

Beyond the Hubble's Law

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Based on the Universe's scale factor introduced by Silva [1], we derive an expression for the receding velocities of arbitrary astronomical objects, which increase linearly up to the lookback distance of 2.1×10^3 Mpc and after that they increase in a positively accelerated way. The linear part corresponds to the Hubble's law.

1 Introduction

In a 2014 paper, Silva [1] introduced an expression for Universe's scale factor to describe the Universe's expansion,

$$a(t) = \exp\left(\frac{H_0 T_0}{\beta} \left(\left(\frac{t}{T_0}\right)^\beta - 1\right)\right), \quad (1)$$

where

$$\beta = 1 + H_0 T_0 \left(-\frac{1}{2} \Omega_m(T_0) + \Omega_\Lambda(T_0) - 1\right), \quad (2)$$

H_0 is the Hubble constant, T_0 is the Universe current age, $\Omega_m(T_0)$ is the cosmic matter density parameter (baryonic + non-baryonic matter), $\Omega_\Lambda(T_0)$ is the cosmic dark energy density parameter [2].

In reference [1] matter and dark energy are treated as perfect fluids and it is shown that it very difficult to distinguish between closed ($k = 1$), flat ($k = 0$) and open ($k = -1$) universes. In this paper we intuitively adopt $k = 1$ and explore the Universe as being closed.

The spacetime metric for $k = 1$ according to Friedmann-Lemaître-Robertson-Walker (FLRW) is [1, 3]

$$ds^2 = \mathfrak{R}^2(T_0) a^2(t) \left(d\psi^2 + \sin^2 \psi \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right) - c^2 dt^2 \quad (3)$$

where ψ , θ and ϕ are the the comoving space coordinates ($0 \leq \psi \leq \pi$, $0 \leq \theta$ and $0 \leq \phi \leq 2\pi$); $\mathfrak{R}(T_0)$ is the current Universe's radius of curvature. This proper time t is the cosmic time.

It is known that at $t = 380,000 \text{ yr} \approx 10^{-4} \text{ Gyr}$, after the Big Bang, the Universe became transparent and the first microwave photons started traveling freely through it. They constitute what is called the Cosmic Microwave Background (CMB).

The observer (Earth) is assumed to occupy position $\psi = 0$ for any time t in the comoving reference system. To reach the observer at the Universe age T the CMB photons leave a specific position ψ_T ($t \approx 10^{-4} \text{ Gyr}$). They follow a null geodesic.

It's time to make the following observation: since we will be dealing with large times values (some *giga* years) we have no loss if we treat $t \approx 10^{-4} \text{ Gyr}$ as $t \approx 0 \text{ Gyr}$ for practical purposes.

For a null geodesic we have:

$$-\frac{c dt}{\mathfrak{R}(0)} = d\psi, \quad (4)$$

$$\psi_T = \frac{c}{\mathfrak{R}(0)} \int_0^T \frac{1}{a(t)} dt. \quad (5)$$

We have seen then that CMB photons emitted at ψ_{T_0} and $t = 0$ should arrive at the observer, $\psi = 0$ and T_0 . Along their trajectory, other emitted photons, at later times, by astronomical objects that lie on the way, join the the photons troop and eventually reach the observer. They form the picture of the sky that the observer "sees". Certainly CMB photons emitted at $\psi > \psi_{T_0}$ will reach the observer at times latter than T_0 .

2 The receding velocity

As the Universe expands, the stretching distance between the observer and any astronomical object at time t is given by

$$\begin{aligned} d(t) &= \mathfrak{R}(0) a(t) (\psi_{T_0} - \psi_t) + ct \\ &= ca(t) \left(\int_0^{T_0} \frac{1}{a(t')} dt' - \int_0^t \frac{1}{a(t')} dt' \right) + ct \\ &= ca(t) \int_t^{T_0} \frac{1}{a(t')} dt + ct. \end{aligned} \quad (6)$$

The receding velocity of any astronomical object with respect to the observer is

$$\begin{aligned} v_{rec}(t) &= \dot{a}(t) c \int_t^{T_0} \frac{1}{a(t')} dt' \\ &= a(t) H(t) c \int_t^{T_0} \frac{1}{a(t')} dt', \end{aligned} \quad (7)$$

where we have used the fact that

$$\dot{a}(t) = a(t) H(t). \quad (8)$$

According to reference [1],

$$H(t) = H_0 \left(\frac{t}{T_0}\right)^{\beta-1} \quad (9)$$

By performing the integration in equation (8) we have

$$v_{\text{rec}}(t) = c \left(\frac{H_0 T_0}{\beta} \right)^{1-\frac{1}{\beta}} \left(\frac{t}{T_0} \right)^{-1+\beta} \exp \left(\frac{H_0 T_0}{\beta} \left(\frac{t}{T_0} \right)^\beta \right) \times \left(\Gamma \left(\frac{1}{\beta}, \frac{H_0 T_0}{\beta} \left(\frac{t}{T_0} \right)^\beta \right) - \Gamma \left(\frac{1}{\beta}, \frac{H_0 T_0}{\beta} \right) \right) \quad (10)$$

where $\Gamma(A, B)$ and $\Gamma(A, C)$ are incomplete Gamma Functions [4].

