nu}\delta^{ab}}{k^2}\left(1 - \frac{a^2 k^2}{1 + a^2 k^2}\right)$$
$$\bar{u}^k(p_1')\gamma^\nu(T^b)_k^\ell u_\ell(p_1)$$

to sum over the color indexes i, j with the color representation like above and over gluon index a we write

$$\sum_{ij} \bar{u}^i(p_2')\gamma^\mu(T^a)_i^j u_j(p_2)$$
$$= \bar{u}(p_2')\gamma^\mu \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}(T^a)\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(p_2)$$

and

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}(T^a)\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{3}\sum_{ij}(T^a)_i^j.$$

Therefore the M matrix elements become

$$M = \frac{1}{9}\sum_a\left(\sum_{ij}(T^a)_i^j\right)^2 g_s^2 \bar{u}(p_2')\gamma^\mu u(p_2)\frac{1}{k^2}\left(1 - \frac{a^2 k^2}{1 + a^2 k^2}\right)$$
$$\bar{u}(p_1')\gamma_\mu u(p_1).$$

The Gell-Mann matrices with nonzero sum of the elements are

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

So

$$\sum_a \left(\sum_{ij} (T^a)_i^j \right)^2 = 3\,(2)^2 = 12.$$

Therefore we have

$$M = \frac{12g_s^2}{9} \frac{1}{k^2} \left(1 - \frac{a^2 k^2}{1 + a^2 k^2} \right) \bar{u}(p_2')\gamma^\mu u(p_2)\bar{u}(p_1')\gamma_\mu u(p_1).$$

We have the potential $\tilde{V}(k)$ in momentum space as we defined

$$iM = -i\tilde{V}(k)J^\mu(p_2', p_2)J_\mu(p_1', p_1)$$
$$= i\frac{12g_s^2}{9} g_s^2 \bar{u}(p_2')\gamma^\mu u(p_2)\frac{1}{k^2}\left(1 - \frac{k^2}{k^2 + 1/a^2}\right)\bar{u}(p_1')\gamma_\mu u(p_1)$$

with the transferred currents $J^\mu(p_2', p_2) = \bar{u}(p_2')\gamma^\mu u(p_2)$ and $J^\mu(p_1', p_1) = \bar{u}(p_1')\gamma^\mu u(p_1)$. So we have

$$\tilde{V}(k) = -\frac{4g_s^2}{3}\frac{1}{k^2}\left(1 - \frac{k^2}{k^2 + 1/a^2}\right).$$

Making the Fourier transformation to the space XYZ, we have the static potential $U(x)$ ($k_0 = 0$) like the electric potential [1]

$$U(x) = \int \frac{d^3 k}{(2\pi)^3} \tilde{V}(k) e^{ik\cdot x}$$
$$= -\frac{4g_s^2}{3} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2}\left(1 - \frac{k^2}{k^2 + 1/a^2}\right)e^{ik\cdot x}$$
$$= -\frac{4g_s^2}{3 \times 4\pi r}\left(1 - \exp\left(-\frac{r}{a}\right)\right) \text{ with } r = \sqrt{x^2 + y^2 + z^2}.$$

For low limited energy we have $ap > 1$ (Fig. 2) so $r < a$, the static potential becomes

$$U(r) = -\frac{4g_s^2}{3 \times 4\pi r}\left[1 - \exp\left(-\frac{r}{a}\right)\right] = -u_0 + a_1 r - a_2 r^2 + \dots$$

with

$$u_0 = \frac{4}{3}\frac{g_s^2}{4\pi a} = \frac{4\alpha_s}{3a},$$
$$a_1 = \sigma = \frac{g_s^2}{3 \times 2\pi a^2} = \frac{2\alpha_s}{3a^2},$$
$$a_2 = \frac{4\alpha_s}{3 \times 6a^3}.$$

To fix $u_0 = 4\alpha_s/3a$ we write it as

$$u_0 = \frac{4\alpha_s}{3a} = \frac{4\alpha_s}{3a^2}a = 2\sigma a$$

fixing the string tension σ and the length $a \to a_0$ at low energy.

This potential appears at low limited energy and prevents the quarks from spreading away, $r < a$ so it holds the quarks inside the hadrons. But starting from the high energies $a \to$

0, although the quarks masses are small but they are created only at high energies where they are free and by dropping the energy the situation $r < a$ appears, the length a would run and becomes higher at low energies, so have $-a^2 k^2 > 1$ for $r < a$ which is the confinement. The confinement (at low limited energy) means when $r \to a$ the two interacting quarks kinetic energy becomes zero (ignore the quark mass), therefore the highest kinetic energy that the quark can get equals σa which relates to the potential $U(r) = -u_0 + \sigma r + \dots$ for $r < a$.

We can make $U(r)$ the potential for all quarks in $r < a$ so $\sigma \to \sum \sigma$ and consider r as average distance between the interacting quarks, so the energy σa becomes the highest kinetic energy of all quarks. When $r \to a$ the potential becomes $U(0) = -u_0 = -4\alpha_s/3a = -\sigma a < 0$ therefore the total quarks energy becomes negative.

In this situation the free quarks disappear, they become condensed in the hadrons. So the role of the potential is reducing the number of free quarks. Therefore the potential $u_0 = \sigma a$ leads to decrease of the free quarks chemical potential μ_0, and we have

$$\mu_0 \to \mu_0 + U(r) = \mu_0 - \frac{\alpha_s}{r}\left(1 - e^{-r/a}\right) = \mu(r)$$
$$\approx \mu_0 - u_0 + \sigma r \text{ for } r < a$$

where we replaced $4\alpha_s/3$ with α_s. We renormalize this step at high energy for the free quarks, quarks plasma.

2 The quarks field dual behavior

To have finite results in the perturbation interaction, we modified the propagator like

$$\overline{\Delta}_{\mu\nu}^{ab}(k^2) = \frac{g_{\mu\nu}\delta^{ab}}{k^2 - i\varepsilon}\left(1 - \frac{a^2 k^2}{1 + a^2 k^2}\right) \text{ for gluons}$$

$$\bar{S}_{ij}(\not p) = \frac{-\not p \delta_{ij}}{p^2 - i\varepsilon}\left(1 - \frac{a^2 p^2}{1 + a^2 p^2}\right) \text{ for quarks}.$$

We saw we can ignore the modification terms $a^2 p^2/(1 + a^2 p^2)$ and $a^2 k^2/(1 + a^2 k^2)$ at high energy, but when the energy drops down to limited energy, those terms take place, we can give them a physical meaning, for that we search for the corresponding terms in the Lagrangian.

To do this, we find the role of those terms in the Feynman diagrams, in self energies, quarks-gluons vertex, ... We find that the terms $a^2 p^2/(1 + a^2 p^2)$ and $a^2 k^2/(1 + a^2 k^2)$ can be related to pairing quark-antiquark that appear as scalar particles with mass $1/a$ and charges $(-1, 0, +1)$ and we can interpret these particles as pions.

That appears in the particles-antiparticles composition in Feynman diagrams which mean for the fields, there is fields dual behavior, free fields and composite fields, this behavior leads to the possibility of separating the particles and possibility for their composition, so the dual behavior of the fields is elementary behavior. In general, for any particle A and its

antiparticle $\bar{A}$, in pertubation interaction, they pair and have a scalar particle $\bar{A}A$, this leads to reduce the currents (charges) of particles and antiparticles.

That is, for each outcoming particle, in Feynman diagrams, there is incoming antiparticle with positive energy and negative mass, depending on the coupling constant behavior (this is at high energy for the electromagnetic interaction and at low energy for the strong interaction, quarks and gluons). Therefore reducing their interactions with the charges in a way leads to finite results in the perturbation results.

Using the gluons modified propagator, the quark self-energy becomes (1.3)

$$i\Sigma_{ij}(\not{p}) = 2g_s^2 C(R)\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not{p}-\not{\ell})}{(p+\ell)^2} \frac{1}{\ell^2}\left(1 - \frac{a^2\ell^2}{1+a^2\ell^2}\right).$$

We can separate it into two parts

1. Quark−gluon part:

$$i\Sigma_{ij}(\not{p}) = 2g_s^2 C(R)\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not{p}-\not{\ell})}{(p+\ell)^2} \frac{1}{\ell^2};$$

2. pairing quarks part:

$$i\Sigma_{ij}(\not{p}) = 2g_s^2 C(R) \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not{p}-\not{\ell})\delta_{ij}}{(p+\ell)^2} \frac{1}{\ell^2}\left(-\frac{a^2\ell^2}{1+a^2\ell^2}\right).$$

It appears that in the pairing part there is a scalar field φ propagator:

$$\frac{1}{i}\frac{1}{\ell^2 + 1/a^2}$$

which is real scalar particles field propagator with mass $1/a$, to preserve the charges, spin, ..., this particle must be condensed of quark-antiquark $|\bar{q}q\rangle$ (particle-antiparticle in general) so we have new diagram (Fig. 3), we rewrite
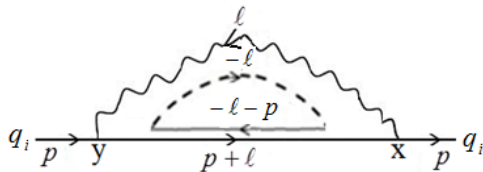

Fig. 3: Representation the dual behavior, joined particle−antiparticle with opposite momentum−energy.

$$i\Sigma_{ij}(\not{p}) = 2(-g_s)^2 C(R) \int \frac{d^4\ell}{(2\pi)^4} \frac{(-\not{p}-\not{\ell})\delta_{ij}}{i(p+\ell)^2} \frac{-i}{(\ell)^2 + 1/a^2}.$$

Therefore we must add new interaction terms to the quarks Lagrangian, the possible terms are:

$$\Delta L = -ig_{\varphi q}\varphi\bar{Q}Q \quad \text{with} \quad g_{\varphi q} = g_s\sqrt{2C(R)}$$

or

$$\Delta L = g_{\varphi q}\varphi\bar{Q}\gamma_5 Q.$$

We expect the pairing particles-antiparticles preserve the flavor symmetry, so the real scalar field φ becomes $|\overline{q_i}q_j\rangle$. For two flavors q_i and q_j we write the quarks field like $Q = \begin{pmatrix} q_i & q_j \end{pmatrix}^T$ so

$$\Delta L = -ig_{\varphi q}\varphi^a\bar{Q}T_2^a Q \quad \text{or} \quad \Delta L = g_{\varphi q}\varphi^a\bar{Q}T_2^a\gamma_5 Q.$$

