

## Exotic Matter: A New Perspective

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In this paper we suggest a possible theoretical way to produce negative energy that is required to allow hyperfast interstellar travels. The term “Exotic Matter” was first coined by K. Thorne and M. Morris to identify a material endowed with such energy in their famous traversable space-time wormhole theory. This possibility relies on the wave-particle dualism theory that was originally predicted by L. de Broglie and later confirmed by electrons scattering experiments. In some circumstances, an electron interacting with a specific dispersive and refracting medium, has its velocity direction opposite to that of the phase velocity of its associated wave. However, it is here shown that a positron placed in the same material exhibits a negative mass. Generalizing the obtained equations leads to an energy density tensor which is de facto negative. This tensor can be used to adequately fit in various “shortcut theories” without violating the energy conditions.

### Introduction

In this paper we show that it is possible to obtain a negative energy provided the associated proper particle’s mass is variable. The basis for this study starts with the associated wave that was originally detected on electrons diffraction experiments [1]. In some circumstances, L. de Broglie showed that a particular homogeneous refractive and dispersive material may cause the tunnelling particle to reverse its velocity with respect to its wave phase propagating velocity [2]. In this case, and under the assumption that the proper mass of the particle is subject to a ultra high frequency vibration synchronized with the wave frequency, it is formally shown that an anti-particle exhibits a negative mass (energy). This energy could be extracted to sustain for example the space-time wormhole, set forth by K. Thorne and M. Morris [3, 4]. To be physically viable, it is well known that it requires a so-called exotic matter endowed with a negative energy density which violates all energy conditions [5]. However, if the exotic matter threading the inner throat of the wormhole is likened to the specific dispersive material wherein circulates a stream of antiparticles, our model does not conflict with classical physics restrictions and can be fully applied.

### Notations

In this paper we will use a set of orthonormal vector basis denoted by  $\{e_0, e_a\}$ , where the space-time indices are  $a, b = 0, 1, 2, 3$ , while the spatial indices are  $\mu, \nu = 1, 2, 3$ . The space-time signature is  $\{-2\}$ .

### 1 Proper mass variation

#### 1.1 Phase velocity and group velocity

It is well known that the classical wave with a frequency  $n$

$$\psi = a(n) \exp [2\pi i(\nu t - \mathbf{k}\mathbf{r})] \quad (1)$$

propagates along the direction given by the unit vector  $N$ . Here  $\mathbf{k}$  is the 3-wave vector,  $\mathbf{k}\mathbf{r} = \phi$  is the wave spatial phase, and  $n$  is the refractive index of the medium. Equation (1) is a solution of the wave propagation equation

$$\Delta\psi = \frac{1}{w^2} \frac{\partial^2\psi}{c^2\partial t^2}, \quad (1)\text{bis}$$

where  $w$  is the wave phase velocity of the wave moving in a dispersive medium whose refractive index is  $n(\nu)$  generally depending of the coordinates, and which is defined by:

$$\frac{1}{w} = \frac{n(\nu)}{c}. \quad (2)$$

In our study, the medium is assumed to be homogeneous but it can be anisotropic and it will depend on the frequency  $\nu$ . In this material, the phase  $\phi$  of the wave is progressing along the given direction with a separation given by a distance

$$\lambda = \frac{w}{\nu} = \frac{c}{n\nu} \quad (2)\text{bis}$$

called the wavelength. Consider now the superposition of two stationary waves along the  $x$ -axis having each close frequencies  $\nu' = \nu + \delta\nu$  and close velocities  $w' = w + (dw/d\nu)\delta\nu$ , so that their superposition can be expressed by:

$$\begin{aligned} & \sin 2\pi\left(\nu t - \frac{\nu x}{w}\right) + \sin 2\pi\left(\nu' t - \frac{\nu' x}{w'}\right) = \\ & = 2 \sin 2\pi\left(\nu t - \frac{\nu x}{w}\right) \cos 2\pi\left[\delta\left(\frac{\nu}{2}\right)t - x \frac{d}{d\nu} \frac{\nu}{w} \delta\frac{\nu}{2}\right]. \end{aligned}$$

