Flyby Anomaly via Least Action

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The observed but unexpected changes in velocity during spacecraft flybys of Earth are examined using the principle of least action in its original dissipative form. In general, the spacecraft's momentum will change when it travels through an energy density gradient of space that is enfolding a gravitating, orbiting and rotating body. When space is understood as a physical substance that embodies quanta of actions, rather than being modeled by a mere metric, it becomes apparent that the changes in momentum couple with flux of quanta from the local system of bodies to the universal surroundings or vice versa. In this way the original least-action principle accounts also for the 'anomalous' change in velocity by an equation of motion which complies with the empirical relation that has been deduced from Earth-flybys.

1 Introduction

Even a slight deviation from a common rule may entail an error in the very rule. Here, in the context of flyby anomaly, the rule – perhaps at stake – is conservation of momentum. It is a corner stone of physics, whence the flyby anomaly is worth attention.

The law of conservation of momentum asserts, for example, that when a spacecraft is passing by a planet, it will gain momentum as much as the planet will lose momentum. The momentum transfer is a minute drop for the massive planet but a giant boost for the tiny spacecraft. The spacecraft's velocity v will change relative to the Sun as much as its flight direction will change relative to orbital velocity u of the planet [1–3]. The gain can be at most $2u$ when the planet is moving straight at the spacecraft which will subsequently swing a full U-turn around the planet. Curiously though, it seems as if spacecraft had acquired more speed during certain flybys than the planet's orbital momentum could possibly grant them [4, 5]. The origin of this anomaly is unknown.

However, it has been inferred from meticulously monitored flybys of Earth [6–10] that the anomalous change in velocity Δv complies closely with relation [5]

$$\frac{\Delta v}{v} = \frac{2\omega_\oplus R_\oplus}{c}(\cos\delta_i - \cos\delta_o), \tag{1}$$

where c is the speed of light, $R_\oplus$ is Earth's radius and $\omega_\oplus$ angular velocity of rotation, δ_i is the spacecraft's inbound and δ_o outbound declination, so that $2\omega_\oplus R_\oplus/c = 0.49\times10^{-6}$. The relationship (Eq. 1) implies that the anomalous gain Δv in the spacecraft's velocity stems from Earth's angular velocity $\omega_\oplus$ depending on how the spacecraft's inbound and outbound asymptotes align relative to the axis of rotation. Yet, the effect of Earth's gravito-magnetic field on the spacecraft's velocity has been calculated to be many orders of magnitude smaller than the measured anomaly [11, 12]. Other explanations have also been considered [13–17] and found feasible [18], but there is currently no consensus what exactly underlies the phenomenon. Also the general validity of Eq. 1 has been questioned [19–22]. Moreover, it should be noted that anomalies, when without radar monitoring, are difficult to detect along flybys of other planetary bodies.

As long as the case is open there ought to be room for attempts to explain the measurements. Thus, we would like to contribute to the puzzle of flyby anomaly by maintaining that the spacecraft does move along a geodesic, i.e., a path of least action, also when it is subject to the unknown force that causes the unaccounted change in momentum. So, it should be possible to infer the cause of anomaly from the principle of least action. However, the familiar Lagrangian form when without dissipation applies only to closed stationary orbits such as ellipses or to ideal paths with symmetrical inbound and outbound trajectories. In contrast, the general form of the least action principle by Maupertuis [23–25] accounts also for open paths, most notably for hyperbolic flyby trajectories that are asymmetric relative to the planet's rotation. Furthermore, we are motivated to apply this general principle that distinguishes itself from particular models of celestial mechanics, because it has already accounted for anomalous perihelion precession [26], rotation of galaxies [27], geodetic and frame dragging drift rates [28] as well as for frequency shifts and bending of light [29], as well as for propagation of cosmic rays [30] and the thrust of electromagnetic drive [31]. Thus, our examination of the flyby anomaly using the universal principle is not a standalone study. It can be seen as a further test of our approach yet in another physical situation.