Taking into account that

$$\Gamma(A, B) - \Gamma(A, C) = \Gamma(A, B, C), \quad (11)$$

where $\Gamma(A, B, C)$ are generalized incomplete Gamma Functions [4], we have

$$v_{\text{rec}}(t) = c \left(\frac{H_0 T_0}{\beta} \right)^{1-\frac{1}{\beta}} \left(\frac{t}{T_0} \right)^{-1+\beta} \exp \left(\frac{H_0 T_0}{\beta} \left(\frac{t}{T_0} \right)^\beta \right) \times \Gamma \left(\frac{1}{\beta}, \frac{H_0 T_0}{\beta} \left(\frac{t}{T_0} \right)^\beta, \frac{H_0 T_0}{\beta} \right). \quad (12)$$

3 Comparison to Hubble's law

By replacing t by $T_0 - d_{lb}/c$, where $d_{lb} = ct_{lb}$ is the so called lookback distance, t_{lb} being the lookback time:

$$v_{\text{rec}}(d_{lb}) = c \left(\frac{H_0 T_0}{\beta} \right)^{1-\frac{1}{\beta}} \left(1 - \frac{d_{lb}}{cT_0} \right)^{-1+\beta} \times \exp \left(\frac{H_0 T_0}{\beta} \left(1 - \frac{d_{lb}}{cT_0} \right)^\beta \right) \times \Gamma \left(\frac{1}{\beta}, \frac{H_0 T_0}{\beta} \left(1 - \frac{d_{lb}}{cT_0} \right)^\beta, \frac{H_0 T_0}{\beta} \right). \quad (13)$$

Figure 1 shows that the receding velocities increase as the lookback distance increases, initially in a linear way. Distant astronomical objects are seen to recede at much faster velocities than the nearest ones.

By expanding expression (13) in power series of d_{lb} , and retaining the lowest order term we get

$$v_{\text{rec}}(d_{lb}) = H_0 v_{\text{rec}}(d_{lb}) + \text{higher order terms}. \quad (14)$$

The Hubble's law,

$$v_{\text{rec}}(d_{lb}) = H_0 v_{\text{rec}}(d_{lb}), \quad (15)$$

is an approximation to our just obtained expression. According to the present work, Hubble's law holds up to ~ 7 Gly or, equivalently $\sim 2.1 \times 10^3$ Megaparsecs.

For this work, we have used the following experimental data [5]:

$$H_0 = 69.32 \text{ kms}^{-1} \text{ Mpc}^{-1} = 0.0709 \text{ Gyr}^{-1}, \quad (16)$$

$$T_0 = 13.772 \text{ Gyr}.$$

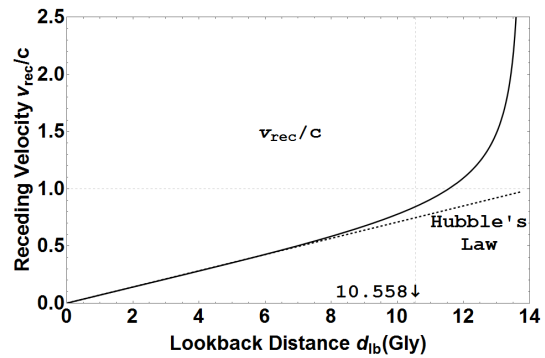


Fig. 1: Receding velocities as function of d_{lb} , the lookback distances from astronomical objects to the observer, on Earth.

As indicated by references [1, 6] the present scale factor predicts that the Universe goes from a matter era to a dark energy era at the age of $T_\star = 3.214$ Gyr. Before that matter dominated, and after that dark energy era dominates.

4 Conclusion

The Universe's scale factor $a(t) = \exp \left(\frac{H_0 T_0}{\beta} \left(\left(\frac{t}{T_0} \right)^\beta - 1 \right) \right)$, with $\beta = 1 + H_0 T_0 \left(-\frac{1}{2} \Omega_m(T_0) + \Omega_\Lambda(T_0) - 1 \right)$ introduced by Silva [1] has been used to find an expression for the receding velocities of astronomical objects caused by the expansion of the Universe. The expression found, equation (13), is a generalization of Hubble's law. This later one should be valid up to $\sim 2.1 \times 10^3$ Megaparsecs.

After such very good results we feel very stimulated with the idea that expression (1) is a very good candidate for describing the geometrical evolution of our Universe.

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