The resulting wave displays a wave packet (or beat) that varies along with the so-called group velocity ( $\mathbf{v} = v^g$ ):

$$\frac{1}{\mathbf{v}_g} = \frac{d}{d\nu} \frac{\nu}{w}. \quad (3)$$

The wave mechanics shows that the momentum 3-vector of an electron of a rest mass  $m_0$  (in vacuum) is given by the de Broglie relation

$$\mathbf{p} = m_0 \mathbf{v} = \frac{h}{\lambda} \tag{4}$$

which completes the Einstein relation  $E = h\nu$ .

### 1.2 The plane wave spinor

Since we deal here with a spin 1/2-fermion, we must introduce the four components wave function  $\Psi_A$  expressed with the non local  $4 \times 4$  Dirac trace free matrices  $\gamma_a$  (capital latin spinor indices are  $A = B = 1, 2, 3, 0$ ). They display here the following real components [8]:

$$\gamma_0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

These matrices are said standard representation as opposed for example to the Majorana representation. Moreover, they verify

$$\gamma_a \gamma_b + \gamma_b \gamma_a = -2\eta_{ab} \mathbf{I} \tag{5}$$

where  $\eta_{ab}$  is the Minkowski tensor and  $\mathbf{I}$  is the unit matrix. In what follows,  $\Lambda^*$  is the complex conjugate of an arbitrary matrix  $\Lambda$ ,  ${}^T \Lambda$  is the transpose of  $\Lambda$ , and  $\tilde{\Lambda}$  is the classical adjoint of  $\Lambda$ .

Introducing now the Hermitean matrix  $\beta = i\gamma_0$

$$\beta = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix},$$

which verifies  $\beta^2 = \mathbf{I}$ , we derive the important relation

$$\beta \gamma_a \beta^{-1} = -\tilde{\gamma}_a \tag{5bis}$$

with  $\beta$  and the spinor  $\Psi$ , we form the Dirac conjugate [9]

$${}^\circ \Psi = t \tilde{\Psi} \beta, \tag{5ter}$$

where  $t$  is the time orientation. or the electron, the Dirac equation is written as

$$[W - (m_0)_{\text{elec}} c] \Psi = 0, \tag{6}$$

where  $W = \gamma_B^A \partial_a$  is the Dirac operator and it is customary to omit the spinor indices  $A, B$  by simply writing  $\gamma_a = \gamma_a^A_B$  so that this operator becomes  $\gamma^a \partial_a$ , or in the slash notation (Feynman),  $\not{\partial}_a$ . The monochromatic wave associated with the

electron can be approximated to a plane wave spinor without loss of generality [10]:

$$\Psi_A = a(x^a) \exp 2\pi i (p_a x^a), \tag{6bis}$$

where

$$p_a x^a = Et - p_\mu x^\mu. \tag{6ter}$$

The 4-vector  $p_a$  is the 4-momentum of the electron. The spinor ‘‘amplitude’’  $a(x^a)$  satisfies the Dirac equation

$$[\gamma^a (p_a)_{\text{elec}}] a = [(m_0)_{\text{elec}} c] a \tag{7}$$

where the operator  $[\gamma^a (p_a)_{\text{elec}}]$  is here substituted to the Dirac operator  $\gamma^a \partial_a$ . We now re-write (6)bis as

$$\Psi = a(x^a) \exp(2\pi i/h) \phi, \tag{7bis}$$

where the global phase is  $\phi = h[\nu - (\alpha x + \beta y + \gamma z)/\lambda] t$  (here  $\alpha, \beta, \gamma$  are the direction cosines). The energy and momentum of the electron located at  $x^a$  are then related with the wave phase by:

$$E = \partial_t \phi, \quad \mathbf{p} = -\text{grad } \phi. \tag{7ter}$$

Now, if the electron moves at a velocity  $\mathbf{v} = \beta c$  within a slight variation  $\beta, \beta + \delta\beta$ , corresponding to the frequency interval  $\nu, \nu + \delta\nu$ ,  $w$  and  $\nu$  are functions of  $\beta$ . The wave phase velocity (in vacuum) can be expressed as  $w = c^2/\nu = c/\beta$  and since  $\nu = (1/h) m_0 c^2 / \sqrt{1 - \beta^2}$ , it is easy to infer that:

$$\mathbf{v}_g = \frac{d\nu}{d\beta} \frac{1}{\frac{d\nu}{d\beta} w} = \beta c = \mathbf{v}. \tag{8}$$