2 The least-action principle

The spacecraft is customarily pictured to move along a hyperbolic path as if it was coming from a distant asymptotic state of free space and returning via periapsis back to the asymptotic state. Per definition this ideal, i.e., fully reversible passage cannot accommodate any net change in momentum in the planet's frame of reference, because the initial and final asymptotic states are taken as indistinguishable from each other in energetic terms. In other words, the Lagrangian having only kinetic and potential energy terms does not allow for any change in the total energy, i.e., dissipation. But in reality the unaccounted increase (or decrease) in kinetic energy reveals that during the flyby the spacecraft does descend down (or move up) along a potential energy gradient, so that the initial and final states are not equal in energetic terms. Therefore, to account for the flyby anomaly as a non-conserved phenomenon we will use Mauperutuis's rather than Lagrange's principle of least action. Then it remains for us to identify among conceivable gradients in energy, the one that lies asymmetrically with respect to the spacecraft's inbound and outbound trajectories, and hence is responsible for the net change in energy.

In all cases, the spacecraft treks at least through the gravitational potential of free space. The all-embracing vacuum potential energy $GM^2/R = Mc^2$ totals from the mass M of all bodies in the Universe within Hubble's radius $R = cT$ at its current age $T = 13.8$ billion years where G is the gravitational constant [32]. In terms of geometry the free space energy density is characterized by the universal L^2-norm [33] that manifest itself in the quadratic form c^2. Physically speaking, the norm means that in the free space there is no shorter path than that taken by light. Thus, the energy density of free space, on the order of one nJ/m^3, is the ultimate reference for any other energy density.

A local potential energy, known as the local gravitational potential energy is in balance with the bound energy density of a body, for example, a planet, just as the universal gravitational potential is energy is in balance with all bodies in the Universe [34]. Thus, the spacecraft when moving past by the planet, will be subject to energy density gradients, i.e., forces that will show as changes in its momentum. We acknowledge that general relativity accounts for the space without energy density due to the gravitational field itself. General relativity expresses gravity in terms of the geometrical properties of spacetime through the Einstein field equations. This mathematical model is excellent for many data, but when without dissipation, it does not account accurately for irreversible changes in momentum, for instance, for the spacecraft anomalous gain in momentum during the flyby.

To work out the energy density gradient responsible for the dissipative change in momentum we will express the local energy density at a distance r from the body relative to the universal energy density by the ratio of light's universal velocity to its local velocity $n = c/v$. The index n has been used earlier to describe the gravitational potential in terms of an optical medium [35] consistently with the fact that gravity and electromagnetism share the same functional forms [34, 36]. The local excess in energy density is miniscule in the vicinity of an ordinary celestial body. This is to say that when light is grazing the planet Earth, its speed $v \leq c$ will hardly deviate from c. Therefore, light will experience only a minute change in momentum that will manifest itself as a tiny blue shift and next-to-negligible bending.

However, the spacecraft with velocity $v \ll c$ will be subject to a marked change in its momentum during its passage through the local potential of space imposed by the gravitating, orbiting and rotating Earth. This is to say that the spacecraft will gain momentum when inbound and conversely it will lose momentum when outbound. The inbound gain and outbound loss will sum up to zero in the case the open hyperbolic trajectory through a spherically symmetric field. A net change in momentum will accrue only if the flight path is open asymmetric relative to energy density gradients of space due to the planet's orbital and rotational motion.

In general the index n for a locus of space can be obtained from the least action principle in its original form by Maupertuis. The principle [23, 25–27, 29] equates a change in kinetic energy $d_t 2K$ with changes in scalar potential energy $\partial_t U$ and vector potential energy $\partial_t Q$,

$$d_t 2K = -\partial_t U + i\partial_t Q, \tag{2}$$

where we emphasize, although self-evidently, the orthogonal relationship between the gradients of scalar and vector potential energy by the imaginary quotient i. The equation of motion (Eq. 2) containing both real and imaginary parts ensures that any (formal) solution is non-conserved. Moreover, orthogonality is familiar from electrodynamics, for instance, as defined by Poynting theorem. Accordingly, when the spacecraft accelerates in the gravitational field of a planet, the quanta will dissipate to the surrounding free space from the local gravitational potential orthogonally to the acceleration.

