The Newtonian Constant G and the Einstein Equations

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The measurement of the Newtonian constant of gravitation G is in an impasse because most results deviate from the average value more than 10 times their estimated measurements uncertainties. Via the Einstein field equations G is related to the cosmological constant Λ and because normal matter, dark matter and dark energy must add up to 100%, Λ is a measure for dark energy. So it follows that G is related to dark matter. The density of the dark matter halo around the earth is influenced by the gravitational attraction of the earth and because the earth is not a perfect sphere, the halo varies along the surface. So we expect a variation of dark matter density with the gravitational acceleration g. These variations in dark matter affect G and indeed we have found a correlation between the constant G and the local value of the gravitational acceleration g.

1 Introduction

The gravitational constant G is commonly measured by using a torsion balance suspended by a wire as has been introduced by Cavendish. The plane of the rotating masses is positioned exactly horizontal and therefore the influence of local gravity variations is supposed to be negligible. However, the horizontal attraction force between the test masses in the apparatus is not only governed by these masses and their distance, but also by the local density of dark matter. We accept that gravitational attraction forces are influenced by dark matter and the local density of dark matter will vary with the local mass variations of the earth. So we expect a correlation between G and the gravitational acceleration g.

2 The correlation between G and g

In the following analyses 16 values of G recommended by CODATA in the period 1999-2014 [1, 2, 3] are represented, as they were measured by 9 institutes. The values of the gravitational acceleration g at 8 different locations are calculated by the website Wolfram Alpha. This calculation method is based on the Earth Gravitational Model, EGM 2008. It is noted that Uci-14 has not been measured at Irvine, California but near Handford, Washington [4].Therefore the value of g is calculated for the nearby city Richland.

Furthermore, the g value of Florence was measured in situ with the Atom Interferometer by the group of Tino [5, 6].

The analysis results in the following table and Figure 1.

G is the gravitational constant in 10^{-11} $m^3kg^{-1}s^{-2}$ and the last column in the table shows the standard uncertainty *u* of the measured value of G.

The graph shows a correlation of the gravitational constant G with the gravitational acceleration g according to the best-fit linear regression line, having a slope of 0.1371 and the coefficient of determination R^2 = 0.6323.

Obviously this effect also results in a dependency of G on the geographical latitude on the earth, as shown in Figure 2.

From 1999 onwards the measured values of G seem to be more reliable than before, so we have included only the values from the year 1999 and after. Where CODATA replaces old measured values by later measurements from the same institute, we have included all values measured in the named

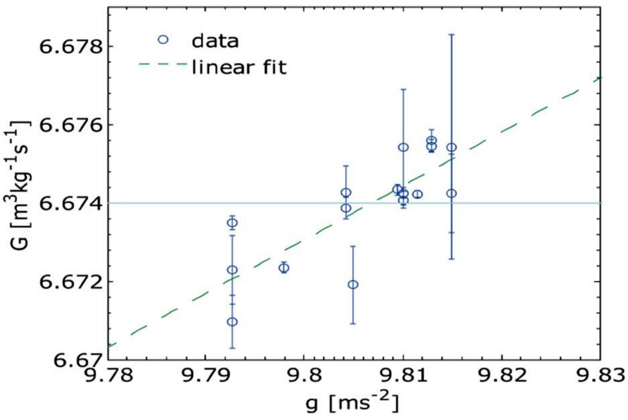

Fig. 1: Correlation of the gravitational constant G with the gravitational acceleration g. G = 0.1371 g + 5.328; R^2=0.6323.

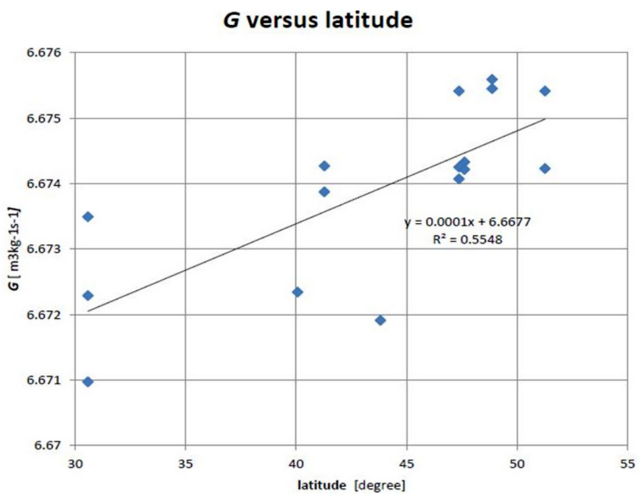

Fig. 2: Dependency of G on the geographical latitude.

