Notes on Extended Lorentz Transformations for Superluminal Reference Frames

Yaroslav I. Grushka

Institute of Mathematics NAS of Ukraine. 3, Tereschenkivska st., Kyiv, 01601, Ukraine. E-mail: grushka@imath.kiev.ua

The present paper is devoted to the analysis of different versions of extended Lorentz transformations, proposed for reference frames moving with the velocity, greater then the velocity of light. In particular we point out some errors of individual authors in this field.

This work is connected with the theory of tachyon movement. Research in this direction were initiated in the papers [1, 2] more than 50 years ago. Then, in the papers of E. Recami, V. Olkhovsky and R. Goldoni [3–5], the *extended Lorentz transformations* for reference frames, moving with the velocity, greater then the velocity of light c were proposed. Latter the above extended Lorentz transformations were rediscovered in [6, 7]. The ideas of E. Recami, V. Olkhovsky and R. Goldoni are still relevant in our time. In particular B. Cox and J. Hill published in [7] a new and elegant way to deduce the formulas of E. Recami, V. Olkhovsky and R. Goldoni's extended Lorentz transformations. Also in paper [8] the extended Lorentz transformations are obtained for the case, where the space of geometrical coordinates may be any real Hilbert space of any dimension, including infinity. Application of the E. Recami, V. Olkhovsky and R. Goldoni's extended Lorentz transformations to the problem of spinless tachyon localization can be found in [9].

In the paper [10] author tries to obtain several variants of new extended superluminal Lorentz transformations, different from transforms obtained by E. Recami, V. Olkhovsky and R. Goldoni. It should be emphasized, that the paper [10], together with incorrect statements, contains also valuable new results. For example, nonlinear extended Lorentz transformations, proposed in [10], may be applied in the theory of kinematic changeable sets [11] for construction some interesting examples or counterexamples. Now we focus on errors, committed by the author of [10].

At first view, the coordinate transformations (3)–(4) and (9)–(10) from [10] look like as new. But, actually, the formulas (3)–(4) and (9)–(10) from [10] are some, not quite correct, representations for well-known classical Lorentz transformations. Hence, these transformations can not be coordinate transformations for reference frames moving with the superluminal velocity.

For example, let us analyze in details the transformations (3)–(4) from [10] for the case of one space dimension:

$$x' = \gamma(v)\,(x - vf(v)t) \qquad \text{(a)}$$

$$t' = \gamma(v)\left(t - \frac{vf(v)x}{c^2}\right), \qquad \text{(b)}$$

where (x, t) are the space-time coordinates of any point in the fixed reference frame l and (x', t') are the space-time coordinates of this point in the moving frame l'.

According to [10], the function $f(v)$ may be any real function, satisfying the following conditions:

1. $f(v) > 0$, $v \in \mathbb{R}$ and $f(0) = 1$;

2. $f(v)$ is even (that is $f(-v) = f(v)$, $v \in \mathbb{R}$);

   The multiplier $\gamma(v)$ in (a)–(b) is connected with the function f by the formula,

   $$\gamma(v) = \left(1 - \frac{v^2 f^2(v)}{c^2}\right)^{-1/2}.$$

   Thus, the following condition must be satisfied:

3. The transformations (a)–(b) are defined for such values $v \in \mathbb{R}$, for which the inequality $|v|\,f(v) < c$ is performed.

   In the paper [10], the parameter v is treated as the velocity of the moving reference frame l'. Thus, to include the subluminal diapason into the set of "allowed velocities", we may apply following condition:

4. $vf(v) < c$ for $0 < v < c$.

Note, that the condition 4 is not strictly necessary, and in the analysis of the transformations (a)–(b) we take into account only the conditions 1–3.

According to the paper [10], the parameter v in transformations (a)–(b) is the velocity of the moving reference frame l'. But now we are going to prove that the last statement is not true. For this purpose we calculate the inverse transform to (a)–(b), by means of solving the system (a)–(b) relatively the variables (x, t):

$$x = \gamma(v)\,(x' + vf(v)t') \qquad \text{(c)}$$

$$t = \gamma(v)\left(t' + \frac{vf(v)x'}{c^2}\right). \qquad \text{(d)}$$

The origin of the moving reference frame l' at any fixed time point τ has the coordinates $(0, \tau)$ in the frame l', and, according to the transformations (c)–(d), it has the coordinates $(\gamma(v)vf(v)\tau,\ \gamma(v)\tau)$ in the frame l. Consequently, the origin of the moving frame l' will overpass the distance $\gamma(v)vf(v)\tau$ during the time interval $[0,\ \gamma(v)\tau]$ (where we select any $\tau \neq 0$). Hence, the velocity u of the moving reference frame l' is equal to the following value:

$$u = \frac{\gamma(v)vf(v)\tau}{\gamma(v)\tau} = vf(v),$$

which is not v. Thus, the parameter v in (a)–(b) is expressed via the actual velocity u of the reference frame $\mathfrak{l}'$ by means of the formula, $v = \frac{u}{f(v)}$. And the substitution of the value $\frac{u}{f(v)}$ instead of v into transformations (a)–(b) leads to the classical Lorentz transformations.

Hence, we have seen, that *the formulas* (a)–(b) *(or the formulas* (3)–(4) *from* [10]*) are one of the representations for classical Lorentz transformations, and the actual velocity* $u = vf(v)$ *of the moving reference frame, according to the condition 3, can not exceed the velocity of light.*

Also, it should be noted, that the transformations (a)–(b) (or (3)–(4) from [10]) are preserving the Lorentz-Minkowski pseudo-metric:

$$\mathsf{M}_c\,(t, x) = x^2 - c^2 t^2$$

in the Minkowski space-time over real axis $x \in \mathbb{R}$. But *any bijective linear operator in the Minkowski space-time, preserving the Lorentz-Minkowski pseudo-metric, belongs to the general Lorentz group* [12]*, and it can not be coordinate transform for superluminal reference frame.*

The coordinate transformations (9)–(10) from [10], according to the author requirements, also are preserving the Lorentz-Minkowski pseudo-metric in the Minkowski space-time over $\mathbb{R}^3$. Therefore *they also can not be coordinate transformations for superluminal reference frames.* And they can be analyzed in details by a similar way as the transformations (3)–(4) from [10].

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