The group velocity  $\mathbf{v}_g$  of the wave packet associated with the electron of rest mass  $m_0$ , coincides with its velocity  $\mathbf{v}$ . The group velocity is thus also expressed by the Hamiltonian form  $\mathbf{v}_g = \partial E / \partial \mathbf{k}$  which corresponds to the particle’s velocity  $\mathbf{v} = \partial E / \partial \mathbf{p}$ . Recalling (2) and (2)bis to as  $1/w = n(\nu)/c$ ,  $\lambda = w/n\nu$ , we easily infer the Rayleigh’s formulae [11]:

$$\frac{1}{\mathbf{v}_g} = \frac{1}{c} \frac{\partial n\nu}{\partial \nu} = \frac{\partial (\frac{1}{\lambda})}{\partial \nu}. \tag{9}$$

### 1.3 Making the electron vibrate

In the framework of the special theory of relativity, the proper frequency  $\nu_0$  of a plane monochromatic wave is transformed as

$$\nu = \frac{\nu_0}{\sqrt{1 - \mathbf{v}^2/c^2}}. \tag{10}$$

**Constraint A:** We assume that the electron is subject to an ultra high stationary vibration having a proper frequency  $\nu_0$ .

When moving at the velocity  $\mathbf{v}$ , this frequency is known to transform according to:

$$\nu_e = \nu_0 \sqrt{1 - \mathbf{v}^2/c^2}. \tag{11}$$

We clearly see that its frequency  $\nu_e$  differs from that of its associated wave denoted here by  $\nu$ .

If  $N$  is the unit vector normal to the associated wave phase, the electron subject to the frequency  $\nu_0 = m_0 c^2 / h$  has traveled a distance  $dN$  during a time interval  $dt$ , so that we may define an electronic phase  $\phi_e$  which has changed by:

$$d\phi_e = h\nu_0 \sqrt{(1 - \mathbf{v}^2/c^2)} dt = m_0 c^2 \sqrt{(1 - \mathbf{v}^2/c^2)} dt. \quad (12)$$

Simultaneously, the corresponding wave phase variation is

$$d\phi = \partial_t \phi dt + \partial_N \phi dN = (\partial_t \phi + \mathbf{v} \text{grad } \phi) dt \quad (12)\text{bis}$$

and by analogy to the classical formula (7)ter, one may write

$$\mathbf{p} = -\text{grad } \phi = \frac{m_0 \mathbf{v}}{\sqrt{1 - \mathbf{v}^2/c^2}}, \quad E = \partial_t \phi = \frac{m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}}$$

so we find

$$d\phi = \left[ \frac{m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}} - \frac{m_0 \mathbf{v}^2}{\sqrt{1 - \mathbf{v}^2/c^2}} \right] dt. \quad (13)$$

**Constraint B:** We set the following phase synchronization:

$$d\phi = d\phi_e, \quad (14)$$

which leads to:

$$\begin{aligned} \left[ \frac{m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}} - \frac{m_0 \mathbf{v}^2}{\sqrt{1 - \mathbf{v}^2/c^2}} \right] dt &= \\ &= \left[ m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2} \right] dt. \end{aligned} \quad (15)$$

Dividing through by  $dt$ , we retrieve the famous Planck-Laue equation

$$\frac{m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}} = m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2} + \frac{m_0 \mathbf{v}^2}{\sqrt{1 - \mathbf{v}^2/c^2}}, \quad (15)\text{bis}$$

which holds provided the proper mass is slightly variable. (see proof in Appendix A). In the frameworks of our postulate, the ultra high frequency vibration imparted to the electron can be viewed as apparently reflecting its stationary mass variation which is likened to a fluctuation.

