

Using the SALI Method to Distinguish Chaotic and Regular Orbits in Barred Galaxies with the LP-VIcode Program

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The Smaller Alignment Index (SALI) is a new mathematical tool for chaos detection in the phase space of Hamiltonian Dynamical Systems. With temporal behavior very specific to movements ordered or chaotic, the SALI method is very efficient in distinguishing between chaotic and regular movements. In this work, this method will be applied in the study of stellar orbits immersed in a gravitational potential of barred galaxies, once the motion of a test particle, in a rotating barred galaxy model is given by a Hamiltonian function. Using an analytical potential representative of a galaxy with bar (two degrees of freedom), we integrate some orbits and apply SALI in order to verify their stabilities. In this paper, we will discuss a few cases illustrating the trajectories of chaotic and regular orbits accompanied by the graph containing the behavior of SALI. All calculations and integrations were performed with the LP-VIcode program.

1 Introduction

One of the schemes more used to classify galaxies according to their morphology was proposed by Edwin Powell Hubble. Basically, the Hubble fork separates galaxies in two types: regular spirals (S) and barred spirals (SB). The galaxy bar, spiral arms and even galactic rings are structures that can be interpreted as disturbance to axisymmetric potential of the galactic disk.

In this work, we study the nature of some orbits immersed in analytical potentials with two degrees of freedom representing barred galaxies. In order to do this, we applied the Smaller Alignment Index (SALI) [9–13], which is a mathematical tool for distinguishing regular and chaotic motions in the phase space of Hamiltonian Dynamical Systems in analytical gravitational potentials. It is possible because the motion of a test particle in a rotating barred galaxy model is given by a Hamiltonian function.

The orbits integration and the SALI calculation were performed using the LP-VIcode program [2]. The LP-VIcode is a fully operational code in Fortran 77 that calculates efficiently 10 chaos indicators for dynamic systems, regardless of the number of dimensions, where SALI is one of them. To construct our barred galaxies models, two different sets of parameters were extracted from the paper of Manos and Athanassoula [5].

The main purpose of this paper is to show some regular and chaotic orbits, where the stability study was done using the SALI method. Such orbits were taken immersed in a mathematical model for the gravitational potential that simulates a barred galaxy in a system with two degrees of freedom.

2 Methodology

2.1 The SALI method

Considering a Hamiltonian flow (N degrees of freedom), an

orbit in the $2N$ -dimensional phase space with initial condition $P(0) = (x_1(0), \dots, x_{2N}(0))$ and two different initial deviation vectors from the initial point $P(0)$, $w_1(t)$ and $w_2(t)$, we define the Smaller Alignment Index (SALI) by:

$$\text{SALI}(t) = \min \{ \|\widehat{w}_1(t) + \widehat{w}_2(t)\|, \|\widehat{w}_1(t) - \widehat{w}_2(t)\| \} \quad (1)$$

where $\widehat{w}_i(t) = w_i(t)/\|w_i(t)\|$ for $i \in \{1, 2\}$.

In the case of chaotic orbits, $\text{SALI}(t)$ falls exponentially to zero as follows:

$$\text{SALI}(t) \propto e^{-(L_1 - L_2)t} \quad (2)$$

where L_1 and L_2 are the biggest Lyapunov Exponents.

When the behavior is ordered, SALI oscillates in non-zero values, that is:

$$\text{SALI}(t) \approx \text{constant} > 0, t \rightarrow \infty. \quad (3)$$

Therefore, there is a clear distinction between orderly and chaotic behavior using this method.

2.2 Gravitational potential of a barred galaxy

We apply the SALI method in the study of stellar orbits immersed in a gravitational potential of barred galaxies, once the movement of a test particle in a rotating three-dimensional model of a barred galaxy is given by the Hamiltonian:

$$\begin{aligned} H(x, y, z, p_x, p_y, p_z) = \\ = (p_x^2 + p_y^2 + p_z^2) + \Phi_T(x, y, z) + \Omega_b(xp_y - yp_x) \end{aligned} \quad (4)$$

where the bar rotates around z ; x and y contain respectively the major and minor axes of the galactic bar, Φ_T is the gravitational potential (which will be described later), and Ω_b represents the standard angular velocity of the bar.

