On the Question of Acceleration in Special Relativity

Pierre A. Millette

E-mail: PierreAMillette@alumni.uottawa.ca, Ottawa, Canada

In this paper, we consider the question of the impact of acceleration in special relativity. Some physicists claim that acceleration does not matter in special relativity based on the Clock Hypothesis. We find that the experimental support of the Clock Hypothesis usually provided by the Mössbauer spectroscopy experiment of Kündig [5] and the muon experiment of Bailey *et al* [2] is questionable at best. We consider the case for the impact of acceleration in special relativity and derive an expression for the time dilation in an accelerated frame of reference, based on the equivalence principle of general relativity. We also derive an expression for space contraction in an accelerated frame of reference. We note that the presence of acceleration in a frame of reference provides a means of determining the motion of that frame of reference as acceleration can be easily detected compared to constant velocity which cannot. We discuss the "twin paradox" of special relativity and note that this is not truly a special relativity problem for there is no way to avoid acceleration. We note that because of time dilation in accelerated frames of reference, the astronaut will age less than its earth-bound twin, but only during periods of acceleration.

1 Introduction

In a recent paper [1], we showed that time dilation and space contraction in inertial reference frames, that is unaccelerated reference frames moving at a constant velocity, are apparent effects perceived in a frame of reference moving with respect to an object of interest. The real physical time and length are in the frame of reference at rest with the object, and in that frame, there is no time dilation or space contraction as $v = 0$ (and acceleration $a = 0$). This is seen clearly in Fig. 1 where a time dilation is perceived in the frame of reference moving at speed v with respect to the object of interest ($\Delta t'$), while there is no dilation in the object's frame of reference (Δt).

This result would seem to be at odds with the often quoted experimental tests of special relativity confirming time dilation and length contraction. But if we consider, for example, Bailey *et al*'s muon experiment [2], we find that there is no contradiction with the experimental observations: a perceived time dilation is observed in the Earth's laboratory frame of reference while the muon, in its frame of reference has no time dilation – note that no measurements were carried out in the muon's frame of reference in the Bailey experiment.

Careful examination of experimental tests of special relativity also often reveals the presence of acceleration in the experiments, contrary to the conditions under which special relativity applies. The question of how to deal with acceleration in special relativity underlies many of the analytical and experimental conundrums encountered in the theory and is investigated in more details in this paper.

2 Measuring the impact of acceleration in special relativity

The theory of special relativity applies to unaccelerated (constant velocity) frames of reference, known as inertial frames of reference, in a four-dimensional Minkowski spacetime [3], of which the three-dimensional Euclidean space is a subspace. When the Lorentz-Fitzgerald contraction was first introduced, it was considered to be a real physical effect in Euclidean space to account for the null results of the Michelson-Morley experiment. Einstein derived length contraction and time dilation as effects originating in special relativity. These depend on the velocity of the frame of reference with respect to which an object is being observed, not the object's velocity

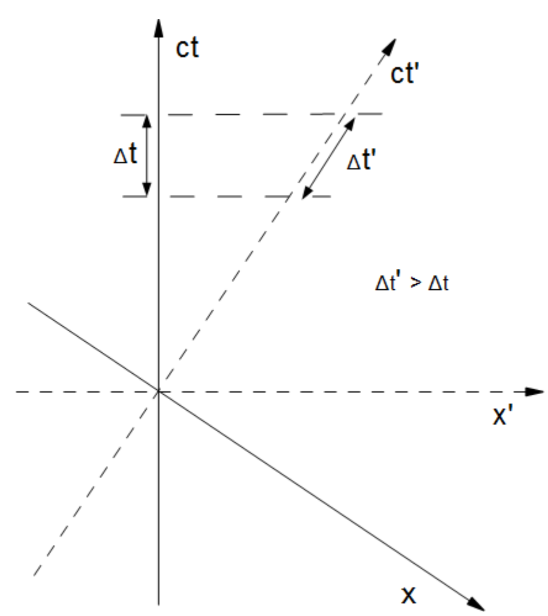

Fig. 1: Physical explanation of time dilation in a Loedel *space−ct* diagram

which can only be relative to another frame of reference, as there is no absolute frame of reference against which to measure the object's velocity. Indeed, if time dilation and length contraction were real effects in special relativity, this would be equivalent to saying that there is an absolute frame of reference against which it is possible to measure an object's velocity, contrary to the theory.