From now on,  ${}^{\#}m_0$  will denote the variable rest mass of the electron so that the Planck-Laue relation becomes:

$$\begin{aligned} {}^{\#}E &= \frac{{}^{\#}m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}} = \\ &= {}^{\#}m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2} + \frac{{}^{\#}m_0 v^2}{\sqrt{1 - \mathbf{v}^2/c^2}}. \end{aligned} \quad (15)\text{ter}$$

This formulae will be required to determine the explicit form of the dispersive material which is the key point of our theory.

## 2 Exotic matter

### 2.1 Dynamics in a refracting material

Let us first recall the relativistic form of the Doppler formulae:

$$\nu_0 = \frac{\nu(1 - \mathbf{v}/w)}{\sqrt{1 - \mathbf{v}^2/c^2}}, \quad (16)$$

where as before,  $\nu_0$  is the wave's frequency in the frame attached to the electron. With the latter equation and taking into account the classical Planck relation  $E = h\nu$ , we find

$$E = \frac{E_0 \sqrt{1 - \mathbf{v}^2/c^2}}{1 - \mathbf{v}/w}. \quad (17)$$

However, inspection shows that the usual equation

$$E = \frac{E_0}{\sqrt{1 - \mathbf{v}^2/c^2}} \quad (18)$$

holds only if

$$1 - \frac{\mathbf{v}}{w} = 1 - \frac{\mathbf{v}^2}{c^2}, \quad (19)$$

which implies

$$w \mathbf{v} = c^2. \quad (20)$$

The latter relation is satisfied provided we set

$${}^{\#}E = \frac{{}^{\#}m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}}, \quad (21)$$

$${}^{\#}\mathbf{p} = \frac{{}^{\#}m_0 \mathbf{v}}{\sqrt{1 - \mathbf{v}^2/c^2}}. \quad (22)$$

**Constraint C:**  ${}^{\#}E$  depends on a specific dispersive and refracting material through which the electron is tunnelling.

Let us define this influence by a function  $Q(n)$  where  $n$  is the refractive index of the material. Note: The variation of the proper mass is independent on  $Q(n)$ . Equation (21) is modified to as

$${}^{\#}E = \frac{{}^{\#}m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}} + Q(n) \quad (23)$$

from which Eq. (22) can be expressed as:

$${}^{\#}\mathbf{p} = \frac{{}^{\#}m_0 \mathbf{v}}{\sqrt{1 - \mathbf{v}^2/c^2}} = \frac{\mathbf{v} [{}^{\#}E - Q(n)]}{c^2}. \quad (24)$$

Now taking into account the Doppler formulae (16), and the Planck-Laue relation (15)ter, we find

$${}^{\#}E - \frac{\mathbf{v}^2 [{}^{\#}E - Q(n)]}{c^2} = {}^{\#}E \left( 1 - \frac{\mathbf{v}}{w} \right) \quad (25)$$

wherefrom is inferred

$$Q(n) = \#E \left(1 - \frac{c^2}{w \mathbf{v}}\right) = h\nu \left(1 - \frac{c^2}{w \mathbf{v}}\right) \quad (26)$$

and with the Rayleigh formulae (4), we eventually obtain the explicit form of  $Q(n)$ :

$$Q(n) = \#E \left[1 - \frac{n\partial(n\nu)}{\partial\nu}\right]. \quad (27)$$

### 2.2 Specific dispersive material

Depending on the nature of the dispersive material, thus its index ( $n$ ), it is well known that the tunnelling electron's 3-velocity  $\mathbf{v}$  can be directed either in the direction of the associated wave phase velocity  $w$  or in the opposite direction. The electron then moves backward through the specific material.

