An Explanation of De Broglie Matter Waves in Terms of the Electron Coupling to the Vacuum State

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This paper examines the de Broglie wave theory derived by Synge who assumed that Hamilton's variational principle in three dimensions applies also to the four-dimensional Minkowski spacetime. Based on the Planck vacuum (PV) view of the electron coupling to the vacuum state, calculations here suggest that the Synge de Broglie waves exist and travel within the PV state.

1 Introduction

In the early part of the twentieth century when it was realized that the massless photon had *particle-like* properties, de Broglie figured therefor that the massive electron must have *wave-like* properties — and the de Broglie matter wave was born [1, p.55].

In circa 1954 Synge [2] published a study on the idea of 3-waves propagating in the 4-dimensional Minkowski spacetime. The study was based on the properties of a *medium-function* which the present author interprets as a vacuum medium. In the PV theory, the occurrence of 3-waves in a 4-dimensional spacetime is symptomatic of an invisible vacuum continuum interacting with a free-space particle.

What follows compares the Compton-(de Broglie) relations derived in the PV theory with the Synge ray-wave diagram in spacetime, and concludes that his de Broglie waves propagate within the vacuum state rather than free space.

2 Compton-(de Broglie) relations

In the PV theory the interaction of the electron with the vacuum state leads to the Compton-(de Broglie) relations [3]

$$r_c \cdot mc^2 = r_d \cdot cp = r_L \cdot E = r_* \cdot m_* c^2 = e_*^2 \; (= c\hbar) \qquad (1)$$

where e_* is the massless bare charge that is related to the electronic charge via $e = \alpha^{1/2} e_*$, and α and m_* are the fine structure constant and the Planck particle mass. The radii $r_c\,(= e_*^2/mc^2)$ and $r_*\,(= e_*^2/m_*c^2)$ are the electron and Planck-particle Compton radii respectively and m is the electron mass. The magnitudes of r_* and m_* are equal to the Planck length and mass respectively [4, p. 1234]. It is because $r_* \neq 0$ that the PV state is a quasi-continuum.

The de Broglie radii, r_L and r_d, are derived from r_c and the Lorentz invariance of the vanishing electron/PV coupling force (A1) at $r = r_c$

$$i\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right) = 0 \qquad (2)$$

and result in

$$r_L = \frac{r_c}{\gamma} \qquad \text{and} \qquad r_d = \frac{r_c}{\beta\gamma} \qquad (3)$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. From (3) $\beta = r_L/r_d$ yields the relative velocity of the electron in the coupled electron/PV system. The relations in (1) also lead to the relativistic electron energy $E = \sqrt{m^2c^4 + c^2p^2}$.

The relativistic scaler wave equation is [5, p. 319]

$$\left(\frac{\hbar}{mc}\right)^2 \left(\nabla^2 - \frac{\partial^2}{c^2 \partial t^2}\right)\psi = r_c^2 \left(\nabla^2 - \frac{\partial^2}{c^2 \partial t^2}\right)\psi = \psi \qquad (4)$$

while its nonrelativistic counterpart reads

$$\left(\nabla^2 - \frac{\partial^2}{v^2 \partial t^2}\right)\psi = 0 \qquad (5)$$

where $v \ll c$; so the electron radii in (1) and (4) are relativistic parameters.

The planewave solution ($\psi \propto \exp i\phi$) to (4) in the z-direction involves the phase

$$\phi = \frac{Et - pz}{\hbar} = \frac{Ect - cpz}{c\hbar} = \frac{Ect - cpz}{e_*^2}$$

$$\phi = \frac{ct}{r_L} - \frac{z}{r_d} \qquad (6)$$

where the relativistic energies $E\,(= m\gamma c^2)$ and $cp\,(= cm\gamma v)$ from (1) are used in the final expression. The normalization of ct and z by the de Broglie radii r_L and r_d is a characteristic of the PV model of the vacuum state, and is related to the Synge primitive (planewave) quantization of spacetime to be discussed below.

3 De Broglie waves

The Ray/3-Wave diagram that represents the Synge de Broglie wave propagation in spacetime [2, p. 106] is shown in Fig. 1, where the electron propagates upward at a uniform velocity v along the Ray. The vertical axis is ict and the horizontal axes are represented by (x, y, z). The need for quantizing the Synge vacuum waves (adding the parameters r_c, r_L, and r_d to Fig. 1) is explained in the following quote. "So far the [variational-principle] theory has been confined to the domain of geometrical mechanics. The de Broglie waves have no phase, no frequency, no wave-length. It is by *quantization*