Increasingly, special relativity has been applied to accelerated frames of reference for which the theory does not apply. Some physicists claim that acceleration does not matter in special relativity and that it has no impact on its results, but there are many indications that this is not the case. The Clock Hypothesis (or Postulate) is used to justify the use of accelerated frames in special relativity: "when a clock is accelerated, the effect of motion on the rate of the clock is no more than that associated with its instantaneous velocity – the acceleration adds nothing" [4, p. 9], and further postulates that if the Clock Hypothesis applies to a clock, " then the clock's proper time will be proportional to the Minkowski distance along its worldline" [4, p. 95] as required.

Two experimental confirmations of the Clock Hypothesis are usually given. The postulate is claimed to have been shown to be true for accelerations of $\sim 10^{16} g$ in a Mössbauer spectroscopy experiment by Kündig [5] and of $\sim 10^{18} g$ in Bailey *et al*'s muon experiment [2], which uses rotational motion of particles to generate the acceleration – one obtains the quoted acceleration for a particle velocity close to the speed of light. However, a close examination of these experiments shows that they don't quite provide the experimental confirmation they are purported to give.

Kholmetskii *et al* [6] reviewed and corrected the processing of Kündig's experimental data and obtained an appreciable difference of the relative energy shift $\Delta E/E$ between emission and absorption resonant lines from the predicted relativistic time dilation $\Delta E/E = -v^2/2c^2$ (to order c^{-2}), where v is the tangential velocity of the resonant radiation absorber. Writing the relative energy shift as $\Delta E/E = -k v^2/c^2$, they found that $k = 0.596 \pm 0.006$ instead of $k = 0.5$ as predicted by special relativity and Kündig's original reported result of $k = 0.5003 \pm 0.006$. They then performed a similar Mössbauer spectroscopy experiment [7] with two absorbers with a substantially different isomer shift to be able to correct the Mössbauer data for vibrations in the rotor system at various rotational frequencies. They obtained a value of $k = 0.68 \pm 0.03$, a value similar to $2/3$. Since then Kholmetskii and others [8–12] have performed additional experimental and theoretical work to try to explain the difference, but the issue remains unresolved at this time, and is a clear indication that acceleration is not compatible with special relativity.

In their experiment of the measurement of the lifetime of positive and negative muons in a circular orbit, Bailey *et al* [2] obtained lifetimes of high-speed muons which they then reduced to a mean proper lifetime at rest, assuming that special relativity holds in their accelerated muon experimental setup. This experiment was carried out at CERN's second Muon Storage Ring (MSR) [13, 14] which stores relativistic muons in a ring in a uniform magnetic field. The MSR was specifically designed to carry out muon $(g-2)$ precession experiments (g is the Landé g-factor) with muons of momentum 3.094 GeV$/c$ corresponding to a γ-factor of 29.3 (effective relativistic mass [1]), so that the electrons emitted from muon decay in the lab frame were very nearly parallel to the muon momentum. The decay times of the emitted electrons were measured in shower counters inside the ring to a high precision, and the muon lifetimes in the laboratory frame were calculated by fitting the experimental decay electron time spectrum to a six-parameter exponential decay modulated by the muon spin precession frequency, using the maximum likelihood method – one of the six parameters is the muon relativistic lifetime.

It is important to note that the decay electrons would be ejected at the instantaneous velocity of the muon ($0.9994c$ from the $\gamma = 29.3$ factor) tangential to the muon's orbit. Thus the ejected electron moves at the constant velocity of ejection to the shower counter and acceleration does not play a role. Even though the muons are accelerated, the detected electrons are not, and the experiment is not a test of the Clock Hypothesis under acceleration as claimed. There is thus no way of knowing the impact of acceleration from the experimental results as acceleration is non-existent in the detection and measurement process.

It should also be noted that Hafele *et al* [17] in their time dilation "twin paradox" experiment applied a correction for centripetal acceleration to their experimental results. in addition to a gravitational time dilation correction, to obtain results in agreement with Lorentz time dilation. The effect of acceleration cannot be disregarded in that experiment. This will be considered in more details in section 4. We thus find that the experimental support of the Clock Hypothesis is questionable at best.