The equation for the dissipative changes in energy [25, 31] (Eq. 2) corresponds to Newton's second law of motion for a change in momentum $\mathbf{p} = m\mathbf{v}$ when multiplied with velocity $\mathbf{v}$, i.e.,

$$\begin{aligned} \mathbf{F} &= d_t\mathbf{v} \quad | \cdot \mathbf{v} \\ \mathbf{F}\cdot\mathbf{v} &= d_t(m\mathbf{v})\cdot\mathbf{v} = \mathbf{v}\cdot m\mathbf{a} + v^2\partial_t m \\ d_t 2K &= -\mathbf{v}\cdot\nabla U + i\partial_t Q, \end{aligned} \tag{3}$$

where kinetic energy, i.e., vis viva is $2K = mv^2$, and where the spatial gradient of U relates to the familiar term $m\mathbf{a}$ of acceleration and the change in mass $dm = dE/c^2$ equals dissipation $n^2 d_t Q = d_t E$ to the free space. As usual, the mass-energy equivalence converts mass-bound energy to energy E of freely propagating photons in the vacuum. In short, Eqs. 2 and 3 simply state that at any position along the spacecraft's least-time path the momentum will follow the force $\mathbf{F} = -\nabla U + i\nabla Q$, where the energy density gradient subsumes both the scalar and vector components. In this way our account on gravity is physical rather than merely mathematical and consistent with electromagnetism. However, in what follows, the orthogonality of the two components remains only implicit when we work out only the magnitude of the total potential energy in any given position along the spacecraft's path.

3 Passages through gradients

The general principle of least action in its original form allows us to examine the flyby trajectories by specifying the energy density of space by the index n at a particular position r of space from the center of a gravitating body with mass $M_\oplus$. Also earlier the gravitational field has been described in terms of an optical medium [35], but we do not model space by an explicit metric, instead present it in energetic terms. When approximating the total potential energy U only with the local gravitational potential energy $GmM_\oplus/r$, Eq. 3 can be solved for the index of space

$$\begin{aligned} & d_t(mv^2) = -\partial_t \frac{GmM_\oplus}{r} + i\partial_t mc^2 \\ & n^2 = \frac{c^2}{v^2} = \left(1 - \frac{GM_\oplus}{c^2 r}\right)^{-1} \approx 1 + \frac{GM_\oplus}{c^2 r} = 1 + \varphi_\oplus \end{aligned} \quad (4)$$

at a locus r. The squared index sums the universal density (unity) and the local excess $\varphi_\oplus$ as experienced by a test body of vanishing mass, i.e., a photon. The first order approximation means that n^2 does not differ much from the asymptotic ($r \to r_\infty$) unity of free space. Explicitly, a ray of light will bend hardly at all even when grazing the Earth of radius $R_\oplus$, since $\varphi_\oplus = GM_\oplus/c^2 R_\oplus \approx 0.7 \times 10^{-9}$.

However, the spacecraft with its minute velocity v relative to the speed of light, i.e., $v^2/c^2 \ll 1$, will accelerate considerably when traversing through the gradient $d(n^2)/dr = \nabla\varphi_\oplus = -GM_\oplus \mathbf{r}_o/c^2 r^2$ where the unit vector $\mathbf{r}_o = \mathbf{r}/r$ points to the center of mass. According to Eqs. 2 and 3 the spacecraft will fly past by the planet when $\mathbf{v} \cdot d_t\mathbf{p}/c^2 > -\mathbf{v} \cdot \nabla\varphi_\oplus$. Conversely, when $\mathbf{v} \cdot d_t\mathbf{p}/c^2 < -\mathbf{v} \cdot \nabla\varphi_\oplus$, the spacecraft will spiral down to a crash on the planet. Eventually, when $\mathbf{v} \cdot d_t\mathbf{p}/c^2 = -\mathbf{v} \cdot \nabla\varphi_\oplus$, Eq. 2 can be integrated to a closed form. Then the net flux from to the system to its surroundings vanishes $d_t Q = 0$, and hence the integration yields the familiar stationary condition $2K + U = 0$, i.e., the virial theorem. This is to say, the spacecraft has settled on a stable Keplerian orbit about the planet.