The real scalar fields φ^a could interact with itself and have real non-zero ground value v then $\langle\varphi\rangle = v$ so we can renormalize it like

$$\varphi^a T_2^a \rightarrow v - iv\pi^a T_2^a + ...$$

then we have

$$\Delta L = -ig_{\varphi q}\bar{Q}(v - iv\pi^a T_2^a + ...)Q$$
$$= -ig_{\varphi q}v\bar{Q}Q - g_{\varphi\pi}\pi\bar{Q}Q + ... \text{ Chiral symmetry breaking}$$

or

$$\Delta L = g_{\varphi q}\bar{Q}(v - iv\pi^a T_2^a + ...)\gamma_5 Q \rightarrow g_{\pi q}\bar{Q}\gamma_5 Q - ig_{\pi q}\pi\bar{Q}\gamma_5 Q + ...$$

Here the particles $\pi^a T_2^a \rightarrow \pi = (\pi^0, \pi^-, \pi^+)$ are the pions. The unusual terms $-ig_{\varphi a}v\bar{Q}Q$ and $g_{\pi q}\bar{Q}\gamma_5 Q$ are not hermitian and violate the symmetries, so they let the quarks disappear, damping at low energy $r < a$:

$$e^{i\Delta Et}|Q\rangle = e^{-i\Delta Lt}|Q\rangle = e^{-g_{\varphi q}v\bar{q}qt}|Q\rangle$$
$$= \sum_n e^{-g_{\varphi q}v(\bar{q}q)t}|E_n\rangle\langle E_n |Q\rangle \rightarrow |0\rangle\langle 0 |Q\rangle .$$

E_n is the energy of the quarks in state $|n\rangle$ and $e^{i\hat{H}t}|Q\rangle$ is the eigenstate of the quarks field operator $\hat{Q}(t)$ in Heisenberg picture, $\hat{Q}(t) = e^{i\hat{H}t}\hat{Q}e^{-i\hat{H}t}$.
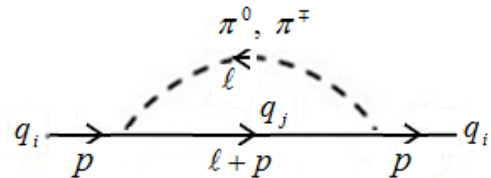

Fig. 4: The quarks interaction with pions as a result of dual behavior.

That damping in the states is because of the pairing quark-antiquark at low energy $a \neq 0$, this pairing reduces the charges (currents) of free quarks (Fig. 5). We can see that if we relate the minus sign in $-a^2\ell^2/(1 + a^2\ell^2)$ to the fermions propagator:

$$S(x - y) = \int \frac{d^4p}{(2\pi)^4}\frac{-\not{p}}{p^2}e^{ip(x-y)} \text{ (propagator from } y \text{ to } x)$$

so

$$-S(x - y) = -\int \frac{d^4p}{(2\pi)^4}\frac{-\not{p}}{p^2}e^{ip(x-y)} = \int \frac{d^4p}{(2\pi)^4}\frac{+\not{p}}{p^2}e^{ip(x-y)}$$

change $p \rightarrow -p$ (propagator from x to y)

$$-S(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{-\slashed{p}}{p^2} e^{-ip(x-y)} = \int \frac{d^4 p}{(2\pi)^4} \frac{-\slashed{p}}{p^2} e^{ip(y-x)} .$$

So it is equivalent to invert the propagator $y \rightarrow x$ to $x \rightarrow y$ with positive energy and negative mass. Therefore it reduces the charges, currents, energies, ... of the particles and antiparticles, we have
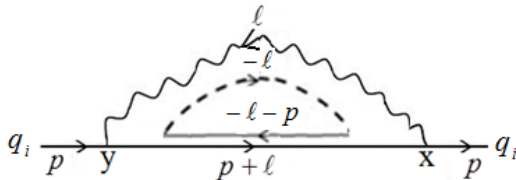

Fig. 5: Omitting the distance x-y from the propagator.

$$(p + \ell) + (-p - \ell) = 0 \text{ and } (-\ell) + (\ell) = 0$$

so incoming with p and outcoming with p, it is like to say the particles jump from y to x, in other words the distance $y - x$ is removed from the interaction. We expect the fields dual behavior takes place in negative potential. If there is no negative potential the paired particles would not survive (never condense). For the quarks, the case $0 < r < a$ must associate with negative potential u and $E + u < 0$. Because the behavior of the strong interaction coupling constant at low energy α_s is high, we expect negative potential at low energy $E + u < 0$ $(E > 0, u < 0)$, so the quarks condense.

Because of the dual behavior of the quarks field which leads to quarks composite in scalar charged particles like the pions (π^-, π^0, π^+) and because of their quantized charges $(-1, 0, +1)$ we expect the hadrons charges to be also quantized $(-Q, -Q+1, \ldots 0, +1, \ldots, +Q)$ this quantization relates to the dual behavior of the quarks field in different hadrons, pairing quarks of different hadrons, so these condensed quarks; pions, kaons, ... are shared between the hadrons, so we put them together with the hadrons in groups, like the pions $(-1, 0, +1)$ which can be inserted in $SU(2)$ generators which can represent the proton-neutron pairing. Therefore the protons and neutrons Lagrangian contains the terms $-ig_{\pi N}\pi^\alpha \bar{N} T_2^\alpha N$ with the nucleon field $N = \begin{pmatrix} p \\ n \end{pmatrix}$.

3 The quarks plasma

We tried before to explain how the quarks are confined, for the strong interaction, we have the condition $r < a \neq 0$ at low limited energy and the condition $r > a \rightarrow 0$ at high energies for free quarks where the length a is removed from the propagators. But it appears to be fixed at low limited energy. In the last section we showed there is dual behavior for the quarks field, but when the length a is fixed, the result is scalar

particles (pions) with mass $1/a_0$ at low limited energy and the result is the chiral symmetry breaking. We found the length a appears in the quark-quark strong interaction (gluons exchanging) potential $U(r)_{r<a} < 0$, so it relates to interaction strength. That is because the behavior of the length a is like the behavior of the coupling constant α_s. The confinement (at low energy $r < a$) means when $r \rightarrow a$ the two interacting quarks kinetic energy becomes zero (ignore the quark mass), therefore the highest kinetic energy the quark can get equals σa which relates to the potential $U(r) = -u_0 + \sigma r + \ldots$ for $r < a$ (at low limited energy). When $r \rightarrow a$ the potential becomes $U(0) = -u_0 = -4\alpha_s/3a < 0$ therefore the total quarks energy becomes negative. In this situation the free quarks disappear ($\mu_0 \rightarrow 0$), they become condensed in the hadrons.

We try here to use statistical thermodynamics to show how the free quarks disappear at low energies (low temperatures) where the length a becomes fixed, so the chiral symmetry breaking and the quarks condensation. One of the results is that the confinement phase (3.14) not necessarily associates with chiral symmetry breaking, that is, the chiral symmetry breaking appears at the end of the cooling process when the expanding and cooling are ended and the length a becomes fixed, therefore the chiral symmetry breaking occurs and the pions become massive $m = 1/a_0$.

We start with the massless quarks, their energy in volume V is

$$E = c \int_{a^3} d^3 r \int_0^\infty d\varepsilon \, g(\varepsilon) \, \varepsilon \, \frac{1}{e^{\beta(\varepsilon - \mu(r))} + 1}$$

$$: g(\varepsilon) = g_q \frac{V}{2\pi^2} \varepsilon^2 \qquad (3.1)$$

where $\mu(r) = \mu_0 + u(r)$ with $u(r) = -\frac{4\alpha_s}{3r}\left(1 - e^{-r/a}\right)$.

Here we inserted the quark-quark strong interaction potential $U(r)$ in the chemical potential (for decreasing the free quarks energy, as we think, the quarks potential reduces the free quarks chemical potential and make them condense at low energy) and because $r < a$ we integrate over the volume a^3: r is the distance between the interacting quarks. We can replace $4\alpha_s/3 \rightarrow \alpha_s$.

The constant c is determined by comparing with free quarks high energy where the potential $U(r) \rightarrow 0$ and $\alpha_s \rightarrow 0$ (decoupling) at high energies, so the length $a \rightarrow 0$ that is as we said before, the behavior of the length a is like the behavior of the coupling constant g_s therefore the quarks become free at high energies.

By integrating over the energy (Maple program) we have:

$$E = cg_q \frac{V}{2\pi^2} \int_{a^3} d^3 r \int_0^\infty d\varepsilon \frac{\varepsilon^3}{e^{\beta(\varepsilon - \mu(r))} + 1}$$

$$= cg_q \frac{V}{2\pi^2 \beta^4} \int_{a^3} d^3 r \left[\frac{7\pi^4}{60} + \frac{\pi^2}{2} u_0(r)^2 + \frac{1}{4} u_0(r)^4 + \right.$$

$$\left. + 6 \sum_{k=1}^\infty \frac{(-1)^k e^{-k\beta\mu(r)}}{k^4} \right]$$

with $u_0(r) = \beta\mu(r) = \beta(\mu_0 + u(r))$. By integrating over r (the distance between the interacting quarks) we have

$$
E = cg_q \frac{2Va^3}{\pi x^4}\left[3.78 + 2\,(\beta\mu_0)^2\left(0.82 - 1.16\,\frac{\alpha_s}{a\mu_0} + \right.\right.
$$
$$
\left. + 0.41\left(\frac{\alpha_s}{a\mu_0}\right)^2\right) + (\beta\mu_0)^4\left(0.08 - 0.23\,\frac{\alpha_s}{a\mu_0} + \right.
$$
$$
\left.\left. + 0.25\left(\frac{\alpha_s}{a\mu_0}\right)^2 - 0.12\left(\frac{\alpha_s}{a\mu_0}\right)^3 + 0.02\left(\frac{\alpha_s}{a\mu_0}\right)^4\right) + \right.
$$
$$
\left. + 6\sum_{k=1}^{\infty}\int_0^1 x^2 dx\,\frac{(-1)^k e^{-k\beta\mu(x)}}{k^4}\right].
$$

g_q is the quarks degeneracy number and $x = \beta\mu_0$. For easiness we write $\alpha_s/a\mu_0 = 2\sigma a/\mu_0 = y$ in the energy relation. So it becomes

$$
E = cg_q \frac{2Va^3}{\pi x^4}\left[3.78 + 2(\beta\mu_0)^2\left(0.82 - 1.16y + 0.41y^2\right)\right.
$$
$$
+ (\beta\mu_0)^4\left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4\right)
$$
$$
\left. + 6\sum_{k=1}^{\infty}\int_0^1 x^2 dx\,\frac{(-1)^k e^{-k\beta\mu(x)}}{k^4}\right]. \tag{3.2}
$$

at high energy: $x = \beta\mu_0 = \mu_0/T \rightarrow 0$. To find the constant c we compare with quarks high energy where they are free massless particles:

$$
E_{high} = g_q V \frac{7\pi^2}{240} T^4.
$$

When T is high, $x = (\mu_0/T) \rightarrow 0$ and $y \rightarrow 0$ therefore $\beta\mu(x) \rightarrow 0$ so we expand $e^{-k\beta\mu(x)}$ near $\beta\mu(x) = 0$, we have:

$$
E_{high} = cg_q\frac{2a^3V}{\pi x^4}\left[3.78 - 1.88 + O(x, y)\right] \rightarrow cg_q\frac{2a^3V}{\pi x^4}1.9
$$
$$
\rightarrow g_q\frac{7\pi^2 V}{240}T^4 = cg_q\frac{2a^3V}{\pi x^4}1.9 \rightarrow c = \frac{\pi}{2a^3\,1.9}\frac{7\pi^2}{240}\mu_0^4 \tag{3.3}
$$