3 The case for the impact of acceleration in special relativity

Having determined that there is little experimental support for the validity of the Clock Hypothesis in accelerated frames of reference in special relativity, we consider the case for the impact of acceleration in special relativity. Einstein developed general relativity to deal with accelerated frames of reference – if acceleration can be used in special relativity, why bother to develop a more general theory of relativity? Inspection of an accelerated worldline in a Minkowski *space-ct* diagram shows that indeed there is no basis for the Clock Hypothesis, as seen in Fig. 2. The accelerated worldline suffers an increasing rate of time dilation, somewhat like gravitational time dilation where increasing height in the gravitational potential results in increasing time dilation.

This brings to mind Einstein's equivalence principle introduced in the analysis of accelerated frames of reference in general relativity. The simplest formulation of this principle states that on a local scale, the physical effects of a gravitational field are indistinguishable from the physical effects of an accelerated frame of reference [15] (*i.e.* an accelerated frame of reference is locally equivalent to a gravitational field). Hence, as displayed graphically for the accelerated worldline in the Minkowski *space-ct* diagram of Fig. 2, an accelerated frame of reference undergoes time dilation similar to gravitational time dilation [15]. Indeed, assuming that acceleration has no impact in special relativity cannot be correct as it violates the equivalence principle of general relativity.

We explore the connection between gravitational time dilation and the time dilation in an accelerated frame of reference in greater details. Gravitational time dilation can be derived starting from the Schwarzschild metric with signature (+ - - -) [16, p. 40]

$$c^2 d\tau^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 - \\ - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) , \tag{1}$$

where τ is the proper time, (r, θ, φ, t) are the spherical polar coordinates including time, G is the gravitational constant, M is the mass of the earth and c is the speed of light in vacuo. The gravitational time dilation is obtained from the dt^2 term to give

$$\Delta t = \left(1 - \frac{2GM}{rc^2}\right)^{-\frac{1}{2}} \Delta t_0 \, , \tag{2}$$

where Δt_0 is the undilated (proper) time interval and Δt is the dilated time interval in the earth's gravitational field. This can be rewritten as

$$\Delta t = \left(1 - \frac{2GMr}{r^2 c^2}\right)^{-\frac{1}{2}} \Delta t_0 \, , \tag{3}$$

where the term GM/r^2 is an acceleration a equal to g for $r = R$, the earth's radius, and finally

$$\Delta t = \left(1 - \frac{2ar}{c^2}\right)^{-\frac{1}{2}} \Delta t_0 \, . \tag{4}$$

By the equivalence principle, this is also the time dilation in an accelerated frame of reference. For small accelerations, using the first few terms of the Taylor expansion, this time dilation expression can be written as

$$\Delta t \simeq \left(1 + \frac{ar}{c^2}\right) \Delta t_0 \, . \tag{5}$$

The impact of acceleration on time dilation for small acceleration will usually be small due to the c^{-2} dependency.

We note in particular the expressions for centripetal acceleration $a = v^2/r$ in the case of circular motion

$$\Delta t = \left(1 - \frac{2v^2}{c^2}\right)^{-\frac{1}{2}} \Delta t_0 \, , \tag{6}$$

which becomes for small accelerations, again using the first few terms of the Taylor expansion,

$$\Delta t \simeq \left(1 + \frac{v^2}{c^2}\right) \Delta t_0 \, . \tag{7}$$

In this case, the impact can be significant, of the same order as the relativistic Lorentz time dilation. Hence there is no doubt that accelerated frames of reference also undergo time dilation compared to unaccelerated (inertial) frames of reference.

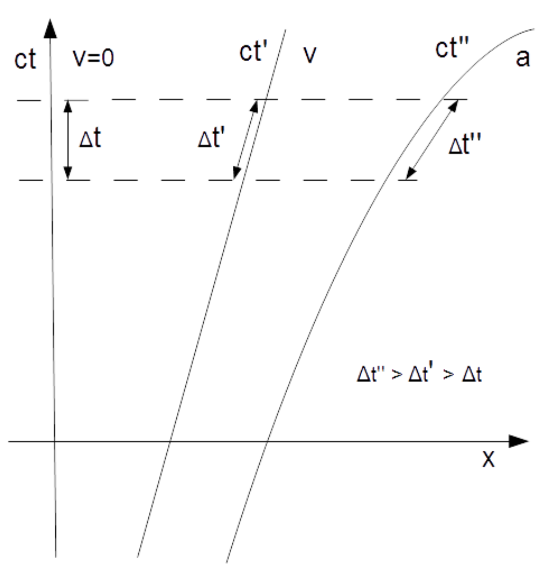

Fig. 2: Physical explanation of an accelerated worldline in a Minkowski *space−ct* diagram

4 The consequences of acceleration in special relativity

The presence of acceleration in a frame of reference provides a means of determining the motion of that frame of reference as acceleration can be easily detected compared to constant velocity which cannot. Whereas in an inertial frame of reference there is no way of determining one's velocity, this limitation disappears in accelerated frames of reference.