When the planet is not only gravitating but both orbiting and rotating, then the excess in energy density of space at r is in balance also with energy that is bound in both the orbital and rotational motion as much as $\mathbf{r}_o$ aligns along the planet's orbital u and rotational $\mathbf{w}_\oplus = \omega_\oplus R_\oplus$ velocities, denoted by $u_r = \mathbf{u} \cdot \mathbf{r}_o$ and $w_r = \|\mathbf{w}_\oplus \times \mathbf{r}_o\|$, i.e.,

$$\begin{aligned} n^2 &= \frac{c^2}{v^2} = \left(1 - \frac{GM_\oplus}{c^2 r} - \frac{u_r^2 R_\oplus}{c^2 r} - \frac{w_r^2 R_\oplus}{c^2 r}\right)^{-1} \\ &\approx 1 + \varphi_\oplus + \varphi_u + \varphi_w . \end{aligned} \quad (5)$$

Again the first order approximation means that n^2 does not differ much from the free space unity. Explicitly when setting for the Earth with $r \approx R_\oplus$ and $u_r = u_\oplus$, the orbital $\varphi_u = u_\oplus^2/c^2 \approx 10^{-8}$ and rotational $\varphi_w = w_\oplus^2/c^2 \approx 0.6 \times 10^{-13}$ contributions are tiny. This means that the Earth hardly drags the vacuum along with its orbital and rotational motion.

However, the spacecraft with velocity $v^2/c^2 \ll 1$ will acquire momentum markedly during its way through the gradient $\nabla\varphi$. The gain in momentum from the orbital motion is the well-known gravity assist. Obviously this gravitational slingshot cannot be used when the spacecraft moves too slowly to catch the planet, i.e., $\mathbf{v} \cdot d_t\mathbf{p}/c^2 < -\mathbf{v} \cdot \nabla\varphi_u$. Eventually, when $\mathbf{v} \cdot d_t\mathbf{p}/c^2 = -\mathbf{v} \cdot \nabla\varphi_u$, dissipation vanishes, and hence Eq. 2 can be integrated to the stationary state condition $2K + U = 0$. It means that the spacecraft has settled on a stable Lagrangian point where it is coorbiting Sun along with Earth.

In addition to the gain in momentum from the planet's orbital motion, the spacecraft may gain a detectable amount of momentum when traversing through the gradient $\nabla\varphi_w$ due to the planet's rotation about its axis. Obviously this velocity excess will be deemed as anomalous when left unaccounted. Conversely, when the gradients along the inbound and outbound trajectories are opposite and equal, i.e., symmetric about the planet's rotation, there is no net dissipation and no net change in momentum. Eventually, when dissipation vanishes, $\mathbf{v} \cdot d_t\mathbf{p}/c^2 = -\mathbf{v} \cdot \nabla\varphi_w$, and hence Eq. 2 reduces to the steady-state condition $2K + U = 0$. It means that the spacecraft has settled on a geostationary orbit. When the spacecraft is in synchrony with the planet's rotation, obviously it will not be exposed to any energy density gradients due to the rotation.

4 Anomalous change in velocity

The above classification of spatial energy density in the gravitational, orbital and rotational terms (Eq. 5) serves us to specify the equation for the "anomalous" gain in velocity Δv. It accrues during the flyby through the energy density gradient of space $\nabla\varphi_w$ imposed by the rotating planet. In general the change in the spacecraft's momentum at any point along the trajectory is, according to Eq. 3, equal to the force $\mathbf{F} = d_t\mathbf{p} = d_t(m\mathbf{v}) = mc^2\nabla\varphi_w$. When the minute change in mass dm is neglected, the anomalous gain in velocity Δv due to the gradient ∂_w of rotational contribution φ_w can be obtained by summing up the changes in velocity dv

$$\begin{aligned} \Delta v &= \int_{v_i}^{v_o} \partial_w \varphi_w dv = \int_{\theta_i}^{\theta_o} v \frac{\partial(\omega_\oplus R_\oplus \cos\delta/c)^2}{\partial(\omega_\oplus R_\oplus \sin\delta/c)} \frac{R_\oplus}{r} d\delta \\ &= \int_{\delta_i}^{\delta_o} v \frac{2\omega_\oplus R_\oplus \sin\delta}{c} \frac{R_\oplus}{r} d\delta \\ &\approx v \frac{2\omega_\oplus R_\oplus}{c} (\cos\delta_i - \cos\delta_o) \end{aligned} \quad (6)$$