The energy becomes:

$$
E = \frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4 g_q\frac{V}{(\beta\mu_0)^4}\left[3.78 + 2(\beta\mu_0)^2(0.82 - 1.16y + \right.
$$
$$
+ 0.41y^2) + (\beta\mu_0)^4(0.08 - 0.23y + 0.25y^2 - 0.12y^3 +
$$
$$
\left. + 0.02y^4) + 6\sum_{k=1}^{\infty}\int_0^1 x^2 dx\,\frac{(-1)^k e^{-ku_0(x)}}{k^4}\right].
$$

Now we see the effects of the length a on the energy, at high energy, by fixing $x = \mu_0/T$ and varying $y = \sigma a/2\mu_0 < 1$:

$$
E_{high} = \frac{1}{1.9}\frac{7\pi^2}{240}g_q V\mu_0^4 x^{-4}\left[1.9 + x(1.8 - 1.24y) + \right.
$$
$$
+ x^2(0.82 - 1.18y + 0.42y^2) + x^3(0.23 - 0.47y +
$$
$$
+ 0.33y^2 - 0.08y^3) + x^4(0.04 - 0.12y + \tag{3.4}
$$
$$
\left. + 0.13y^2 - 0.07y^3 + 0.01y^4) + \dots\right]_{x=\beta\mu_0 \rightarrow 0}.
$$

We expanded $e^{-k\beta\mu(x)}$ near $\beta\mu(x) = 0$ and fixed the tension σ as we assumed before, so we have Fig. 6.
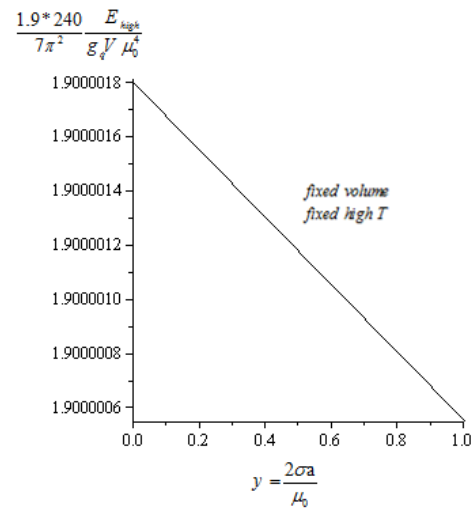

Fig. 6: Decreasing the high energy with increasing y.

It appears in the figure that the high energy quarks lose energy when the length a increases although the temperature is fixed. That means, when the length a increases the number of the excited quarks decreases. That is because of the attractive linear potential $\sigma r \dots$ between the quarks, that potential absorbs an energy ($r < a$ confinement, section 1), so the quarks are cooled faster by the expansion. As we said before, the behavior of length a is like the behavior of the coupling constant α_s so when the energy dropped to lowest energy, the length a increased extremely and this is fast cooling (extreme cooling). That occurs when the particles spread away, the length a, as a distance between the quarks, increases.

To determine the end, we search for the balance situations, such as zero pressure, confinement condition, ... First we find the high energy pressure including the effects of the potential σa. Starting from the general pressure relation:

$$
p = -\frac{\partial}{\partial V}F \text{ where } F = -T\ln Z = -\frac{1}{\beta}\ln Z
$$

here we use the relation:

$$
\ln Z = c\int_{a^3} d^3 r\int_0^{\infty} d\varepsilon\,g(\varepsilon)\ln\left(e^{-\beta(\varepsilon - \mu(r))} + 1\right)
$$
$$
: g(\varepsilon) = g_q\frac{V}{2\pi^2}\varepsilon^2
$$

and the pressure becomes

$$
P = \frac{1}{3}\frac{\partial}{\partial V}E
$$

so for high energy $x = \beta\mu_0 \to 0$ we have the pressure:

$$
\begin{aligned}
P_{high} &= \frac{1}{3}\frac{\partial}{\partial V}E_{high}\\
&= \frac{\partial}{\partial V}\frac{1}{3\times 1.9}\frac{7\pi^2}{240}g_q V\mu_0^4 x^{-4}\big[1.9 + x(1.8 - 1.24y) +\\
&\quad + x^2(0.82 - 1.18y + 0.42y^2) + x^3(0.23 - 0.47y +\\
&\quad + 0.33y^2 - 0.08y^3) + x^4(0.04 - 0.12y +\\
&\quad + 0.13y^2 - 0.07y^3 + 0.01y^4) + \dots\big].
\end{aligned}
$$

Now the key point is, we want to include the potential effect on the pressure so we replace the volume V with the volume $a^3 \sim y^3$ so

$$
\begin{aligned}
P_{high} \to &\frac{\partial}{\partial y^3}y^3\frac{1}{3\times 1.9}\frac{7\pi^2}{240}g_q \mu_0^4 x^{-4}\big[1.9 + x(1.8 - 1.24y) +\\
&+ x^2(0.82 - 1.18y + 0.42y^2) + x^3(0.23 - 0.47y +\\
&+ 0.33y^2 - 0.08y^3) + x^4(0.04 - 0.12y +\\
&+ 0.13y^2 - 0.07y^3 + 0.01y^4) + \dots\big]
\end{aligned}
\tag{3.5}
$$

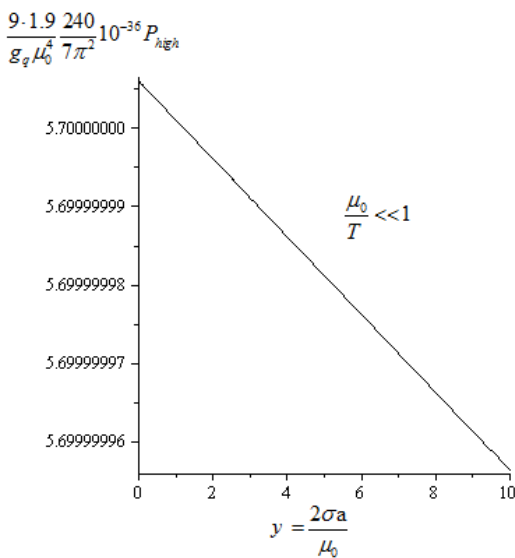which is represented in Fig. 7, without conditions on y or on the length a.

Fig. 7: The effects of potential σa on the pressure.

It is clear (without conditions on y) the pressure decreases with increasing the length a (decreasing the quarks energy $-p^2$) until it becomes zero, then negative. That becomes clear at low energy where there are conditions on y and so on the length a.

For the low energy quarks, $T \to 0$ so $\beta\mu(x) \to \infty$ so $e^{-k\beta\mu(x)} \to 0$. The energy becomes:

$$
\begin{aligned}
E_{low} = &\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4 g_q \frac{V}{(\beta\mu_0)^4}\big[3.78 +\\
&+ 2(\beta\mu_0)^2\left(0.82 - 1.16y + 0.41y^2\right) +\\
&+ (\beta\mu_0)^4\left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4\right)\big].
\end{aligned}
\tag{3.6}
$$

Making $x = T/\mu_0$ so

$$
\begin{aligned}
E_{low} = &\frac{1}{1.9}\frac{7\pi^2}{240}\mu_0^4 g_q V x^4\big[3.78 + 2x^{-2}\left(0.82 - 1.16y + 0.41y^2\right)\\
&+ x^{-4}\left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4\right)\big].
\end{aligned}
$$

Now the key point, we want to show the effect of the potential σa on the energy so we see the behavior of the energy in the volume a^3 with respect to $y = 2\sigma a/\mu_0$ the diagram is given in Fig. 8. That is extreme behavior after $y = 0.6$ where the
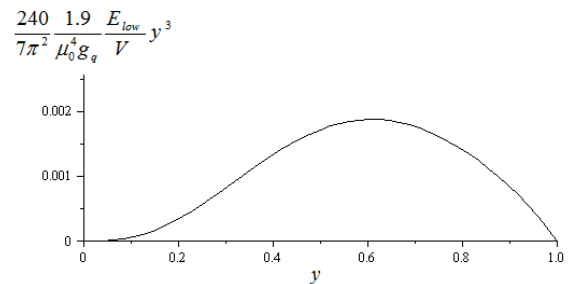

Fig. 8: The extremely decreasing in quarks low energy in the strong interaction.

energy $(E/V)a^3$ decreases when the volume a^3 increases, the end in $y = 1$ where the free quarks disappear for $y > 1$.

Now we can distinguish between the confinement and the chiral symmetry breaking, when $y > 0.6$ there is confinement: extreme cooling, negative pressure. But when reach $y = 1$ there is chiral symmetry breaking where the length a becomes fixed, and from the quarks field dual behavior there are scalar charged particles with mass $1/a$ appear when the length a is fixed with non-zero value a_0. Here the evidence for fixing the length a is the lowest limited quarks energy, that is as we said before, the behavior of the length a is like the behavior of the coupling constant α_s so when the quarks energy dropped (extreme cooling) the length a increases extremely to reach the highest value when $y = 1$ which is equivalent to smallest energy $E = 0$ (the cooling end). Another evidence for fixing the length a (chiral symmetry breaking) is the low energy pressure:

$$
P_{low} = \frac{1}{3}\frac{\partial}{\partial V}E_{low} \to \frac{1}{3}\frac{\partial}{\partial y^3}\frac{E_{low}}{V}y^3 .
$$

To include the potential effect we study the pressure using the

volume $a^3 \sim y^3$ therefore

$$P_{low} \rightarrow \frac{1}{3} \frac{\partial}{\partial y^3} \frac{1}{1.9} \frac{7\pi^2}{240} g_q \mu_0^4 y^3 x^4 \left[3.78 + \right.$$
$$+ 2x^{-2} \left(0.82 - 1.16y + 0.41y^2 \right)$$
$$\left. + x^{-4} \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) \right]$$

and therefore

$$\frac{P_{low}}{\mu_0^4} = \frac{1}{9 \times 1.9} \frac{7\pi^2}{240} g_q \left[3 \times 3.78 \, x^4 + \right.$$
$$+ 3 \times 2 \times x^2 \left(0.82 - 1.16y + 0.41y^2 \right) \qquad (3.7)$$
$$+ 3 \times \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) +$$
$$+ 2yx^2 \left(-1.16 + 0.82y \right) +$$
$$\left. + y \left(-0.23 + 0.5y - 0.36y^2 + 0.08y^3 \right) \right].$$
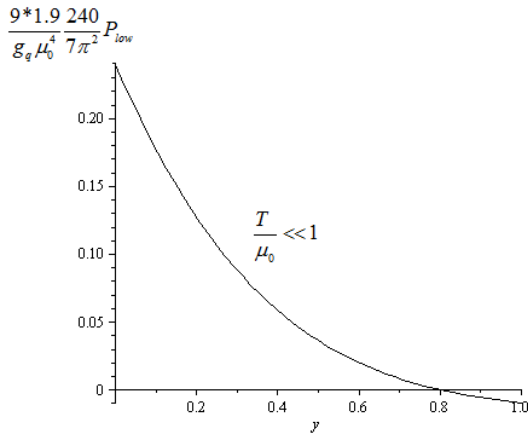
We see its behavior in Fig. 9 below

Fig. 9: The extremely decreasing in the pressure at low energy.