Let  $N$  be the 3-unit vector directed to the wave phase direction (chosen positive) so that the wave number is given by:

$$\mathbf{k} = \frac{Nh}{\lambda}. \quad (28)$$

By applying the Rayleigh formulae (4) to this particular case where  $\mathbf{v}$  is opposite to the wave phase propagation, we have  $\mathbf{v} < 0$ . Hence, from  $Q(n) = \#E (1 - c^2/w \mathbf{v})$ , we find

$$\#E - Q(n) = \frac{\#Ec^2}{w \mathbf{v}} \quad (29)$$

which is negative.

Then, with  $\mathbf{p} = \#m_0 \mathbf{v} / \sqrt{1 - \mathbf{v}^2/c^2}$ , we infer from (24):

$$\frac{\#m_0}{\sqrt{1 - \mathbf{v}^2/c^2}} = \frac{\#E - Q(n)}{c^2}. \quad (29)\text{bis}$$

In order to maintain the variable proper mass  $\#m_0$  positive i.e.

$$\#m_0 = \sqrt{(1 - \mathbf{v}^2/c^2)} \frac{\#E}{w \mathbf{v}} > 0 \quad (30)$$

we must have necessarily:  $\mathbf{p} = -\mathbf{k}$ .

### 2.3 Matching the exotic matter definition

Now consider a stream of electrons and positrons placed in the specific material whose respective associated wave (positive) direction is given by the same unit vector  $N$  (i.e.  $w > 0$ ). From the Dirac theory, we know that the electron momentum 3-vector  $\mathbf{p}_{\text{elec}}$  and that of the positron momentum 3-vector  $\mathbf{p}_{\text{pos}}$  are opposed. (See proof in Appendix B). Therefore we have here  $\mathbf{p}_{\text{pos}} = \mathbf{k}$ , however the dispersive material yet imposes  $\mathbf{v}_{\text{pos}} < 0$ , hence, we are led to the fundamental conclusion:

*A positron moving at the backward velocity  $\mathbf{v}_{\text{pos}}$  through the specific dispersive refracting material defined above and*

*subject to Constraints A, B and C, will exhibit a negative mass given by:*

$$(\#m_0)_{\text{pos}} = \sqrt{1 - \mathbf{v}_{\text{pos}}^2/c^2} \frac{\#E}{w \mathbf{v}} < 0, \quad (30)\text{bis}$$

where  $\#E - Q(n) = \#Ec^2/w_{\text{pos}} < 0$  in accordance with Eq. (29).

Let us write the mass (30)bis as:

$$(\#m_0)_{\text{pos}} = \int (\#\rho_0)_{\text{pos}} \sqrt{-g} dV, \quad (31)$$

where  $(\#\rho_0)_{\text{pos}}$  is the variable proper density of the positronic massive flow. The integral is performed over the 3-volume  $V$  delimiting the variable proper mass  $(\#m_0)_{\text{pos}}$  boundary. We then readily infer the familiar form of the energy density tensor in the static case

$$(\#T_0^0)_{\text{pos}} = (\#\rho_0)_{\text{pos}} c^2, \quad (32)$$

which is de facto negative.

So, within the scheme of the wave-particle picture, we have been able to give a consistent picture of what could be the united conditions to reach our goal :

*The so-called "exotic matter" required to assemble a space-time distortion can be provided by the negative energy extracted from a stream of vibrating antifermions interacting with a specific dispersive refracting material adequately engineered.*

### 3 Concluding remarks

Without going into details of a sound engineering, we have here only scratched the surface of a basic theory describing the ability of a system composed of antiparticles to interact with a specific refracting and dispersive material in order to exhibit a dynamical negative mass.

Thus, our approach mainly relies on de Broglie's theory which has been verified for the electron.

Upon Constraints A, B, and C, we might as well consider other heavier particles such as the antiproton to produce negative energy.

Once these conditions are fulfilled, the concept of hyperfast interstellar travel is viable if one can "handle" routinely antimatter, and envision a sufficient amount of negative energy density. These orders of magnitude are beyond the scope of this text.

Without any doubt, some advanced civilizations have already long mastered the negative energy obtained by this process, to achieve superluminal travels as described by space-time warp drive theories [12–14].

For us, a huge research work is still ahead, but if we have contributed to open a small door, then the challenge is widely available for physicists.