Physical time dilation due to acceleration is a reality, as is physical space contraction, which, from (1), is seen to have the inverse of the functional form of (4), to give the acceleration space contraction relation

$$\Delta x = \left(1 - \frac{2ar}{c^2}\right)^{\frac{1}{2}} \Delta x_0 \tag{8}$$

which for small accelerations, using the first few terms of the Taylor expansion, becomes

$$\Delta x \simeq \left(1 - \frac{ar}{c^2}\right) \Delta x_0 \, . \tag{9}$$

Till now, we have not discussed the so-called "twin paradox" of special relativity. This is not truly a paradox for there is no way to avoid acceleration in the problem and it is thus not a special relativity problem. Assume that by some miracle we have twins moving at constant velocity with respect to one another from departure to return with no acceleration and that they are able to compare their age. It is important to notice that in their inertial frames of reference, both proper times $d\tau$, the one in the frame of reference at rest with the earth, and the one in the frame of reference at rest with the spaceship, are equal to the physical time in both the frame of reference at rest with the earth and the frame of reference at rest with the spaceship. From the earth, it looks like the spaceship's time is dilated, and from the spaceship, it looks like the earth's time is dilated. It doesn't matter as the time dilation in one location as seen from the other location is apparent as seen in [1]. When the spaceship comes back to earth, the twins would see that indeed they have the same age.

The problem can be recast in a simpler fashion. Suppose instead of the earth and a spaceship, we have two spaceships moving at constant relativistic speed with respect to one another from start to finish with no acceleration, and that the twins are able to compare their age at the start and the finish. One spaceship moves slowly because of engine problems, while the other moves at relativistic speeds. The resolution would be as described in the previous paragraph: the twins would see that indeed they have the same age at the finish.

The complication in this problem is that forces have to be applied to accelerate the spaceship, then decelerate it to turn around, accelerate it again and finally decelerate it when it comes back to the earth. The problem then needs to be treated using accelerated frames of reference for those periods on the spaceship. As we have seen in section 3, because of time dilation in accelerated frames of reference, the astronaut will age less than its earth-bound twin, but only during periods of acceleration. During periods of unaccelerated constant velocity travel, there will be no differential aging between the twins. However, the earth-bound twin is itself in an accelerated frame of reference the whole time, so its time will also be dilated. The details of who is older and younger will depend on the details of the acceleration periods, with the earth-bound twin's time dilation depending on (2) and (6), and the spaceship-bound twin's time dilation depending on (4).

Comparing how these findings line up with the results of Hafele's circumglobal experiment [17, 18], it is important to note that Hafele's experiment was done the whole time in a non-inertial accelerated frame of reference. Its results were corrected for gravitational time dilation and centripetal acceleration time dilation, the latter correction clearly showing that acceleration has an impact on special relativity. The centripetal acceleration time dilation correction used by Hafele *et al* [17] is similar to (6). One side effect of the experiment being conducted in gravitational and accelerated frames of reference is that it was possible to determine their motion, contrary to special relativity. The Lorentz time dilation would then become a real effect in this purported test of the "twin paradox". There was no symmetry in the relative motions that would have seen the plane stationary and the earth moving given that gravitational and centripetal accelerations clearly showed who was moving and at what velocity.

5 Discussion and conclusion

In this paper, we have considered the question of the impact of acceleration in special relativity. Some physicists claim that acceleration does not matter in special relativity – this view is part of the Clock Hypothesis which is used to justify the use of accelerated frames in special relativity. We have found that the experimental support of the Clock Hypothesis usually provided by the Mössbauer spectroscopy experiment of Kündig [5] and the muon experiment of Bailey *et al* [2] is questionable at best.

We have considered the case for the impact of acceleration in special relativity and have derived an expression for the time dilation in an accelerated frame of reference, based on the equivalence principle of general relativity. We have also derived an expression for space contraction in an accelerated frame of reference.

As a consequence, we have noted that the presence of acceleration in a frame of reference provides a means of determining the motion of that frame of reference as acceleration can be easily detected compared to constant velocity which cannot – whereas in an inertial frame of reference there is no way of determining one's velocity, this limitation disappears in accelerated frames of reference.