It is clear from the figure, when $y > 0.6$ the quarks pressure becomes negative. We expect the condensed quarks phase (confinement quarks) has positive pressure, so the preferred phase is the condensed quarks phase. So when $y > 0.6$ the quarks condense until $y = 1$: $a \rightarrow a_0 \approx 1/(135 - 140 \, \text{Mev})$ the quarks disappear, the scalar charged particles (pions) appear instead of them, that is because of the quarks dual behavior (free-condensed quarks), but at low limited energy the condensed phase has a big chance instead the free phase.

3.1 The confinement phase

In this paper we study two quarks (up and down) condensation in the pions (π^0, π^+, π^-) and baryons (n, p^+, p^-), so the degeneracy number is $g_q = 2_{flavor} \times 2_{charge} \times 2_{spin} \times 3_{color} = 24$.

We need more clarifying for determining if the quarks could stay free particles or they condense in hadrons. We

can think they could be free if their energy is enough for covering the strong interaction potential and stay free particles with least possible energy (at 0 temperature). Unless they condense in the hadrons.

To cover the strong interaction potential means to lose an energy E_u which is transferred to the exchanged static gluons and pions which are created between the low energy quarks. So the remaining energy in the volume $4\pi a^3/3$ is

$$\frac{E_{q,low}}{V} \frac{4\pi}{3} a^3 - \frac{E_u}{V} \frac{4\pi}{3} a^3 . \qquad (3.8)$$

This energy must be enough for the least possible free quarks. Therefore we must determine the chemical potential μ_0 of the free quarks with smallest possible density at 0 temperature.

According to the quarks confinement $r < a$ at low limited energy, which means the highest possible distance between the two interacting quarks is a, we expect the least quarks density is two quarks in the volume $4\pi (a/2)^3/3$.
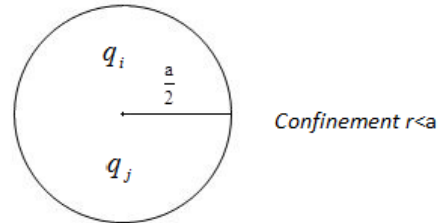

Fig. 10: The quarks confinement at low energy.

From this view we can calculate the least quarks chemical potential μ_0 of free quarks:

$$2 \left(\frac{4\pi}{3} \left(\frac{a}{2} \right)^3 \right)^{-1} = \frac{1}{V} \int_0^{\mu_0} g(\varepsilon) \, d\varepsilon = g_q \frac{\mu_0^3}{6\pi^2}$$
$$\rightarrow \left(\mu_0 \frac{a}{2} \right)^3 = \frac{9\pi}{g_q}$$
$$\rightarrow (\mu_0 a)^3 = \frac{8 \times 9\pi}{g_q}$$

$1/a$ is the pion mass when $a \rightarrow a_0$ in the end of free quarks phase so $1/a \rightarrow (135 - 140) \, \text{Mev}$. So the least free quarks energy density in 0 temperature is

$$\frac{\varepsilon_{free}}{V} = \frac{1}{V} \int_0^{\mu_0} g(\varepsilon) \, \varepsilon \, d\varepsilon = g_q \frac{\mu_0^4}{4 \times 2\pi^2} .$$

The smallest energy of the free quarks in the volume $4\pi a^3/3$ is

$$\varepsilon_{free,a^3} = g_q \frac{\mu_0^4}{4 \times 2\pi^2} \frac{4\pi a^3}{3} = \frac{4\pi}{3} g_q \frac{\mu_0}{4 \times 2\pi^2} (\mu_0 a)^3$$
$$= \frac{4\pi}{3} g_q \frac{\mu_0}{4 \times 2\pi^2} \frac{8 \times 9\pi}{g_q} = \frac{4\pi}{3} \frac{9}{\pi} \mu_0 \qquad (3.9)$$

therefore

$$\frac{\varepsilon_{free,a^3}}{2\mu_0} = \frac{4\pi}{3} \frac{9}{2\pi} = \frac{4\pi}{3} \times 1.43 \,.$$

Because the chemical potential $\mu \sim 1/a$ and $\mu \to \mu_0$ when $a \to a_0$ and because $y \sim a$ so we modified $\mu_0 \to \mu_0/y$ so

$$\frac{4\pi}{3} \times 1.43 \to \frac{4\pi}{3} \times \frac{1.43}{y} \,. \tag{3.10}$$

Now we find the least energy E_u which is transferred to the static exchanged gluons and pions according to the potential

$$u(r) = -\frac{4\alpha_s}{3r}\left(1 - e^{-r/a}\right) \approx -u_0 + \sigma r : r < a \,.$$

We absorbed $4/3$ to α_s so and made $\alpha_s/a\mu_0 = 2\sigma a/\mu_0 = y$ the constant σ is the string tension. This potential is inserted to reduce the chemical potential μ_0 and the energy is renormalized at high energy. So we have $\mu_0 \to \mu_0 + u(r)$:

$$\mu(r) = \mu_0 - \frac{\alpha_s}{r}\left(1 - e^{-r/a}\right) \approx \mu_0 - u_0 + \sigma r : r < a \,.$$

Therefore we can calculate the least absorbed energy by this potential, by calculating the changes on the energy density at 0 temperature

$$\frac{\varepsilon(\alpha_s/a)}{V} = \frac{\varepsilon(y)}{V} = \frac{1}{V} c \int_0^a 4\pi r^2 dr \int_0^{\mu(r)} g(\varepsilon)\,\varepsilon\,d\varepsilon$$

$$= c \int_0^a 4\pi r^2 dr\, g_q \frac{\mu(r)^4}{4 \times 2\pi^2} \,.$$

The constant c is determined

$$c = \frac{\pi}{2a^3 1.9} \frac{7\pi^2}{240}$$

so the interaction energy is

$$\frac{\varepsilon(\alpha_s/a)}{V} = \frac{\varepsilon(y)}{V} = g_q \frac{\pi}{2a^3 1.9} \frac{7\pi^2}{240} \frac{4\pi}{4 \times 2\pi^2} \int_0^a r^2 dr \mu(r)^4$$

$$= g_q \frac{7\pi^2}{4 \times 1.9 \times 240 a^3} \int_0^a r^2 dr \mu(r)^4 \,.$$

This becomes

$$\frac{\varepsilon(y)}{V} = g_q \frac{7\pi^2}{4 \times 1.9 \times 240 a^3} \int_0^a r^2 dr \left[\mu_0 - \frac{\alpha_s}{r}\left(1 - e^{-r/a}\right)\right]^4$$

$$= g_q \frac{7\pi^2}{4 \times 1.9 \times 240 a^3} (\mu_0)^4 \int_0^a r^2 dr \left[1 - \frac{\alpha_s}{\mu_0 r}\left(1 - e^{-r/a}\right)\right]^4$$

Using the change $r = ax$ so

$$\frac{\varepsilon(y)}{V} = g_q \frac{7\pi^2}{4 \times 1.9 \times 240 a^3} (\mu_0)^4$$

$$\int_0^1 a^3 x^2 dx \left[1 - \frac{\alpha_s}{\mu_0 ax}\left(1 - e^{-x}\right)\right]^4$$

therefore

$$\frac{\varepsilon(y)}{V} = g_q \frac{7\pi^2}{4 \times 1.9 \times 240} (\mu_0)^4 \int_0^1 x^2 dx \left[1 - \frac{y}{x}\left(1 - e^{-x}\right)\right]^4 \,.$$

The spent energy for the interaction in the volume $4\pi a^3/3$ is

$$\varepsilon_{u,a^3} = \frac{\varepsilon(1) - \varepsilon(0)}{V} \frac{4\pi a^3}{3}$$

$$= \frac{4\pi}{3} g_q \frac{7\pi^2 (\mu_0 a)^3 \mu_0}{4 \times 1.9 \times 240} \tag{3.11}$$

$$\left(\int_0^1 x^2 dx \left[1 - \frac{1}{x}\left(1 - e^{-x}\right)\right]^4 - \int_0^1 x^2 dx\right)$$

and it becomes

$$\varepsilon_{u,a^3} = -\frac{4\pi}{3} g_q \frac{7\pi^2}{4 \times 1.9 \times 240} (\mu_0 a)^3 \mu_0\, 0.33$$

$$= -\frac{4\pi}{3} g_q \frac{7\pi^2}{4 \times 1.9 \times 240} \frac{8 \times 9\pi}{g_q} \mu_0 \times 0.33 \,.$$

Therefore

$$\varepsilon_{u,a^3} = -\frac{4\pi}{3} g_q \frac{7\pi^2}{4 \times 1.9 \times 240} (\mu_0 a)^3 \mu_0 \times 0.33 \tag{3.12}$$

$$= -\frac{4\pi}{3} \frac{7 \times 8 \times 9 \times 0.33\pi^3}{4 \times 1.9 \times 240} \mu_0 = -\frac{4\pi}{3} \times 2.82 \mu_0 \,.$$