along the flight path from the inbound asymptotic velocity v_i to the outbound asymptotic velocity v_o. The equation 6 integrates the gradient ∂_w of the rotational contribution φ_w

given by Eq. 5 from the inbound asymptote with declination δ_i to the outbound asymptote with declination δ_o along the spacecraft's path. The gain in velocity will accrue only when the inbound and outbound trajectories through the energy gradients due to the planet's rotation are asymmetric. The trigonometric form of the energy density gradient $\partial_w = \partial/\partial(\omega_\oplus R_\oplus \sin\delta/c)$, where δ denotes declination (Figure 1), for the integration of declination from the inbound to outbound asymptote has been derived earlier [16]. It is easy to check by inspecting the following two points. At the Equatorial plane $\delta = 0$, where the quadratic factor $(\omega_\oplus R_\oplus \cos\delta/c)^2$ of φ_w peaks, the energy density gradient vanishes. Conversely, at poles $\delta = \pm\pi/2$, where φ_w in turn vanishes, the gradient in space due to the planet's rotation peaks. In addition to the declination by $\sin\delta$, the gradient depends on the angular velocity $\omega_\oplus$ and radius $R_\oplus$ relative to c. The product form of the three factors ensures the obvious fact that if any one of them vanishes, the gradient does not exist. The transformation from one variable of integration to another $dv = vd\delta$ follows from $vdt = rd\delta$, e.g., defining $dv = adt$ via acceleration $a = v^2/r$.

We motivate the approximation $R_\oplus/r \approx 1$ in Eq. 6 to recognize the empirical equation (Eq. 1) because the radial gradient of φ falls as $1/r^2$, and hence most of Δv accumulates when the spacecraft is near the periapsis whereas contribution from the long opposite inbound and outbound legs is negligible. The polar coordinate representation $R_\oplus/r = 1/2(1-R_\oplus/C+\cos\theta)/(1-R_\oplus/2C)$ reveals the decreasing contribution of a path position r in the sum (Eq. 6) as a function of increasing polar angle θ. The distance from the center of mass to the intersection of the inbound and outbound asymptotes of the hyperbola is denoted with C. Specifically, Eq. 6 yields the maximum change $\Delta v/v = 2\omega_\oplus R_\oplus/c$ for the flight along the rectangular hyperbola from the inbound arm $\delta_i = \pi$ to the outbound arm $\delta_o = -\pi/2$ via the periapsis at $\delta_a = \pi/4$ for a low altitude $r \to R_\oplus$ passage. Conversely, for a high altitude path, such as that of Rosetta's last flyby, the approximation $r \to R_\oplus$ underlying the empirical equation is less motivated, and hence the anomaly is negligible.

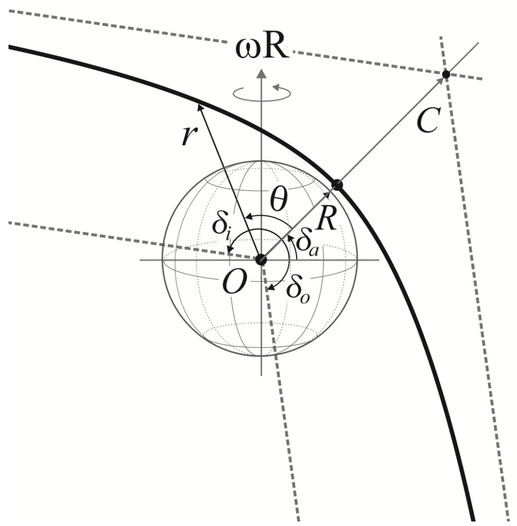

Fig. 1: Equatorial view of a grazing flyby trajectory. The hyperbolic flight path is defined by the planet's radius R extending nearly to the periapsis (solid dot) at declination δ_a and the distance C from the center of mass at the origin O to the intersection of inbound and outbound asymptotes (dashed lines) with declinations δ_i and δ_o. The path's radial coordinate is given by r and polar angle by θ as measured from δ_a. The planet's axis of rotation with angular velocity ωR stands upright.