We have discussed the "twin paradox" of special relativity and have noted that this is not truly a paradox for there is no way to avoid acceleration in the problem and it is thus not a special relativity problem. We have noted that because of time dilation in accelerated frames of reference, the astronaut will age less than its earth-bound twin, but only during periods of acceleration, while during periods of unaccelerated constant velocity travel, there will be no differential aging between the twins. However, as the earth-bound twin is itself in an accelerated frame of reference the whole time, the details of who is older and who is younger will depend on the details of the acceleration periods of both twins. Finally we have reviewed how these findings line up with the results of Hafele's circumglobal experiment [17, 18] and find no contradiction.

Received on September 28, 2017

References

1. Millette P. A. On Time Dilation, Space Contraction and the Question of Relativistic Mass. *Progress in Physics*, 2017, v. 13, 200–203.
2. Bailey J., Borer K., Combley F., Drumm H., Krienen F., Lange F., Picasso E., von Ruden W., Farley F. J. M., Field, J. H., Flegel, W., Hattersley P. M. Measurements of Relativistic Time Dilatation for Positive and Negative Muons in a Circular Orbit. *Nature*, 1977, v. 268, 301–305.

3. Minkowski H. Space and Time. 80^{th} *Assembly of German Natural Scientists and Physicians*. Cologne, 21 September 1908. English translation reprinted in Lorentz H. A., Einstein A., Minkowski H, and Weyl H. The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. Dover Publications, New York, 1952, pp. 73–91.
4. Brown H. Physical Relativity: Spacetime Structure from a Dynamical Perspective. Oxford University Press, Oxford, 2005.
5. Kündig W. *Phys. Rev.*, 1963, v. 129, 2371.
6. Kholmetskii A. L., Yarman T. and Missevitch O. V. Kündig's Experiment on the Transverse Doppler Shift Re-analyzed. *Phys. Scr.*, 2008, v. 77, 035302–035306.
7. Kholmetskii A. L., Yarman T., Missevitch O. V. and Rogozev B. I. A Mössbauer Experiment in a Rotating System on the Second-order Doppler Shift: Confirmation of the Corrected Result by Kündig. *Phys. Scr.*, 2009, v. 79, 065007.
8. Kholmetskii A. L., Yarman T., Missevitch O. V. and Rogozev B. I. Mössbauer Experiments in a Rotating System on the Time Dilation Effect. *Int. J. Phys. Sci.*, 2011, v. 6, 84–92.
9. Yarman T., Kholmetskii A. L., Arik M., Akku, B., Öktem Y., Susam A., Missevitch O. V. Novel Mössbauer Experiment in a Rotating System and the Extra Energy Shift Between Emission and Absorption Lines. arXiv: physics.gen-ph/1503.05853.
10. Yarman T., Kholmetskii A. L., Arik M. Mössbauer Experiments in a Rotating System: Recent Errors and Novel Interpretation. *The European Physical Journal Plus*, 2015, v. 130, 191.
11. Corda C. The Mössbauer Rotor Experiment and the General Theory of Relativity. arXiv: gr-qc/1602.04212.
12. Kholmetskii A. L., Yarman T., Yarman O., Arik M. Unabridged Response to "The Mössbauer Rotor Experiment and the General Theory of Relativity" by C. Corda: General Relativity Cannot Supply an Answer to the Extra Time Dilation in Rotor Mössbauer Experiments. arXiv: physics.gen-ph/1610.04219.
13. Combley F., Farley F. J. M., and Picasso E. The CERN Muon $(g-2)$ Experiments. *Physics Reports (Review Section of Physics Letters)*, 1981, v. 68 (2), 93–119.
14. Miller J. P., de Rafael E., and Roberts B. L. Muon $(g-2)$: Experiment and Theory. *Rep. Prog. Phys.*, 2007, v. 70, 7953–881.
15. Nolan P.J. The Fundamentals of the Theory of Relativity. Farmingdale State College, New York, pp. 7-2, 7-13.
16. Ciufolini I. and Wheeler J. A. Gravitation and Inertia. Princeton University Press, Princeton, NJ, 1995.
17. Hafele J. C. and Keating R. E. Around-the-World Atomic Clocks: Predicted Relativistic Time Gains. *Science*, 1972, v. 177, 166–168.
18. Hafele J. C. and Keating R. E. Around-the-World Atomic Clocks: Observed Relativistic Time Gains. *Science*, 1972, v. 177, 168–170.