So we have

$$\frac{\varepsilon_{u,a^3}}{2\mu_0} = -\frac{4\pi}{3} \times 1.41 \,.$$

As for E_{free} we replace

$$\frac{4\pi}{3} \times 1.41 \to \frac{4\pi}{3} \times \frac{1.41}{y} \,.$$

Now we find the confinement condition at any temperature, if the quarks energy is not enough to cover the interaction energy E_u and give free quarks with smallest density, at 0 temperature, then they become confinement ($r < a$), so the confinement condition

$$E(T,y) - \varepsilon_u - \varepsilon_{free} < 0 \,. \tag{3.13}$$

Then

$$\frac{E(T,y)}{V} \frac{4\pi a^3}{3} - \frac{\varepsilon_u}{V} \frac{4\pi a^3}{3} - \frac{\varepsilon_{free}}{V} \frac{4\pi a^3}{3} < 0$$

or

$$\frac{E(T,y)}{2\mu_0 V} \frac{4\pi a^3}{3} - \frac{\varepsilon_u}{2\mu_0 V} \frac{4\pi a^3}{3} - \frac{\varepsilon_{free}}{2\mu_0 V} \frac{4\pi a^3}{3} < 0 \,.$$

We consider

$$\sigma_{a^3} a = \frac{\varepsilon_{free}}{V} \frac{4\pi a^3}{3}$$

as critical energy of free quarks for lowest energy, the tension σ_{a^3} here is the volume tension. Therefore this critical energy

is transferred to the produced hadrons and photons. Using the quarks low energy

$$E_{low} = \frac{1}{1.9} \frac{7\pi^2}{240} g_q \mu_0^4 V x^4 \left[3.78 + \right.$$

$$+ 2 x^{-2} \left(0.82 - 1.16y + 0.41y^2 \right) +$$

$$\left. + x^{-4} \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) \right].$$

With $x = T/\mu_0 \ll 1$, the confinement condition becomes

$$\frac{1}{2} \frac{1}{1.9} \frac{7\pi^2}{240} \mu_0^3 g_q x^4 \frac{4\pi a^3}{3} \left[3.78 + \right.$$

$$+ 2x^{-2} \left(0.82 - 1.16y + 0.41y^2 \right) +$$

$$\left. + x^{-4} \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) \right] -$$

$$- \frac{\varepsilon_u}{2\mu_0 V} \frac{4\pi a^3}{3} - \frac{\varepsilon_{free}}{2\mu_0 V} \frac{4\pi a^3}{3} < 0 \,.$$

It becomes

$$\frac{1}{2} \frac{1}{1.9} \frac{7\pi^2}{240} g_q \frac{4\pi (\mu_0 a)^3}{3} \left[3.78x^4 + \right.$$

$$+ 2x^2 \left(0.82 - 1.16y + 0.41y^2 \right) +$$

$$\left. + \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) \right] -$$

$$- \frac{4\pi}{3} \times \frac{1.41}{y} - \frac{4\pi}{3} \times \frac{1.43}{y} < 0 \,.$$

We had the relation

$$(\mu_0 a)^3 = \frac{8 \times 9\pi}{g_q} \,,$$

therefore, the condition becomes

$$\frac{1}{2} \frac{1}{1.9} \frac{7\pi^2}{240} \frac{4 \times 8 \times 9\pi^2}{3} \left[3.78x^4 + \right.$$

$$+ 2 x^2 \left(0.82 - 1.16y + 0.41y^2 \right)$$

$$\left. + \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) \right] -$$

$$- \frac{4\pi}{3} \times \frac{1.41}{y} - \frac{4\pi}{3} \times \frac{1.43}{y} < 0 \,.$$

It becomes

$$3.78x^4 + 2x^2 \left(0.82 - 1.16y + 0.41y^2 \right) +$$

$$+ \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) - \quad (3.14)$$

$$- 0.16y^{-1} < 0$$

with the curve of Fig. 11.

The critical situation x_c with $y \to 1$ (the end of the extreme cooling)

$$3.78x_c^4 + 2 \times 0.07x_c^2 - 0.16574 = 0 \to x_c = 0.438 \,.$$
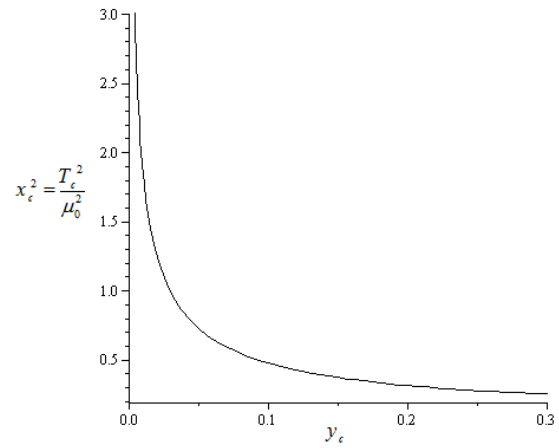

Fig. 11: The critical $x_c^2 y_c$ curve separates the free and confinement quarks phases.

So the critical temperature of the confinement condition when $y \to 1$ from $x_c = T_c/\mu_0$ is $T_c = 0.438\mu_0$. We determine μ_0 from

$$(\mu_0 a)^3 = \frac{8 \times 9\pi}{g_q}$$

when $y \to 1$ so $a \to a_0$ we set $1/a_0$ = pion mass = $(135 - 140)$ Mev so therefore, the condition becomes

$$\mu_0 = \frac{1}{a} \left(\frac{8 \times 9\pi}{g_q} \right)^{\frac{1}{3}} \to 135 \left(\frac{8 \times 9\pi}{24} \right)^{\frac{1}{3}}$$

$$= 285.15 \, \text{Mev} \quad \text{for} \quad \frac{1}{a_0} = 135 \, \text{Mev} \,. \tag{3.15}$$

Hence the critical temperature is $T_c = 0.438 \times 285.15 = 124.9$ Mev.

Now we try to find the produced hadrons, after covering the potential (3.12), the quarks critical energy (possible smallest energy) E_{free} (3.9) is transferred to the produced hadrons and photons. The key idea here is: because the cooling is an extreme cooling, it is expanding $a : 0 \to a_0 = 1/(135 - 140 \, \text{Mev})$ so this process is thermally isolated from the other fields (adiabatic change), therefore the produced particles are in $T_c = 124.9$ Mev. We assume that the produced particles are hadrons (fermions and bosons) and photons. When $a : 0 \to a_0 : y \to 1$ the pions become massive $m = 1/a_0$ so we expect the other hadrons become massive at this stage, we assume that is in $T \to T_c$.

Therefore we assume when $T > T_c$ massless hadrons and $T < T_c$ massive hadrons. Anyway in $x_c y_c$ curve we find the confinement is possible at high energy $(T \gg T_c : a \to 0)$. First we write using (3.9)

$$\frac{\varepsilon_{free}}{V} = \frac{\varepsilon_{free,a^3}}{4\pi a^3/3} = g_q \frac{\mu_0^4}{4 \cdot 2\pi^2} \tag{3.16}$$

$$= \frac{\sigma_{a^3} a}{4\pi a^3/3} \to \frac{E_{hadrons} + E_{photons}}{V} \quad \text{below } x_c y_c \text{ curve}$$

or

$$\frac{\sigma_{a^3} a}{4\pi a^3/3} = g_q \frac{\mu_0^4}{4 \cdot 2\pi^2} \rightarrow \varepsilon_f + \varepsilon_b + \varepsilon_{ph} \,.$$

With the densities

$$\varepsilon_f = \frac{E_f}{V} \,,\ \varepsilon_b = \frac{E_b}{V}\ \text{and}\ \varepsilon_{ph} = \frac{E_{ph}}{V}$$

for spin 1/2 hadrons (fermions p^+, p^-, n), spin 0 hadrons (bosons π^0, π^-, π^+) and photons densities. For massless phase $T \gg T_c$ and $y_c \approx 0$ ignoring the chemical potential we have

$$n_f = \frac{N_f}{V} = g_f \frac{3\zeta(3)}{4\pi^2} T^3 \,,$$

$$n_b = \frac{N_b}{V} = g_b \frac{\zeta(3)}{\pi^2} T^3 \ \text{and} \qquad (3.17)$$

$$\varepsilon_{ph} = \frac{E_{ph}}{V} = g_{ph} \frac{\pi^2}{30} T^4 \,.$$

Now the key point, because the cooling is extreme cooling, to take all the particles (quarks) from high temperature and put them at low temperature, so the same structure at high energy will be at low energies, like the charges ratios, energy distribution over the particles, spins, ... At $T \rightarrow T_c$ and $y_c = 1$ the hadrons become massive, we approximate: for bosons (pions with mass $1/a_0 = 135 - 140$ Mev) the energy density becomes:

$$\varepsilon_b = g_b \frac{\pi^2}{30} T^4 \rightarrow \varepsilon_b = g_b \frac{\pi^2}{30} T^4 + m_{pion} n_b$$

$$\text{with } n_b = \frac{N_b}{V} = g_b \frac{\zeta(3)}{\pi^2} T^3 \text{ and } m_{pion} = \frac{1}{a_0} \,.$$

And for fermions (let them be p^+, p^-, n) we approximate (ignoring the chemical potential)

$$\varepsilon_f = g_f \frac{7}{8} \frac{\pi^2}{30} T^4 \rightarrow \varepsilon_f = g_f \frac{7}{8} \frac{\pi^2}{30} T^4 + m_f n_f$$

$$\text{with } n_f = \frac{N_f}{V} = g_f \frac{3\zeta(3)}{4\pi^2} T^3 \,.$$

So (3.15) becomes

$$\frac{\sigma_{a^3} a}{4\pi a^3/3} = g_q \frac{\mu_0^4}{4 \cdot 2\pi^2} = \varepsilon_f + \varepsilon_b + \varepsilon_{ph} \qquad (3.18)$$

$$= g_f \frac{7}{8} \frac{\pi^2}{30} T_c^4 + m_f n_f + g_b \frac{\pi^2}{30} T_c^4 + \frac{1}{a_0} n_b + g_{ph} \frac{\pi^2}{30} T_c^4 \,.$$

with $g_{quarks} = 2_{flavor} \times 2_{charge} \times 2_{spin} \times 3_{color}$, $g_f = 3_{charge} \times 2_{spin}$, $g_b = 3_{charge}$ and $g_{ph} = 2_{polarization}$.

Now we calculate (3.17) for $1/a_0 = 135$ Mev (π^0), $\mu_0 = 285.15$ Mev, and $T_c = 124.9$ Mev we have

$$2.0096 \times 10^9 \text{ Mev}^4 = 6 \times \frac{7}{8} \frac{\pi^2}{30} (124.9)^4 +$$

$$+ m_f 6 \times \frac{3\zeta(3)}{4\pi^2} (124.9)^3 + 3 \times \frac{\pi^2}{30} (124.9)^4 +$$

$$+ 135 \times 3 \times \frac{\zeta(3)}{\pi^2} (124.9)^3 + 2 \times \frac{\pi^2}{30} (124.9)^4 \,.$$

Its solution is $m_f = 1023$ Mev. We keep 2.0096×10^9 Mev^4 as smallest possible energy density.