Obviously the derived formula (Eq. 6) is not only an explicit approximation by $R_\oplus/r \approx 1$, but also implicit in modeling the planet as a rigid homogenous sphere. Moreover, the derivation also neglects apparent forces that are imposed on the spacecraft, such as a drag due to atmospheric friction. However, our study does not aim at producing a formula to calculate Δv due to the atmospheric drag or planet's geoid, instead it targets by the derivation of $\Delta v/v$ to explain the phenomenological formula (Eq. 1) and to identify the anomalous gain in momentum to result from the spacecraft traversing through the energy density gradient of space imposed by the rotation of the planet. Undoubtedly, when more flyby data accumulates, the empirical formula (Eq. 1) will be verified or falsified, thereby giving also a verdict on this study.

5 Discussion

The mathematical correspondence between the empirical relationship (Eq. 1) and the derived formula (Eq. 6) is reassuring, but not alone an explanation for the anomalous gain in velocity. Namely, the obtained consistency in energetic terms is by itself not a tangible explanation, because energy as such does not exist but it is an attribute of its carrier. Thus, the profound question is: What is the carrier substance that embodies the universal density of space and local gravitational potentials that the spacecraft is subject to during its flyby? Of course, this query is not relevant when general relativity is used as a mathematical model for measurements. But when one is after the cause, i.e., the force responsible for the flyby anomaly, the physical form of space must be considered.

The carrier of gravitational force has been sought for long. Nonetheless the graviton of quantum field theory remains a hypothetical elementary particle. In the past the photon was considered as the carrier, because gravity and electromagnetism share similar functional forms [34, 36, 38] as well as because the squared speed of light in the vacuum relates to the absolute electromagnetic characteristics of free space via $c^2 = 1/\varepsilon_o\mu_o$. Also the free space gauge $\partial_t\phi + c^2\nabla\cdot\mathbf{A} = 0$ implies physical existence of scalar ϕ and vector $\mathbf{A}$ potentials, so that ϕ will decrease with time when quanta move down

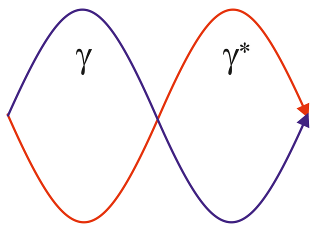

Fig. 2: The photon γ (blue) is the undividable quantum of action whose momentum resides on its wavelength, and equivalently, whose energy is within its period of time. The photon with opposite polarization (red) is the antiphoton γ^*. When γ and γ^* copropagate, the net electromagnetic force will vanish, but the compound boson continues to carry momentum and energy. These compound actions embody space universally and locally. Thus, the associated energy density appears as the universal gravitational potential energy, known as the vacuum density, which is in balance with all bodies. Likewise, a local potential energy is in balance with a local gravitating, orbiting and rotating body.

along the gradient of **A** or vice versa. Recently the old tenet of photon-embodied space has been revived so that the photons are considered to propagate in pairs of opposite polarization, and hence the pairs are without electromagnetic forces [28, 31, 39]. This destructive interference is, of course, familiar from diffraction. By the same token, Aharonov-Bohm experiment demonstrates how an applied vector potential will increase the energy density without introducing fields along the path [40]. According to this percept the two quanta of light do not vanish for nothing when interfering destructively, instead they continue in copropagation with opposite phases, and hence continue in carrying energy and momentum (Figure 2).

Our portrayal of the physical vacuum reminds of de Broglie's theory [41] about a spatially extended, particle centered pilot wave [42]. This view of the physical vacuum, as ours, makes sense of quantum mechanical phenomena without conceptual challenges [43]. In view of that, it has been understood also earlier that c, ε_o and μ_o are not constants, but properties when the vacuum has been considered to embody continuously appearing and disappearing fermion pairs [44, 45]. Instead of accounting for the vacuum's electromagnetic properties by transiently appearing paired charges we reason that when a charge appears in the vacuum, a corresponding force will appear. The force will move the paired photons away from the out-of-phase relation, and hence an electromagnetic field will appear around the charge. Thus, when an atom ionizes, the photons of the electromagnetic field will not appear out of the blue, but they have been around all the time, however in the out-of-phase configuration that manifests itself only as energy density.