### Appendix A: The Planck-Laué relation

The Planck-Laué relation is a relativistic equation which has been derived when the proper mass is assumed to slightly fluctuate. This proper mass is here denoted by  ${}^{\#}m_0$ . Under this circumstance, the relativistic dynamics of  ${}^{\#}m_0$  can now be extended as follows.

We first write the Lagrange function for an observer who see the particle moving at the velocity  $\mathbf{v}$

$$L = -{}^{\#}m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2}$$

so that the least action principle applied to this function is still expressed by

$$\delta \int_{t_0}^{t_1} L dt = \delta \int_{t_0}^{t_1} -{}^{\#}m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2} = 0.$$

From this principle the equations of motion

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_a} \right) = \frac{\partial L}{\partial x_a}, \quad \dot{x}_a = \frac{dx_a}{dt},$$

are inferred, which lead to

$$\frac{d{}^{\#}\mathbf{p}}{dt} = -c^2 \sqrt{1 - \mathbf{v}^2/c^2} \text{grad } {}^{\#}m_0 \quad (\text{A.1})$$

(since  ${}^{\#}m_0$  is now variable). Hence, by differentiating the relativistic relation  ${}^{\#}E^2/c^2 = {}^{\#}p^2 + {}^{\#}m_0^2 c^2$ , we obtain

$$\frac{d{}^{\#}E}{dt} = c^2 \sqrt{1 - \mathbf{v}^2/c^2} \frac{\partial {}^{\#}m_0}{\partial t}. \quad (\text{A.2})$$

Combining (A.1) and (A.2) readily gives

$$\frac{d{}^{\#}E}{dt} - \mathbf{v} \cdot \frac{d{}^{\#}\mathbf{p}}{dt} = c^2 \sqrt{1 - \mathbf{v}^2/c^2} \frac{d{}^{\#}m_0}{dt}, \quad (\text{A.3})$$

where  $d{}^{\#}m_0/dt = \partial {}^{\#}m_0/\partial t + \text{grad } {}^{\#}m_0$  is the variation of the mass in the course of its motion. On the other hand, we have

$$\begin{aligned} \frac{d({}^{\#}\mathbf{p} \cdot \mathbf{v})}{dt} &= \frac{\mathbf{v} \cdot d{}^{\#}\mathbf{p}}{dt} + {}^{\#}m_0 c^2 \frac{(\mathbf{v}/c) d(\mathbf{v}/c) dt}{\sqrt{1 - \mathbf{v}^2/c^2}} = \\ &= \mathbf{v} \cdot \frac{d{}^{\#}\mathbf{p}}{dt} - {}^{\#}m_0 c^2 \frac{d}{dt} (1 - \mathbf{v}^2/c^2) \end{aligned} \quad (\text{A.4})$$

i.e.

$$\begin{aligned} \frac{d}{dt} [{}^{\#}m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2}] &= \\ &= c^2 \sqrt{1 - \mathbf{v}^2/c^2} \frac{d{}^{\#}m_0}{dt} + {}^{\#}m_0 c^2 \frac{d}{dt} \sqrt{1 - \mathbf{v}^2/c^2} \end{aligned}$$

hence (A.3) can be re-written as

$$\frac{d}{dt} [{}^{\#}E - \mathbf{v} \cdot {}^{\#}\mathbf{p} - {}^{\#}m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2}] = 0 \quad (\text{A.5})$$

which is satisfied when the particle is at rest, that is:  $\mathbf{v} = 0 \Rightarrow {}^{\#}E_0 = {}^{\#}m_0 c^2$ . Therefore, we must always have:

$${}^{\#}E = \frac{{}^{\#}m_0 c^2}{\sqrt{1 - \mathbf{v}^2/c^2}} = {}^{\#}m_0 c^2 \sqrt{1 - \mathbf{v}^2/c^2} + \frac{{}^{\#}m_0 \mathbf{v}^2}{\sqrt{1 - \mathbf{v}^2/c^2}}. \quad (\text{A.6})$$

It is important to note that this variable (proper) mass,  ${}^{\#}m_0$ , is purely intrinsic, i.e. its motion is unaffected.