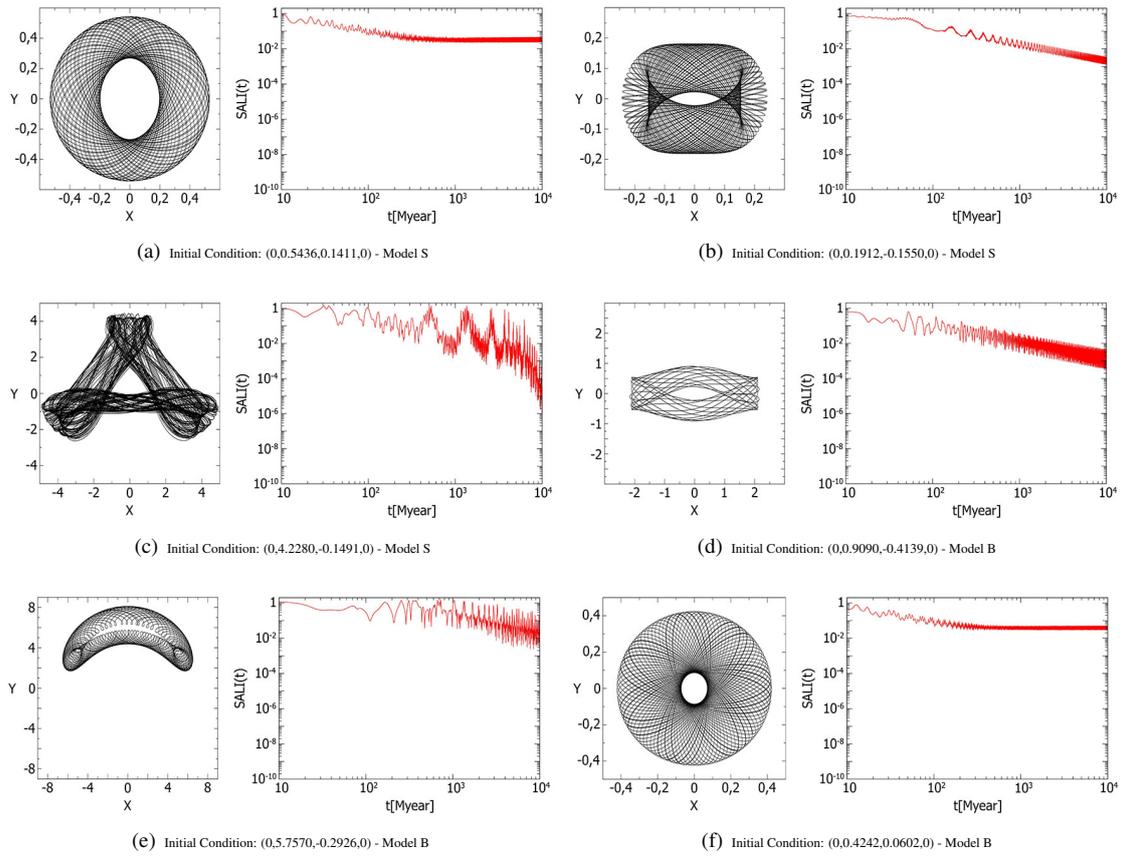


Fig. 1: Six orbits, each one with its SALI diagram. All orbits were integrated up to 10,000 Myr. Only the first 500 Myr were plotted in (a), (b), (d) and (f), for clarity.

For this Hamiltonian, the corresponding equations of motion and the corresponding variational equations that govern the evolution of a deviation vector can be found in [4]. With such equations it is possible to follow the temporal evolution of a moving particle immersed in the potential Φ_T , as well as verify if this orbit is chaotic or regular, following the evolution of deviation vectors by the SALI method.

In this work, the total potential Φ_T is composed by three components, representing the galactic bulge, disk and bar:

$$\Phi_T = \Phi_{Bulge} + \Phi_{Disk} + \Phi_{Bar}. \quad (5)$$

We represent the bulge by the Plummer Model [8]

$$\Phi_{Bulge} = -\frac{GM_S}{\sqrt{x^2 + y^2 + z^2 + \epsilon_S^2}}, \quad (6)$$

where ϵ_S is the length scale and M_S is the bulge mass.

We represent the disk by the Miyamoto-Nagai Model [6]

$$\Phi_{Disk} = -\frac{GM_D}{\sqrt{x^2 + y^2 + (A + \sqrt{z^2 + B^2})^2}} \quad (7)$$

where A and B are respectively the radial and vertical scale lengths, and M_D is the disk mass.

We represent the bar by the Ferrers Model [3]. In this model, the density is given by

$$\begin{cases} \rho_B(x, y, z) = \rho_c (1 - m^2)^2, & m < 1 \\ \rho_B(x, y, z) = 0, & m \geq 1 \end{cases} \quad (8)$$

where the central density is

$$\rho_c = \frac{105}{32\pi} \frac{GM_B}{abc},$$

M_B is the bar mass and

$$m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2},$$

where $a > b > c > 0$ are the semi-axes of the ellipsoid which represents the bar.