The photon-embodied vacuum is understood to emerge from various processes, such as annihilation, where constituents of matter with opposite charge transform to mere radiation. For example, the annihilation of electron with positron will yield, in addition to the two readily observable photons of opposite polarization and directions of propagation, also pairs of co-propagating photons. Conversely, the photon-embodied vacuum is the source of quanta for pair production [37,39,46]. Likewise, electron capture where a proton turns to a neutron, pairs of co-propagating photons will emerge from annihilation of the constituents with opposite charges. When the space is understood to embody the oppositely paired photons, it is easy to envision that space around a body of high energy density houses a radially decreasing energy density, known as the local gravitational potential energy. In this way gravity can be understood as force, like any other force, to result from the energy density difference over a distance, i.e., from a gradient. Ensuing motions consume the free energy in least time. This evolution is expressed by the principle of least action in its original form (Eq. 2). Namely, all bodies move from one state to another along geodesics to diminish density gradients in the least time.

The least-time imperative means that the two bodies will move toward each other when the surrounding universal space is sparse enough to accept the paired quanta that are released from the dense gravitational potential of the bodies to the surrounding free space along the paths of least time. For example, an object falls straight down on the ground, i.e., along the least-time path, to consume the energy density difference between the local gravitational potential and the sparse surrounding vacuum. When the body is falling down, the oppositely paired photons are released from the local gravitational potential to the surrounding universal vacuum also along their paths of least time. Conversely, the two bodies will move away from each other when the surrounding potential is rich enough to grant paired quanta with energy to the local potential about the bodies.

In the same manner it is inescapable that it takes some form of free energy, ultimately carried by the photons that have been acquired from insolation, to lift up the fallen object from the ground back up on its initial height. So, the logic of reversibility says that the photons that were absorbed when the object was lifted up must have been emitted when the object was falling down. Thus, gravity is a dissipative phenomenon. When the bodies move toward each other, there is an efflux of quanta with energy to the surroundings, and conversely when the bodies move away from each other, there is an influx of quanta with energy from the surroundings. Manifestly, there is no net flux, i.e., no net dissipation from the system of bodies at a stationary state corresponding to an energetic balance with its surroundings.

This insight to gravity allows us to describe the spacecraft's flyby as an energy transfer process where quanta move from the local system of bodies to the surrounding space or vice versa. Flyby mission data show temporary maxima and minima in energy transfer that moderate toward the spacecraft's asymptotic courses [4]. We remind that oscillations are characteristics of least-time transitions from one state to another [47]. The oscillations are pronounced when the rate of energy transfer is rapid compared with the bound energy.

With this insight to gravity as a dissipative phenomenon, let us first consider the flight past a gravitating spherical body. The spacecraft treks along its inbound trajectory through an increasing energy density of space, i.e., the $1/r^2$-force field when the distance r closes toward the body. The increase in the spatial potential energy is balanced, according to Eqs. 2 and 3, by an increase in kinetic energy as well as by efflux of the oppositely paired quanta from the local gravitational potential comprising the body and the spacecraft to the universal gravitational potential due to all bodies in the Universe. The flux of quanta is often overlooked because the oppositely paired quanta without net electromagnetic field cannot be detected readily. However, the dissipation can be inferred recalling that the total gravitational potential energy of the body and the spacecraft at the periapsis is not exactly equal to the total potential energy when the spacecraft is at a point on the arm of hyperbola. The emission of quanta will cease, i.e., dissipation will vanish $d_tQ = 0$ momentarily, when the spacecraft arrives at the periapsis, where kinetic energy $2K$ matches the scalar potential energy U. Thereafter, along the outbound asymptote $2K$ will exceed U, and hence the paired quanta will be acquired from the surrounding vacuum to the local gravitational potential so that the balance with the surrounding density will be eventually regained far away from the planet. Since the passage from the inbound asymptotic state via the periapsis to the outbound asymptotic state is symmetric, the emission of quanta from the local system and the absorption to the local system match perfectly, and hence the net dissipation vanishes. Thus, the momentum of the two-body system is conserved.