Equation (A.6) is known as the Planck-Laué formula.

### Appendix B: Dirac currents

Let us consider the real Dirac current as

$$J^a = i ({}^{\circ}\Psi \gamma^a \Psi) = (J^a)_1 - (J^a)_2,$$

where

$$(J^a)_1 = i {}^{\circ}\Psi_A \gamma_B^{aA} \Psi^B, \quad (J^a)_2 = i \Psi^B \gamma_B^{aA} {}^{\circ}\Psi_A.$$

The charge conjugate of  $J^a$  is first calculated

$$[(J^a)_1]^{(C)} = i \Psi_A^* \gamma_B^{aA} \Psi^{*B} = i t^T \Psi_A \beta_B^A \gamma_C^{aB} \Psi^{*C}$$

i.e.

$$[(J^a)_1]^{(C)} = i t \Psi^A \beta_B^A \gamma_B^{aC} \Psi^{*C}.$$

From the antisymmetry of  $\beta$ , and remembering that the  $\gamma^a$  are here *real*, we have

$${}^T \gamma^a \beta = -\tilde{\gamma}^a \beta = \beta \gamma^a$$

from which we infer

$$[(J^a)_1]^{(C)} = i t \Psi^A \gamma_A^{aB} \beta_B^C \tilde{\Psi}_C = i \Psi^A \gamma_A^{aB} {}^{\circ}\Psi_B$$

hence, we see that

$$[(J^a)_1]^{(C)} = (J^a)_2$$

and similarly

$$[(J^a)_2]^{(C)} = (J^a)_1$$

therefore, we obtain the most important relation:

$$-(J^a)^{(C)} = J^a \quad (\text{B.1})$$

The Dirac current orientation is opposed to that of its Dirac conjugate [15]. The Dirac conjugate  ${}^{\circ}\Psi$  of the plane wave spinor (6)bis is here:

$${}^{\circ}\Psi = {}^{\circ}a \exp -2\pi i (p_a x^a). \quad (\text{B.2})$$

With the Dirac conjugate spinor amplitude  ${}^{\circ}a = a^* \gamma^0$ , that is equivalent to (5)ter, we first set the normalization condition:

$${}^{\circ}a a = m_0 c. \quad (\text{B.3})$$

Besides, the Dirac equation reads:

$$(\gamma^a p_a) {}^{\circ}a = m_0 c {}^{\circ}a. \quad (\text{B.4})$$

Due to the property of  $(\gamma^a)^2$ , Equations (7) and (B.4) are both satisfied for:

$$(p_a)^2 = (m_0 c)^2. \quad (\text{B.5})$$

Multiplying now Equation (7) on the left with  ${}^\circ a$ , we obtain with (B.2) and (B.5)

$$({}^\circ a \gamma^a a) p_a = (m_0 c)^2 = (p_a)^2 \quad (\text{B.6})$$

from which we infer:

$${}^\circ a \gamma^a a = p^a. \quad (\text{B.7})$$

The Dirac current density vector  $J^a = {}^\circ \Psi \gamma^a \Psi$  will here yield

$$J^a = {}^\circ a \gamma^a a = p^a \quad (\text{B.8})$$

with

$$p^a = m_0 c^2 + p^\mu \quad (\text{B.9})$$

( [16]: compare with formulae (23.6) there).

From the charge conjugate  $\Psi^{(C)}$  corresponding to the positron plane spinor, we define the Dirac current for the positron  $(J^a)^{(C)}$ . However, it was shown that  $(J^a)^{(C)} = -J^a$ . Therefore, assuming that  $(m_0)_{\text{elec}} = (m_0)_{\text{posit}}$  in vacuum, we must then have

$$(J^\mu)^{(C)} = (p^\mu)_{\text{posit}} = -(p^\mu)_{\text{elect}}. \quad (\text{B.10})$$

This clearly means that in vacuum,  $\mathbf{v}_{\text{posit}} = -\mathbf{v}_{\text{elect}}$ .

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