The potential created by the galactic bar is calculated with the Poisson equation (see [1]):

$$\Phi_{Bar} = -\pi G abc \frac{\rho_c}{3} \int_{\lambda}^{\infty} \frac{du}{\Delta(u)} (1 - m^2(u))^3 \quad (9)$$

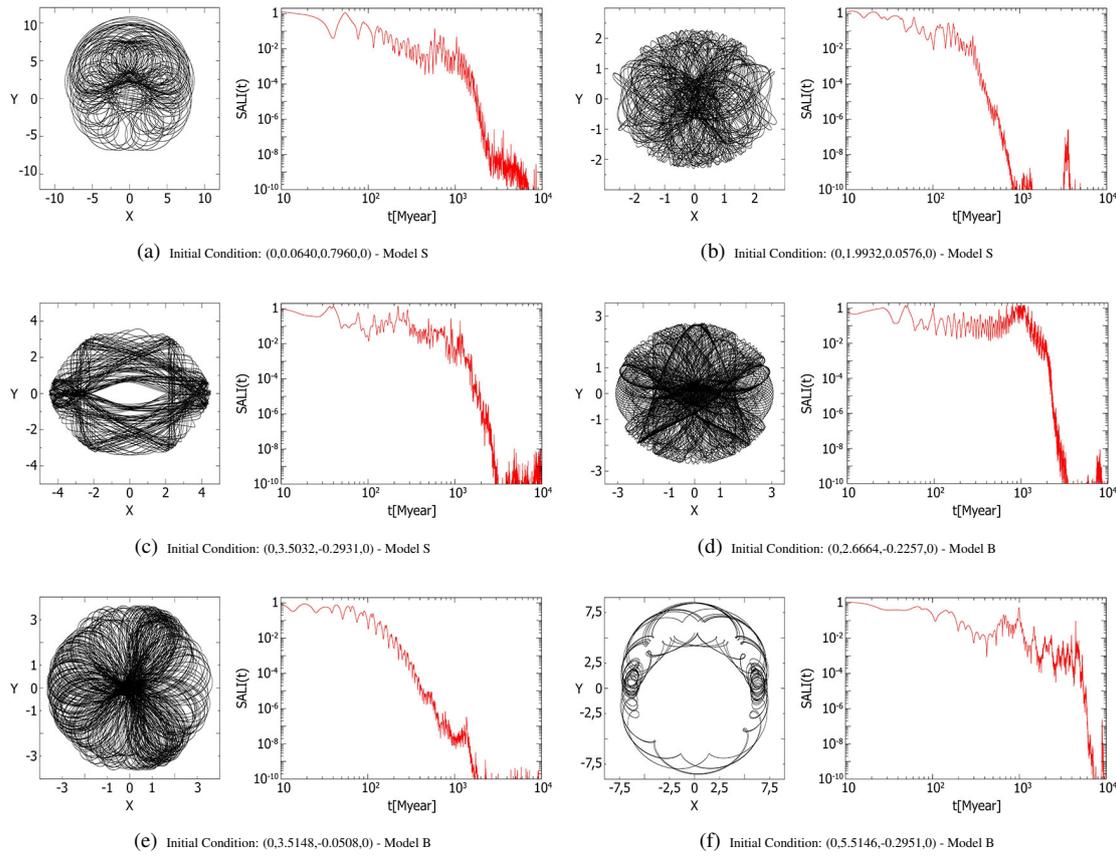


Fig. 2: The SALI graphics has both axes in logarithmic scale. All orbits were integrated into 10,000 Myr. Only the first 5,000 Myr were plotted in (b), for clarity.

where

$$m^2(u) = \frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u},$$

$$\Delta^2(u) = (a^2 + u)(b^2 + u)(c^2 + u)$$

and λ is the positive solution of $m^2(\lambda) = 1$ for the region outside the bar ($m \geq 1$) and $\lambda = 0$ for the region inside the bar ($m < 1$).

2.3 The LP-Vicode program with minor adjustments

To perform the orbits integrations and the SALI calculation, we used the LP-Vicode program [2], which is an operational code in Fortran 77 that calculates efficiently 10 chaos indicators for dynamical systems, including SALI.

In this program, the user must provide the expressions of the potential as well the expressions of motion and variational equations. However, the general structure of motion and variational equations previously written in the main program, take into account only a static reference frame, and it is known that in order to model the galactic bar potential, it is necessary to consider a coordinate system that rotates along with the bar.

In this context, considering Ω_b the bar angular velocity, our reference frame should also rotate with angular velocity Ω_b . This affects the motion and variational equations since, as can be seen in [4], they depend on Ω_b . In order to solve this problem, adjustments were made to the main program to include the rotation in the coordinate system with the same angular velocity of the bar.

