Oscillating Massless Neutrinos

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The phenomenon of neutrino oscillations requires that not only should neutrinos be massive but that these masses be unique. How they acquire this mass remains an *open question*. Various mechanisms have been proposed to explain this phenomenon of neutrino oscillations. Herein, we propose — *the simplest imaginable* — alternative mechanism which operates *via* coupling the massless neutrino to a massive Dirac scalar. This massive Dirac scalar is a new hypothetical particle that we — *unfortunately* — can not observe directly because of its point-particle nature. Further, this massive Dirac scalar comes in as an integral part of the neutrino system — it [massive Dirac scalar] oscillates between three states, thus leading to the observed neutrino oscillations. This model predicts neutrinos are Dirac in nature and not Majorana.

"Just by studying mathematics we can hope to make a guess at the kind of mathematics that will come into the physics of the future."

— Paul A. M. Dirac (1902-1984)

1 Introduction

According to Albert Einstein (1879–1955)'s Special Theory of Relativity (STR) [1], the energy E and momentum p of a massless ($m_0 = 0$) are related by the energy-momentum equation ($E = pc$), where c is the speed of Light in *vacuo*. In accordance with the dictates of wave mechanics/phenomenon, the group velocity, $v_{\rm g}$:

$$v_{\rm g} = \frac{\partial E}{\partial p}, \tag{1}$$

of a particle whose energy and momentum are related by ($E = pc$) is equal to the speed of Light in *vacuo*, *i.e.* ($v_{\rm g} = c$). All indications are that the neutrino travels at the *vacuo* speed of Light, c, thus prompting physicists to assume that the neutrino is massless. Be that as it may, a massless neutrino pauses a problem to the physicist in that one can not explain the all-important experimentally [2–5] verified and common-place phenomenon of *neutrino oscillation*.

First predicted [6, 7] in 1957 by the Italian nuclear physicist — Bruno Pontecorvo (1913–1993), and observed in 1968 by the America chemist and physicist — Raymond Davis Jr. (1914–2006) et al. [8], neutrino oscillation is a quantum mechanical phenomenon whereby a neutrino created with a specific lepton flavour (electron ν_e, muon ν_μ, or tau ν_τ) can be measured at a latter time to have a different flavour. The probability of measuring a particular flavour for a neutrino varies between the three known flavour states (ν_e, ν_μ, ν_τ) as it propagates through the intestacies of space. From a theoretical standpoint, two conditions are necessary for neutrinos to oscillate — *i.e.*, to change from one type to the other, *e.g.*, from an Electron-neutrino (ν_e) to a Muon-neutrino (ν_μ) or *vice-verse*, and these conditions are:

1. Neutrinos *must* have a *non-zero mass*, and this mass *cannot be identical* for all the three neutrino flavours (ν_e, ν_μ, ν_τ).
2. There *must be no rigorous law forbidding* a transition between neutrino species, the meaning of which is that these transitions are purely probabilistic in nature.

Since the coming to light or since the "conception" of this important question *i.e.*, the question of how neutrino masses arise — this question, has not been answered conclusively [9]. In the *Standard Model* of particle physics, fermions only have mass because of interactions with the *Higgs Field*. Do neutrinos generate their mass *via* the *Higgs Mechanism* [10–12] as-well? This is a question that needs an answer. We here do not claim to give a definitive answer to this question, but merely a suggestion — *perhaps*, a suggestion that one might consider worthy of their attention.

That said, we must here at the penultimate of this introductory section make clear the scope of the present letter — *i.e.*, while this letter presents — *in our feeble view*, a new model whose endeavour is to explain neutrino oscillations, we present this model only as an alternative to existing explanations on this phenomenon. We deliberately avoid an in-depth comparative analysis of these models with the present and this we have done in-order that our ideas are clearly presented without overshadowing them with existing ideas on the same endeavour.

2 Massless Dirac particle

First considered by the German mathematician, mathematical physicist and philosopher — Hermann Klaus Hugo Weyl (1885–1955); a massless Dirac particle is described by the following Dirac-Weyl [13] equation:

$$\imath\hbar\gamma^\mu\partial_\mu\psi = 0, \tag{2}$$

where ($\imath = \sqrt{-1}$), ∂_μ is the four spacetime partial derivatives, $\hbar$ is the normalized Planck constant, γ^μ are the four 4×4 Dirac matrices and ψ is the usual 4×1 component Dirac wavefunction.