For $1/a_0 = 140$ Mev, $\mu_0 = 295.7$ Mev so $T_c = 129.5$ Mev the mass m_f becomes $m_f = 798.4$ Mev. Therefore it must be 135 Mev $< 1/a_0 <$ 140 Mev.

For $1/a_0 = 136.8$ Mev we have $T_c = 126.56$ Mev then the mass m_f becomes $m_f \approx 938$ Mev so the fermions (hadrons) are the baryons (p^+, p^-, n).

Therefore we fix it $1/a_0 = 136.8$ Mev, we use it to cancel the dark matter. Maybe there is an external pressure $-P_{ex}$ so the lost energy is $P_{ex} 4\pi a^3/3$.

Now we try to calculate the ratio N_q/N_h. From the condensation relation

$$N_q \delta\mu_q + N_h \delta\mu_h = 0$$

N_h is the hadrons (consider only the fermions) and μ_h is their chemical potential.

We assumed before the relation for the quarks chemical potential

$$\mu(r) = \mu_0 + u(r) \ \text{ with } u(r) = -\frac{\alpha_s}{r}\left(1 - e^{-r/a}\right)$$

$$\text{so } \delta\mu_q(r) = u(r) = -\frac{\alpha_s}{r}\left(1 - e^{-r/a}\right) \,.$$

The effect of this changing appeared in $y = \alpha_s/a\mu_0$ in the results. For the hadrons we have

$$\delta\mu_h = -\frac{N_q}{N_h} \delta\mu_q = -\frac{N_q}{N_h} u(r) \,.$$

That is right if we consider the hadrons are massless, that is when $T \gg T_c$ and $y \ll 1$ (in the condensation phase, below the curve $x_c y_c$) so we have the chemical potential for the hadrons

$$\mu_h(r) = \mu_{0h} - u(r) \ \text{ with } u(r) = -\frac{\alpha_s}{r}\left(1 - e^{-r/a}\right)$$

therefore we replace $y \rightarrow (-N_q\mu_{0q}/N_h\mu_{0h}) y$ in the quarks energy to get the hadrons energy. The energy of the hadrons becomes

$$E_{H,low} = \frac{1}{1.9} \frac{7\pi^2}{240} \mu_{0h}^4 g_h V x^4 \Bigg[3.78 +$$

$$+ 2x^{-2}\left(0.82 + 1.16\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)y + 0.41\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^2 y^2\right) +$$

$$+ x^{-4}\left(0.08 + 0.23\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)y + 0.25\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^2 y^2 +$$

$$+ 0.12\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^3 y^3 + 0.02\left(\frac{N_q\mu_{0q}}{N_h\mu_{0h}}\right)^4 y^4\right)\Bigg] \,.$$

Assume $\mu_{0h} = \mu_{0q}$ and $y = 1$ so

$$E_{H,low} = \frac{1}{1.9} \frac{7\pi^2}{240} \mu_{0q}^4 g_h V \left[3.78x^4 + 2x^2 \left(0.82 + \right. \right.$$
$$+ 1.16 \left(\frac{N_q}{N_h} \right) + 0.41 \left(\frac{N_q}{N_h} \right)^2 \right) + \left(0.08 + 0.23 \left(\frac{N_q}{N_h} \right) + \right.$$
$$\left. \left. + 0.25 \left(\frac{N_q}{N_h} \right)^2 + 0.12 \left(\frac{N_q}{N_h} \right)^3 + 0.02 \left(\frac{N_q}{N_h} \right)^4 \right) \right].$$

So the chemical potential μ_h of the hadrons becomes

$$\mu_h^4 = \mu_{0q}^4 \left(1 + \frac{0.23}{0.08} \left(\frac{N_q}{N_h} \right) + \frac{0.25}{0.08} \left(\frac{N_q}{N_h} \right)^2 + \right.$$
$$\left. + \frac{0.12}{0.08} \left(\frac{N_q}{N_h} \right)^3 + \frac{0.02}{0.08} \left(\frac{N_q}{N_h} \right)^4 \right).$$

When $T < T_c$ the hadrons become massive, as we assumed before, so for massive hadrons with $m_f = 938$ Mev we expect $\mu_h = m_f = 938$ Mev when they cooled with small densities. Therefore

$$(938)^4 = (285.15)^4 \left(1 + \frac{0.23}{0.08} \left(\frac{N_q}{N_h} \right) + \frac{0.25}{0.08} \left(\frac{N_q}{N_h} \right)^2 + \right.$$
$$\left. + \frac{0.12}{0.08} \left(\frac{N_q}{N_h} \right)^3 + \frac{0.02}{0.08} \left(\frac{N_q}{N_h} \right)^4 \right).$$

Its positive solution is $N_q/N_h = 3.1$ so they are the baryons (fermions with three quarks). For 0 temperature fermions the chemical potential is approximated by

$$\mu_0^2 = m^2 + \left(\frac{N}{V} \frac{6\pi^2}{g_f} \right)^{2/3}.$$

For low hadrons density we ignored the term

$$\left(\frac{N}{V} \frac{6\pi^2}{g_f} \right)^{2/3}.$$

4 The nuclear compression

The cooled hadrons have high density, so there is hidden high pressure, that pressure makes influence δa so δy near $y = 1$ or it makes $y = 1 + \delta y$: $\delta y \approx 0.005$ so the cooled quarks inside the hadrons fluctuate, this depends on the energy, if the energy is high then there are new hadrons. These processes let the interacting hadrons lose kinetic energy and form the pions.

Because the number of quarks increases although the hadrons are fixed, therefore the hadrons energy decreases and they cannot spread away. We can see how the chemical potential of the interacting hadrons changes under the fluctuation $\delta y \sim \delta a$ (due to the quarks interaction) from the condensation

relation $N_q \delta \mu_q + N_h \delta \mu_h = 0$ we have $\delta \mu_h = -N_q \delta \mu_q / N_h$ for the fluctuation δy we have

$$\delta \mu_h = -\frac{N_q}{N_h} \frac{\partial \mu_q}{\partial y} \delta y$$

from quarks chemical potential (4.4), we find

$$\frac{\partial \mu_q}{\partial y} < 0 \quad \text{so} \quad -\frac{\partial \mu_q}{\partial y} > 0$$

therefore we have

$$\delta \mu_h = \frac{N_q}{N_h} \left(-\frac{\partial \mu_q}{\partial y} \right) \delta y < 0 \quad \text{when} \quad \delta y < 0$$

which is the quarks compressing, when the hadrons collide together this leads to $\delta y < 0$ (compression) so the hadrons lose energy and new hadrons are created. And when they try to extend (spread away) $\delta y > 0$ so $\delta \mu_h > 0$, there will be a negative potential.

For the interacting hadrons pressure we have the phase changing relation $V_q \delta P_q + V_h \delta P_h = 0$: V volume, we have

$$\delta P_h = -\frac{V_q}{V_h} \delta P_q = -\frac{V_q}{V_h} \frac{\partial P_q}{\partial y} \delta y$$

because $\partial P_q / \partial y < 0 \rightarrow -\partial P_q / \partial y > 0$ therefore when the hadrons collide together $\delta y < 0$ so their pressure decreases, they lose energy, so new hadrons are created.

We have

$$\delta y = \left(-\frac{V_q}{V_h} \frac{\partial P_q}{\partial y} \right)^{-1} \delta P_h \quad \text{at} \quad y = 1.$$

So the hadrons chemical potential becomes

$$\delta \mu_h = \frac{N_q}{N_h} \left(-\frac{\partial \mu_q}{\partial y} \right) \left(-\frac{V_q}{V_h} \frac{\partial P_q}{\partial y} \right)^{-1} \delta P_h \quad : \quad y = 1.$$

It becomes

$$\delta \mu_h = \frac{N_q V_h}{N_h V_q} \left(\frac{\partial \mu_q}{\partial y} \right) \left(\frac{\partial P_q}{\partial y} \right)^{-1} \delta P_h \quad : \quad y = 1. \tag{4.1}$$

We can relate this changing to a constant nuclear potential. Like to write

$$\delta \mu_h = -V_0. \tag{4.2}$$

V_0 is the potential for each hadron.

So when the hadron (fermions, like protons or neutrons) join, their density increases $\delta \mu_h > 0$ so their pressure rises $\delta P_h > 0$, therefore there is a negative potential $V_0 < 0$. At low energies this potential prevents them from spreading away.

Now we calculate

$$\delta \mu_h = \frac{N_q V_h}{N_h V_q} \left(\frac{\partial \mu_q}{\partial y} \right) \left(\frac{\partial P_q}{\partial y} \right)^{-1} \delta P_h \quad : \quad y = 1.$$

We use the pressure at low energy (3.7)

$$
P_{low}/\mu_0^4 = (9 \times 1.9 \times 240)^{-1}\, 7\pi^2 g_q \Big[3 \times 3.78 x^4 +
$$
$$
+ 3 \times 2 \times x^2 \left(0.82 - 1.16y + 0.41y^2 \right) + 3\,(0.08 - 0.23y +
$$
$$
+ 0.25y^2 - 0.12y^3 + 0.02y^4) + 2yx^2\,(-1.16 + 0.82y) +
$$
$$
+ y \left(-0.23 + 0.5y - 0.36y^2 + 0.08y^3 \right) \Big]
$$

and we get

$$
\frac{\partial P_q}{\partial y} = -\frac{0.076 \times 7 \times \pi^2 \times g_q \mu_0^4}{240 \times 3 \times 1.9} : x_c = 0.438,\ y = 1. \quad (4.3)
$$

Using the relation

$$
\mu_0 = \frac{1}{a_0}\left(\frac{8 \times 9\pi}{g_q} \right)^{\frac{1}{3}}
$$

we have

$\mu_0 = 285.15\,\text{Mev}$ for $g_q = 24$ and $1/a_0 = 135\,\text{Mev}$ for π^0
$\mu_0 = 295.7\,\text{Mev}$ for $1/a_0 = 140\,\text{Mev}$ for π^- and π^+.