Table 1: The 16 values of G recommended by CODATA in the period 1999-2014.

g	G	Institute	Location	Latitude	std
-	$\times 10^{-11}$	-	-	Degree	$\times 10^{-11}$
9.7927	6.67097	hust 99	Wuhan	30.58	0.00067
9.7927	6.67229	hust 05	Wuhan	30.58	0.00087
9.7927	6.67349	hust 09	Wuhan	30.58	0.00018
9.79795	6.67234	jila 010	Boulder	40.07	0.00014
9.80422	6.67427	msl 99	New Zeland	41.28	0.00067
9.80422	6.67387	msl 03	New Zeland	41.28	0.00027
9.80492	6.67191	lens 14	Florence	43.82	0.00099
9.80943	6.67433	uci 14	Richland	47.62	0.00013
9.81007	6.67542	uzur 99	Zurich	47.37	0.00147
9.81007	6.67407	uzur 02	Zurich	47.37	0.00022
9.81007	6.67425	uzur 06	Zurich	47.37	0.00013
9.81145	6.67422	uwash 00	Seattle	47.62	0.00009
9.81289	6.67559	bipm 01	Paris	48.87	0.00027
9.81289	6.67545	bipm 13	Paris	48.87	0.00016
9.81498	6.67542	uwup 99	Wuppertal	51.26	0.00287
9.81498	6.67423	uwup 02	Wuppertal	51.26	0.00100

period. The horizontal line in the graph at G = 6.674×10^{-11} $m^3kg^{-1}s^{-1}$ represents the average value calculated by CODATA in the year 2010. However, the correlation between G and g as we have found, renders it not useful to calculate an average value for G.

3 Further measurements

It has been raised by Quinn [7] that the Newtonian constant may be too difficult to measure, as the measured values spread 10 times more than the uncertainties of most measurements. However, we maintain that the problem is not the difficulty of the measurement but ignorance about the correlation of G and g.

Further compelling evidence for the named correlation can be obtained by doing several measurements with one and the same apparatus at different locations. Then the measured values can be compared better, because their accuracy is the same and no differences occur due to different measuring methods and different devices. It is also necessary to measure g in situ instead of calculating that value. More clarity can be obtained by taking additional measurements at places where g has an extreme value, for instance far away from the equator (e.g. at Helsinki) and nearby (e.g. at Quito). The group of Tino [5, 6] has developed a small apparatus based on atom interferometry. Such apparatus would be quite suitable for measuring both G and g.

4 Conclusion

Our analysis shows a correlation between G and g. This correlation suggests that the value of G depends on the place where it is measured, and thus G is not a universal constant of nature.

5 Appendix

The original Einstein field equations are:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

The right hand part of the equation is the energy/momentum tensor and governs the curvature of space-time. The left hand part describes the measure of this curvature.

This set of equations generates no stationary solution, and therefore Einstein made a correction by adding an extra term with the cosmological constant Λ. The corrected field equations are:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

which can generate a stationary solution by inserting a suitable value for Λ.

At the end of the 20th century dark matter and dark energy were introduced in order to understand the uneven expansion of the universe and since then Λ is considered to be a measure of dark energy. When dark energy dominates dark matter, there is an accelerated expansion of the universe, and when dark matter dominates, the expansion is decelerated.

The cosmological constant Λ is linked to the gravitational constant G by the corrected field equations of Einstein. At the same time dark energy, dark matter and normal matter must add up to 100%. So dark energy and dark matter are dependent. In the field equations Λ and G are dependent as well. This means that we can rewrite the corrected field equations in the original form, without Λ, realizing that G depends on place and time. The field equations then become:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G(r,t)}{c^4} T_{\mu\nu}.$$

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