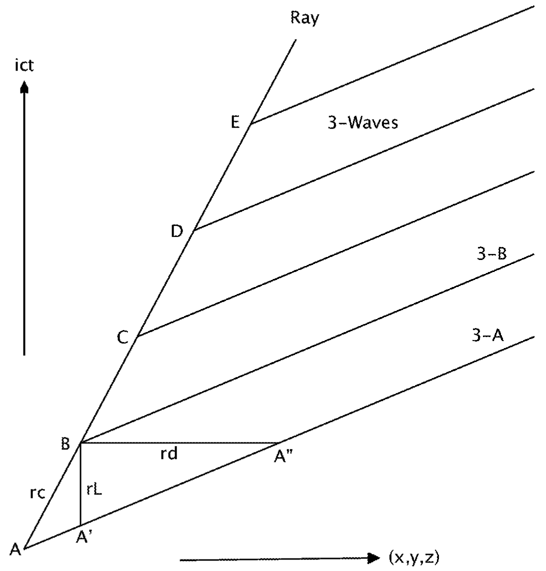

Fig. 1: Planewave quantization of de Broglie Waves in Spacetime. The figure consists of a single Ray and a partial picture of the corresponding 3-Waves propagating toward the upper left. The Ray and their 3-Waves are orthogonal in the 4-dimensional spacetime sense. The quantization consists of the normalization constants r_c, r_L, and r_d.

that these things are introduced, in much the same way as they are introduced in the transition from geometrical optics to physical optics" [2, p. 105].

The parameters in the figure correspond to rc $= r_c$, rL $= r_L$, and rd $= r_d$; the Compton and de Broglie radii from (1). In the quantization, the Synge theory utilizes the wavelengths $2\pi r_c$, $2\pi r_L$ and $2\pi r_d$. Thus there is a one-to-one correspondence between the electron/PV coupling radii from (1) and the Synge wave theory regarding Fig. 1. In the PV theory the electron radii (r_c, r_L, r_d) are parameters generated by the electron/PV interaction — thus it is reasonable to conclude that the de Broglie waves travel within the vacuum state.

Appendix A: Coupling force

In its rest frame, the coupling force the electron core $(-e_*, m)$ exerts on the PV quasi-continuum is [3]

$$i\left(\frac{e_*^2}{r^2} - \frac{mc^2}{r}\right) \tag{A1}$$

where the radius r begins at the electron core. The spacetime coordinates are denoted by

$$x_\mu = (x_0, x_1, x_2, x_2) = (ict, x, y, z) \tag{A2}$$

where $\mu = (0, 1, 2, 3)$ and $r = (x^2 + y^2 + z^2)^{1/2}$.

The force (A1) vanishes at the electron Compton radius r_c and leads to:

$$\frac{i}{r_c}\left(\frac{e_*^2}{r_c} - mc^2\right) = 0 \tag{A3}$$

in the electron rest frame; and when (A3) is Lorentz transformed it results in the two coupling forces

$$\frac{i}{r_L}\left(\frac{e_*^2}{r_c} - mc^2\right) = 0 \tag{A4}$$

and

$$\frac{1}{r_d}\left(\frac{e_*^2}{r_c} - mc^2\right) = 0 \tag{A5}$$

in the uniformly moving frame.

To complete the PV perspective, the space and time derivatives in the scaler wave equation

$$r_c^2\left(\nabla^2 - \frac{\partial^2}{c^2\partial t^2}\right)\psi = \psi \tag{A6}$$

and the Dirac electron equation [6, p. 74]

$$ic\hbar\left(\boldsymbol{\alpha}\cdot\nabla + \frac{\partial}{c\partial t}\right)\psi = mc^2\beta\psi$$

or

$$ir_c\left(\boldsymbol{\alpha}\cdot\nabla + \frac{\partial}{c\partial t}\right)\psi = \beta\psi \tag{A7}$$

are quantized (normalized, scaled) by the same constant r_c, where the 4×4 vector matrix $\boldsymbol{\alpha}$ accounts for electron spin. [The β in (A7) is a 4×4 matrix, not a relative velocity.]

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References

1. De Broglie L., Brillouin L. Selected Papers on Wave Mechanics. 1928, London and Glasgow, Blackie.
2. Synge J.L. Geometrical Mechanics and de Broglie Waves. 1954, Cambridge University Press.
3. Daywitt W.C. The de Broglie relations derived from the electron and proton coupling to the Planck vacuum state. *Progress in Physics*, 2015, v. 11, Issue 2 (April), 189. See also www.planckvacuum.com.
4. Carroll B.W. Ostlie D.A. An Introduction to Modern Astrophysics. 2007, Addison-Wesley, San Francisco—Toronto.
5. Schiff L.I. Quantum Mechanics. 1955, McGraw-Hill Book Co., Inc., Second Edition, New York.
6. Gingrich D.M. Practical Quantum Electrodynamics. 2006, CRC, The Taylor & Francis Group, Boca Raton, London, New York.