So the chemical potential μ_0 is in the range from 285.15 Mev to 295.7 Mev therefore

$$
\frac{\partial P_q}{\partial y} = -6.06 \times 10^8\,\text{Mev}^4\ \text{ for } \mu_0 = 285.15\,\text{Mev}
$$
$$
\frac{\partial P_q}{\partial y} = -7.01 \times 10^8\,\text{Mev}^4\ \text{ for } \mu_0 = 295.7\,\text{Mev}.
$$

Now we try to calculate $\partial \mu_q / \partial y$, according to low energy

$$
E_{low} = (1.9 \times 240)^{-1}\, 7\pi^2 \mu_0^4\, g_q\, V\, x^4 \Big[3.78 +
$$
$$
+ 2x^{-2} \left(0.82 - 1.16y + 0.41y^2 \right)
$$
$$
+ x^{-4} \left(0.08 - 0.23y + 0.25y^2 - 0.12y^3 + 0.02y^4 \right) \Big]
$$

we can equivalence

$$
\mu^4 = \mu_0^4 \left(1 - \frac{0.23}{0.08}y + \frac{0.25}{0.08}y^2 - \frac{0.12}{0.08}y^3 + \frac{0.02}{0.08}y^4 \right). \quad (4.4)
$$

But $\partial \mu / \partial y \to \infty$ when $y \to 1$ so we replace

$$
\frac{\partial \mu_q}{\partial y} \to \frac{\mu_{y=1} - \mu_{y=0}}{1 - 0} = \frac{0 - \mu_{y=0}}{1 - 0} = -\mu_{y=0} = -\mu_0.
$$

Therefore we have

$$
\delta \mu_h = \frac{N_q V_h}{N_h V_q} \left(\frac{\partial \mu_q}{\partial y} \right) \left(\frac{\partial P_q}{\partial y} \right)^{-1} \delta P_h
$$
$$
= \frac{N_q V_h}{N_h V_q} \mu_0 \left(0.09\mu_0^4 \right)^{-1} \delta P_h
$$
$$
= \frac{N_q V_h}{N_h V_q} \left(0.09\mu_0^3 \right)^{-1} \delta P_h.
$$

So we have

$$
\delta \mu_h = \frac{N_q V_h}{N_h V_q} \left(0.09\,\mu_0^3 \right)^{-1} \delta P_h
$$
$$
= 4.7 \times 10^{-7} \frac{N_q V_h}{N_h V_q} \delta P_h\ \text{ for } \mu_0 = 285.15\,\text{Mev}
$$

and

$$
\delta \mu_h = 4.2 \times 10^{-7} \frac{N_q V_h}{N_h V_q} \delta P_h\ \text{ for } \mu_0 = 295.7\,\text{Mev}. \quad (4.5)
$$

We use them to cancel the dark matter and dark energy.

5 The Big Bang

We assume there were two universal phases, high energies massless particles phase (let them be the quarks plasma) and then the massive low energies particles (let them be the hadrons).

The first phase associated with high energy density (drops from infinity to finite), the time of that stage is $\tau : 0 \to a_0 = 1/(135-140)\,\text{Mev}$ then the massive hadrons phase begins (the time $t : 0 \to \infty$).

In both stages the highest universal expansion must not exceed the light speed, for the first phase, high energies massless quarks phase, the density of the energy is the same in all space points so the universal expansion is the same in every point in the space, we let the speed of that expansion equal the light speed, therefore the Hubble parameter $H(t < a_0)$ of this stage $t < a_0$ is given by (5.2).

To find the Hubble parameter for the massive hadrons phase $H(t > a_0)$, we suggest the geometry transformation (5.3) in which the time $\tau : 0 \to a_0$ for the quarks corresponds to the time $t : 0 \to \infty$ for the massive hadrons phase. We can relate that change in the geometry to the high differences in the energy densities of the two phases. The phase $\tau : 0 \to a_0$ high quarks energy, uniform high energy density, massless, ... The phase $t : 0 \to \infty$ the massive hadrons, low energy density, separated particles, ...

Now we try to explain how the universe exploded and expanded, we start from our assumptions we made before and find the Hubble parameter and try to find the dark energy and matter. We found that the quarks expand to the length $a_0 = 1/(135-140)\,\text{Mev}$ then the hadrons appear instead.

We assume that the universe was created in every point in two dimensional space XY then the explosion in the Z direction. That is by the quarks, in each point in the XY plane the quarks were created and then they expanded in each point XY to the length a_0 then the explosion in the Z direction, the result is the universe in the space XYZ. There was no universal explosion in the XY plane, the universal explosion was only in the Z direction, in the plane XY there was extension due to the quarks expanding from $r = 0$ to $r = a_0 = 1/(135-140)\,\text{Mev}$ the plane XY was infinity before the quarks expansion and it is infinity after that expansion, what happened is an increase

in the number of the XY points, then the explosion in the Z direction. We assume both expansion (XY and Z) occurred with the light speed c.

To find the lost matter, dark matter and dark energy, we use the relation (4.5) we found before:

$$\delta\mu_h = \frac{N_q V_h}{N_h V_q}\left(0.09\,\mu_0^3\right)^{-1}\delta P_h$$
$$= 4.7\times 10^{-7}\,\frac{N_q V_h}{N_h V_q}\,\delta P_h \ \text{ for } \mu_0 = 285.15\,\text{Mev}$$

and

$$\delta\mu_h = 4.2\times 10^{-7}\,\frac{N_q V_h}{N_h V_q}\,\delta P_h \ \text{ for } \mu_0 = 295.7\,\text{Mev}.$$

Here we relate this changing in the pressure δP (independent of time) to the hadrons condensation process to form the nucleuses, where the global pressure $\delta P = \delta P_h$ dropped extremely due to the nuclear attractive potential (make it the nuclear binding energy) $V_0 = (-7-8)\,\text{Mev}$ [3]. This pressure δP_h remains contained in the nucleuses, but globally is not visible.

So there is hidden global pressure δP_h and we have to include that problem in the Friedman equations solutions, we notice that the nuclear attractive potential leads to increasing in the cooled hadrons densities. Therefore the decreasing in the hadrons pressure associated with the increasing of their densities (inside the nucleuses). The result is excess in the local energy density, that effects appear in the equations, that is, the matter density appears to be larger than the right energy density. So there is neither dark matter nor dark energy, it is just global and local densities.

We start from the definition of the scale parameter $R(t)$ for the universe expansion, we write [6]

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right). \qquad (5.1)$$

We set $k = 0$ flat Universe. Now we try to find the Hubble parameter

$$H(t) = \frac{1}{R(t)}\frac{dR(t)}{dt} = \frac{\dot{R}(t)}{R(t)}.$$

There are two phases $t < a_0$ free quarks phase and $t > a_0$ hadrons phase which is the expansion in the Z direction. That means there are two different spacetime geometry, $t < a_0$ and $t > a_0$.

In the first phase $\tau = t < a_0$ the expansion is the same in all space points, so the expansion velocity

$$\frac{dR_1}{dt} = \dot{R}(t)\,r$$

is the same in all space points and equals the light speed $c = \hbar = 1$ here, so

$$1 = \dot{R}(t)r : \ t < a = a_0.$$

Therefore

$$\dot{R}(t) = \frac{1}{r} : \ t < a = a_0.$$

So we can write

$$R(t) = \frac{t}{r} : \ t < a = a_0.$$

So the Hubble parameter becomes

$$H(t) = \frac{\dot{R}(t)}{R(t)} = \frac{1/r}{t/r} = \frac{1}{t} : \ t < a = a_0. \qquad (5.2)$$

Now we want to find the Hubble parameter in the phase $t > a_0$ low energy phase. Actually when the quarks expand from $r = 0$ to $r = a \to a_0$ there will be infinity points expanding, so infinity expanding distance in XY space, but the expansion cannot exceed the light speed $c = 1$ therefore an explosion occurs in the Z direction, so the universal explosion. Therefore the time $t = \tau : 0 \to a_0$ for the free quarks phase will associate with $t : 0 \to \infty$ for the universal expansion, so we make the geometry transformation

$$t = \frac{-c_0}{\tau - a_0} : \ \tau < a_0. \qquad (5.3)$$

c_0 is constant, we can relate that relation to a difference in spacetime geometry. That means if the quarks space $r < a_0 = 1/(135-140)\,\text{Mev}$ is flat, so the hadrons space is not, it is curved space, where we live. It is convenient to consider the quarks space ($r < a_0$ large energy density) is curved not our space (low energy density).

Now we can find the Hubble parameter for the universe $t : 0 \to \infty$. We can find the Hubble parameter $H(t > a_0)$ for the geometry $t : 0 \to \infty$ from $H(t < a_0)$:

$$H(\tau < a) = \frac{1}{R(\tau < a)}\frac{dR(\tau < a)}{d\tau}$$
$$= \frac{1}{f(r,\theta,\varphi)R(t > a)}\frac{d}{d\tau}f(r,\theta,\varphi)R(t > a).$$

We set the geometry transformation

$$R(\tau < a) = f(r,\theta,\varphi)\,R(t > a)$$

so

$$\frac{1}{\tau} = \frac{1}{f(r,\theta,\varphi)R(t > a)}\frac{d}{d\tau}f(r,\theta,\varphi)R(t > a)$$
$$= \frac{1}{R(t > a)}\frac{d}{d\tau}R(t > a) \qquad (5.4)$$

or

$$\frac{1}{\tau} = \frac{1}{R(t > a)}\frac{dt}{d\tau}\frac{d}{dt}R(t > a)$$
$$= \frac{dt}{d\tau}\frac{1}{R(t > a)}\frac{d}{dt}R(t > a) = \frac{dt}{d\tau}H(t > a).$$

Using the geometry transformation

$$t = \frac{-c_0}{\tau - a_0} \; : \; \tau < a \rightarrow a_0 \,,$$

we have the Hubble parameter of the low energy density of the cold Universe

$$H(t > a_0) = \frac{1}{R(t > a_0)} \frac{d}{dt} R(t > a_0)$$
$$= \frac{c_0}{t(a_0 t - c_0)} = \frac{1}{t\left(\frac{a_0}{c_0} t - 1\right)} = \frac{1}{t\left(c'_0 t - 1\right)}$$

where c'_0 is constant.

The Friedman equations can be written, for $k = 0$, like [6]

$$3\frac{\dot{R}^2(t)}{R^2(t)} = 8\pi G_N \rho + \Lambda \qquad (1)$$

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = 4\pi G_N(\rho + p) \qquad (2) \qquad (5.5)$$

$$\frac{d}{dt}(\rho + \delta p) = -3(\rho + p)\frac{\dot{R}(t)}{R(t)} \,. \qquad (3)$$

To control (or cancel) the dark matter and energy, we make the transformations in the Friedman equations which keep the Hubble parameter unchanged

$$3\frac{\dot{R}^2(t)}{R^2(t)} = 8\pi G_N(\rho + \delta P) + \Lambda - 8\pi G_N \delta P \qquad (1')$$

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = 4\pi G_N(\rho + \delta P + p - \delta P) \qquad (2') \qquad (5.6)$$

$$\frac{d}{dt}(\rho + \delta p) = -3(\rho + \delta P + p - \delta P)\frac{\dot{R}(t)}{R(t)} \,. \qquad (3')$$

So we have (for same Hubble parameter we had before)

$$\rho' = \rho + \delta p_h$$
$$p' = p - \delta p_h$$
$$\Lambda' = \Lambda - 8\pi G_N \delta p_h = 0 \,.$$

For the universal nuclear condensation, we assume the universal change $\delta \rho = \delta P = \delta p_h > 0$ is independent of the time.