Next, let us consider the flight past by an orbiting body. Along the inbound trajectory the spacecraft travels through the energy density of space that increases more rapidly than in the case of the merely gravitating body, namely at the rate that the planet orbits straight at the spacecraft. This more rapid increase in the potential energy is balanced, just as reasoned above, by a more rapid increase in kinetic energy concurrently with dissipation of the oppositely paired quanta to the surrounding space. First when at the periapsis, where the spacecraft moves orthogonal to the planetary orbit, dissipation vanishes momentarily. Thereafter, along the outbound asymptote $2K$ will exceed U, and hence quanta will be acquired from the surrounding vacuum to the local gravitational potential energy comprising the body and the spacecraft to regain the balance eventually when far away. Clearly the flyby about the approaching planet and the flyby about the departing planet differ from each by the rates of momentum and energy transfer from the system to the surrounding space. Thus, the spacecraft will pick up momentum in the former case and it will lose momentum in the latter case. The formula for the spacecraft's change in velocity can be derived in the same manner as Eq. 6 was derived. Consistently, also the (very slightly) perturbed planet will regain a stable orbit by processes where the paired quanta carry energy from the surroundings to the local potential and vice versa until the free energy minimum state has been attained.

Finally, let us consider the flight past a rotating planet that imposes an axially symmetric energy density gradient on the surrounding space. When the gradient along the inbound trajectory is equal in magnitude to the gradient along the outbound trajectory but of opposite sign, the emission and absorption of quanta from the system comprising the body and the spacecraft to the surrounding vacuum are equal. Thus, in that case the momentum is conserved, and hence no anomalous gain or loss in velocity will detected. Conversely, when the emission of quanta along the inbound trajectory and the absorption of quanta along the outbound trajectory do not cancel each other exactly, the spacecraft will either pick up or lose momentum depending on the sign of net dissipation. Likewise the concurrent (minute) perturbation of the planet's rotational momentum will damp down toward a stable state of spinning by energy transfer processes from the systemic potential to the surroundings and vice versa until the net dissipation finally vanishes at the free energy minimum state. Perhaps our account on gravity summons up the old abandoned idea of luminiferous ether [48]. Therefore, it is worth emphasizing that the proposed physical vacuum is not a medium that supports propagation of light, instead the photons constitute space. The paired photons without net polarization do not couple in electromagnetic terms, and hence the space is dark, but not illusive or only a mathematical metric. It reacts to every act. Any change in momentum is met with resistance, known as inertia, since the spatial energy density redistributes to regain balance among perturbed bodies [31].

6 Conclusions

We conclude that the flyby anomaly only appears as an odd phenomenon when not all components of force are included in its explanation. Specifically, we maintain that the law of conservation of momentum holds when the system of bodies associated with local potentials of space will in total neither lose nor gain quanta from the surrounding systems. The ultimate surroundings for any local system is the universal free space. It must be taken into account in the explanation of flyby anomaly.

We resort to the old idea that the vacuum is embodied by the quanta of light which pair in opposite polarization. Hence space is dark but it holds an energy density [32] on the order of one nJ/m^3. The non-zero energy density displays itself also in the Aharanov-Bohm experiment [40] and as the Casimir effect [49]. So in any closed system the conservation of momentum is a solid law. In fact, the law may seem universal, since the Universe as a whole may by definition seem like a closed system. However, the quanta of light, that embody the space both in pairs of opposite polarization and solo, are open actions (Figure 2), whose momentum $\mathbf{p}$ may decrease concomitantly with increasing wavelength λ or vice versa so that the measure, known as Planck's constant, $h = \mathbf{p} \cdot \lambda$ remains invariant. Equivalently stated, a decrease in energy E is counterbalanced by an increase in time t, so that $h = Et$ is constant. Indeed, astronomical observations imply that the total energy density of the Universe is decreasing with increasing time. The photon that emerged from the nascent energy-dense Universe has shifted down in frequency $f = 1/t$ when adapting to ever more sparse surrounding densities on its way to us and eventually terminating at absorption to our detector. Conversely, when insisting on that energy is conserved, i.e., by applying a theory that conserves a symmetry, the ensuing interpretation of supernovae data will require an ad hoc patching, for instance, by dark energy [26].

Rules and regularities that are so apparent across scales of nature, are rightfully related to conservation laws. However, to avoid assigning phenomena as anomalous, it is necessary to include everything in an explanation. To this end among the laws of nature the truly superior and solid one is the conservation of the total number of quantized actions in the whole Universe.

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