2.4 Parameters sets

We used the two parameter sets shown in Table 1 for the potential model, taken from the paper by Manos & Athanasoulas [5]. The model units adopted are: 1 kpc for length, 10^3 km s^{-1} for velocity, $10^3 \text{ km s}^{-1} \text{ kpc}^{-1}$ for angular velocity, 1 Myr for time, and $2 \times 10^{11} M_{\text{solar}}$ for mass. The universal gravitational constant G will always be considered 1 and the total mass $G(M_S + M_D + M_B)$ will be always equal to 1.

2.5 Initial conditions

We emphasize that in this paper we study orbits with two degrees of freedom. In order to do that, we consider $z = 0$ and $p_z = 0$ in the three-dimensional Hamiltonian (4).

Table 1: Parameter Sets and the Bars Co-rotation.

	M_S	ϵ_S	M_D	A	B	M_B	a	b	c	Ω_b	CR
Model S	0.08	0.4	0.82	3.0	1.0	0.1	6.0	1.5	0.6	0.054	6.04
Model B	0.08	0.4	0.82	3.0	1.0	0.1	6.0	3.0	0.6	0.054	6.06

The effective potential, which is the sum of the gravitational potential with the potential generated by the repulsive centrifugal force, is given by:

$$\Phi_{eff}(\mathbf{x}) = \Phi_T(\mathbf{x}) - \frac{1}{2}|\Omega \times \mathbf{x}|^2. \quad (10)$$

Written like that, this potential represents a rotating system.

The quantity

$$E_J = \frac{1}{2}|\mathbf{v}|^2 + \Phi_{eff}(\mathbf{x}) \quad (11)$$

is called Jacobi Energy and is conserved in the rotating system.

The curve given by $\Phi_{eff}(0, y, 0) = E_J$ is called Zero Velocity Curve and provides a good demarcation for the choice of initial conditions, since there is only possibility of orbits when $\Phi_{eff} \leq E_J$, in other words, below this curve (see [1]).

Therefore, we generated some random initial conditions initially taking a value to y_0 less than the highest possible value of y for a given energy E_J , taking $x_0 = 0$ and $v_{y_0} = 0$. This done, we could calculate v_x as follows:

$$E_J = \frac{1}{2}(v_{x_0}^2 + v_{y_0}^2) + \Phi_{eff} = \frac{1}{2}v_{x_0}^2 + \Phi_{eff} \quad (12)$$

and this implies

$$v_{x_0} = \pm \sqrt{2(E_J - \Phi_{eff})}. \quad (13)$$

Then we constructed initial conditions $(x_0, y_0, v_{x_0}, v_{y_0})$ to integrate the orbits. As $x_0 = 0$ and $v_{y_0} = 0$, the launched orbits will always be initially over the y axis and will have initial velocity only in the x direction.

Notice that we have two possible velocities from equation (13): one negative and one positive. We decided to take y_0 always positive, so that when v_{x_0} is positive, the orbits are prograde (orbits that rotate in the same direction of the bar) and when v_{x_0} is negative, the orbits are retrograde (orbits that rotate in the opposite direction of the bar).

3 Results

In our computational calculations, we consider $SALI < 10^{-8}$ close enough to zero to consider the movement chaotic.

3.1 Regular orbits

In Fig. 1 we show 6 different orbits, each one with its SALI diagram, from where we can identify them as regular orbits, as explained in section 2.1.

3.2 Chaotic orbits

Fig. 2 shows a sample of 6 chaotic orbits, identified by their SALI indexes that goes to zero after some time, as discussed in section 2.1.

4 Conclusion

In this study, we were able to reproduce a mathematical modeling of the gravitational potential of a barred galaxy and, in order to verify the stability of the orbits within, we applied the SALI method. We were able to prove the SALI efficiency in distinguishing regular or chaotic orbits. In fact, this method offers an easily observable distinction between chaotic and regular behavior.

We also perceive the LP-Vicode efficiency, which proved to be extremely competent in the orbits integration and study of stability with SALI. To make an adjustment in the variational and motion equations programmed in the LP-Vicode, we insert an adaptation in the main code to take into account a rotating system.

Therefore, we conclude that we were successful in calculating these orbits and confirm the SALI method as a new important tool in the study of stellar orbits stability.

Acknowledgements

We acknowledge the Brazilian agencies FAPESP, CAPES and CNPq (200906-2015-1), as well as the Mexican agency CONACyT (CB-2014-240426) for supporting this work. Our sincere thanks to Dr. Pfenniger, who kindly provided us with his Fortran 77 implementation of the Ferrers bar potential. All the numerical work was developed using the Hipercubo Cluster resources (FINEP 01.10.0661-00, FAPESP 2011/13250-0 and FAPESP 2013/17247-9) at IP&D–UNIVAP.

Received on May 12, 2017

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