We can say ρ', p' and $\Lambda' = 0, P' = 0$ are for the located matter, when the hadrons are cooled, they condense and locate in small volumes with high matter density, because of the strong nuclear attractive interaction, so their pressure decreases extremely $P' \approx 0$. That pressure is contained (hidden) in the nucleus. It is like to condense a gas with certain mass m and fixed volume V, the density m/V is the same before and after the condensation, but the real density of the produced liquid is not. Like that we consider ρ the right matter ρ_{matter} and the problems; the increasing $\rho' = \rho + \delta p_h$ and $\Lambda \neq 0$ are because of the phase changing.

We set $\rho' = \rho(t)$ and solve the two equations:

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = 4\pi G_N \rho(t) \qquad (2')$$

$$\frac{d}{dt}(\rho + \delta p) = \dot{\rho}(t) = -3\rho(t)\frac{\dot{R}(t)}{R(t)} \qquad (3')$$

using the Hubble parameter

$$H(t) = \frac{1}{R}\frac{dR}{dt} = \frac{1}{t\left(c'_0 t - 1\right)} \qquad : \; t > a_0 \,.$$

From (3') we have

$$-\frac{1}{3}\frac{R(t)}{\dot{R}(t)}\dot{\rho}(t) = \rho(t)$$

so (2') becomes

$$-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} = -\frac{4\pi G_N}{3}\frac{R(t)}{\dot{R}(t)}\dot{\rho}(t) \,.$$

This equation becomes

$$\frac{\dot{R}(t)}{R(t)}\left(-\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)}\right) = -\frac{4\pi G_N}{3}\dot{\rho}(t)$$

or

$$H(t)\left(-\frac{\ddot{R}(t)}{R(t)} + H^2(t)\right) = -\frac{4\pi G_N}{3}\dot{\rho}(t) \,.$$

Using

$$\frac{d}{dt}\frac{\dot{R}(t)}{R(t)} = \frac{\ddot{R}(t)}{R(t)} - \frac{\dot{R}^2(t)}{R^2(t)}$$

we get

$$H(t)\frac{d}{dt}H(t) = \frac{4\pi G_N}{3}\dot{\rho}(t) \rightarrow \frac{1}{2}H(t)^2 = \frac{4\pi G_N}{3}(\rho(t) - \rho_0) \,.$$

For finite results we put $\rho_0 = 0$ so

$$\frac{1}{2}H(t)^2 = \frac{4\pi G_N}{3}\rho(t) \,.$$

Now we calculate the contributions of the vacuum energy to the total energy using the cosmological constant Λ' from (1')

$$\Omega_{\Lambda'} = \frac{\rho'_\Lambda}{\rho_c} = \frac{\Lambda'}{3H^2} = \frac{3H^2 - 8\pi G_N \rho(t)}{3H^2}$$
$$= 1 - 2\frac{4\pi G_N}{3H^2}\rho(t) = 1 - 2\frac{1}{H^2}\frac{1}{2}H(t)^2 = 0$$

with the critical energy density

$$\rho_c = \frac{3H^2}{8\pi G_N} \,.$$

So the vacuum energy density is canceled, and the total energy is the matter energy $\Omega_{matter} = 1$ so $\rho(t)/\rho_c = 1$. Here

$\rho(t) = \rho_c$ is $\rho(t) = \rho' = \rho_{matter} + \delta p_h$, so $\rho(t)$ is higher than the right matter ρ_{matter}.

Now we see if this relation is satisfied or not. We use the global change on the pressure $\delta p = \delta p_h > 0$ which we derived in (4.5):

$$\delta \mu_h = \frac{N_q V_h}{N_h V_q} \left(0.09 \mu_0^3\right)^{-1} \delta P_h$$

$$= 4.7 \times 10^{-7} \frac{N_q V_h}{N_h V_q} \delta P_h \ \text{ for } \mu_0 = 285.15 \, \text{Mev}$$

and

$$\delta \mu_h = 4.2 \times 10^{-7} \frac{N_q V_h}{N_h V_q} \delta P_h \ \text{ for } \mu_0 = 295.7 \, \text{Mev} .$$

Now we try to find V_q/V_h the quarks volume $V_q = S d_q$ and the hadrons volume $V_h = S d_h$ as shown in Fig. 12 where the
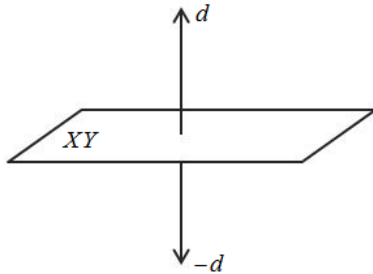

Fig. 12: The universal explosion in Z direction starting from XY flat.

universal explosion is in the $Z = d$ direction. If we assume the explosion speed is the same for both hadrons and quarks, light speed $v = c = 1$, so for the quarks

$$H_q(t) = \frac{\dot{R}(t)_q}{R(t)_q} = \frac{v_q}{d_q} = \frac{1}{d_q} .$$

For the hadrons

$$H_h(t) = \frac{\dot{R}(t)_h}{R(t)_h} = \frac{v_h}{d_h} = \frac{1}{d_h}$$

therefore

$$\frac{V_q}{V_h} = \frac{S d_q}{S d_h} = \frac{d_q}{d_h} = \frac{H_h}{H_q} ,$$

H_h is the universal Hubble parameter, today is

$H = 71 \, \text{km/s/mpc} = 2.3 \times 10^{-18} \, s^{-1}$

$\quad = 2.3 \times 10^{-18} \times 6.58 \times 10^{-22} \, \text{Mev} = 151.34 \times 10^{-41} \, \text{Mev} .$

The quarks Hubble parameter $H_q = 1/\tau \rightarrow 1/a_0 = (135 - 140) \, \text{Mev}$. So we have (for 135 Mev)

$$\frac{H_h}{H_q} = \frac{151.34 \times 10^{-41} \, \text{Mev}}{135 \, \text{Mev}} = 1.127 \times 10^{-41} .$$

Therefore

$$\frac{V_q}{V_h} = \frac{H_h}{H_q} = 1.127 \times 10^{-41} .$$

We set $\delta \mu_h = -V_0 = (7-8) \, \text{Mev}$ the nuclear potential (nucleon binding energy). Therefore, from (4.5), we have

$$\delta \rho = \delta P_h = -\frac{N_h}{N_q} \times 1.127 \times 10^{-41} \times \frac{-V_0}{47} \times 10^8 \, \text{Mev}^4$$

$$\text{for } \frac{1}{a_0} = 135 \, \text{Mev} : \mu_0 = 285.15 \, \text{Mev}$$

and

$$\delta \rho = \delta P_h = -\frac{N_h}{N_q} \times 1.087 \times 10^{-41} \times \frac{-V_0}{42} \times 10^8 \, \text{Mev}^4$$

$$\text{for } \frac{1}{a_0} = 140 \, \text{Mev} : \mu_0 = 295.7 \, \text{Mev} .$$

For $N_h/N_q = 1/5$, like the interaction $P^+ + \pi^- \rightarrow n$ the neutron n appears to have five quarks, that is acceptable according to the fields dual behavior. Therefore

$$\delta \rho = \delta P_h = -\frac{1}{5} \times 1.127 \times 10^{-41} \times \frac{-7}{47} \times 10^8 \, \text{Mev}^4$$

$$= 335.7 \times 10^{-37} \, \text{Mev}^4$$

$$\text{for } \mu_0 = 285.15 \, \text{Mev and } V_0 = -7 \, \text{Mev}$$

and

$$\delta \rho = \delta P_h = -\frac{1}{5} \times 1.087 \times 10^{-41} \times \frac{-8}{42} \times 10^8 \, \text{Mev}^4$$

$$= 414 \times 10^{-37} \, \text{Mev}^4$$

$$\text{for } \mu_0 = 295.7 \, \text{Mev and } V_0 = -8 \, \text{Mev} .$$

So the change $\delta \rho = \delta P_h$ is in the range:

$$\text{from } 335.7 \times 10^{-37} \, \text{Mev}^4 \text{ to } 414 \times 10^{-37} \, \text{Mev}^4 .$$

Therefore the visible matter is in the range

$$\text{from } \rho_{matter} = \rho_c - \delta p_h = 335.7 \times 10^{-37} \, \text{Mev}^4$$
$$\text{to } \rho_{matter} = \rho_c - \delta p_h = 414 \times 10^{-37} \, \text{Mev}^4 .$$

For the critical energy $\rho_c = 406 \times 10^{-37} \, \text{Mev}^4$ the visible matter is in the range

$$\text{from } \rho_{matter} = 0 \text{ to } \rho_{matter} = 70 \times 10^{-37} \, \text{Mev}^4 .$$

The right baryonic matter energy density is

$$\rho_b = 4.19 \times 10^{-31} \text{g/cm}^3 \approx 17.97 \times 10^{-37} \, \text{Mev}^4$$

which belongs to the range 0 to $70 \times 10^{-37} \, \text{Mev}^4$. We can control this and have

$$\rho_{matter} = \rho_c - \delta P_h = 406 \times 10^{-37} - \delta P_h = 17.97 \times 10^{-37} \, \text{Mev}^4$$

by finding r:

$$140r + 135(1-r) = \frac{1}{a_0}$$

where $1/a_0$ satisfies

$$406 \times 10^{-37} \, \text{Mev}^4 - \delta P_h = 17.97 \times 10^{-37} \, \text{Mev}^4.$$

For $1/a_0 = 136.8$ Mev (we used it in (3.17) to have $m_f \approx 938$ Mev), the chemical potential becomes $\mu_0 = 288.95$ Mev. And with $V_0 = 7.776$ Mev we get

$$\delta\rho = \delta P_h = 335.7 \times 10^{-37} \times \left(\frac{288.95}{285.15}\right)^3 \times \frac{7.776}{7} \, \text{Mev}^4$$

$$= 388 \times 10^{-37} \, \text{Mev}^4 \,.$$

The matter density becomes

$$\rho_{matter} = 406 \times 10^{-37} \, \text{Mev}^4 - 388 \times 10^{-37} \, \text{Mev}^4$$

$$= 17.9 \times 10^{-37} \, \text{Mev}^4 \,.$$

which is the right matter (global visible matter density). Therefore we can control the dark matter and dark energy. We can cancel them.

Note that not all of those ideas are contained in the references.

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