In this letter, the gamma matrices shall be assumed to be four vectors the meaning of which is that they transform like vectors *i.e.*:

$$\gamma^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^\mu}\gamma^\mu. \tag{3}$$

This assumption of treating the γ-matrices as four vectors may appear strange and if not completely and outright wrong. Be that as it may, in the letter [14], this idea of treating the γ-matrices as vectors as been justified. As argued therein the said letter [14], once the γ-matrices are four vectors, ψ can take three forms:

1. It [ψ] can be a zero ranks scalar.
2. It [ψ] can be a four 4×1 component scalar where the four components are zero ranks scalar objects.
3. Provided a certain transformational condition is met [*i.e.*, the condition given in equation (28) of [14]], it [ψ] can be the typical Dirac spinor.

In the subsequent section, we shall look at the scalar version.

3 Scalar coupled massive Dirac particle

For a moment, suppose we couple the massless ψ-particle to a massive ϕ-scalar particle, that is to say, we have ψ interfere with ϕ in such a way that the resulting 4×1 component Dirac wavefunction of the interference $\uppsi$, is such that:

$$\uppsi = \phi\psi. \tag{4}$$

The ϕ-particle is a simple (zero-rank) scalar, *i.e.*, unlike the ψ-particle which is a 4×1 component object, ϕ has no components, it is a zero rank mathematical object. Together, ϕ and ψ make a complete quantum mechanical particle *i.e.*, they satisfy the quantum probability normalization condition:

$$\iiint_{\forall\, Space} (\phi\psi)^\dagger\,(\phi\psi)\,dxdydz = 1, \tag{5}$$

and as individuals (ϕ, ψ), they do not satisfy the quantum probability normalization condition required for a complete quantum mechanical particle *i.e.*:

$$0 < \iiint_{\forall\, Space} \phi^\dagger\phi dxdydz < 1, \tag{6}$$

and:

$$0 < \iiint_{\forall\, Space} \psi^\dagger\psi dxdydz < 1. \tag{7}$$

Now, substituting ($\uppsi = \phi\psi$) into equation (2), we will have:

$$\imath\hbar\gamma^\mu\phi\partial_\mu\psi = -\imath\hbar\gamma^\mu\left(\partial_\mu\phi\right)\psi. \tag{8}$$

If ϕ is a massive particle satisfying the equation:

$$-\imath\hbar\gamma^\mu\partial_\mu\phi = m_0c\phi, \tag{9}$$

where ($m_0 \neq 0$), then, equation (8), becomes:

$$\imath\hbar\gamma^\mu\phi\partial_\mu\psi = m_0c\phi\psi, \tag{10}$$

hence:

$$\imath\hbar\gamma^\mu\partial_\mu\psi = m_0c\psi. \tag{11}$$

Equation (11) is the Dirac [15, 16] equation describing a massive particle of mass m_0 and it is this equation that is used to describe neutrino oscillations. Thus, the neutrino as described by $\uppsi$ is now a massive particle — the meaning of which is that one can now describe neutrino oscillations which require a non-zero mass. It is important at this juncture that we state the obvious, namely that — just as the ψ-particle is a spin-1/2 particle, the ϕ-particle is likewise a spin-1/2 particle. As pointed out in the pernultimate of the previous section, we must remind the reader at this point that equation (9) with ϕ as a scalar has been justified in the letter [14]. That is to say, as justified therein the letter [14], the γ-matrices have here been assumed to be four vectors, hence equation (9).

While neutrino oscillations strongly point to the existence of unique non-zero mass for the three neutrino flavours, these oscillations do not directly mean the mass of these neutrinos is non-zero (*e.g.*, [17]). Only direct experimental observations as deliver a definitive answer to the question (*e.g.*, [17]). A number of experiments have been dedicated to this effect and these experiments place upper limits with not definitive and precise value being pinned down.

4 Dirac scalar particle

While the ϕ-scalar particle is operated on by the usual Dirac operator, it is not an ordinary Dirac particle because an ordinary Dirac particle is described by a 4×1 component wavefunction and not a zero rank scalar. Consequently, the question that naturally and immediately comes to mind is whether this Dirac [15, 16] equation (9) describing this scalar particle is a valid equation. To answer this — just as is the case with the Dirac [15, 16] equation, the validity of this equation is to judged on whether or not this equation (9) yields reasonable energy solutions for the case of a free scalar. As usual, the free particle solution of the new hypothetical Dirac scalar is:

$$\phi = \phi_0 e^{\imath p_\mu x^\mu/\hbar}, \tag{12}$$

where ϕ_0 is a normalization constant, p_μ and x^μ are the four momentum and position of this scalar particle. Substituting ϕ as given in equation (12) into equation (9), and thereafter performing some algebraic operations and clean-up, one obtains the following set of four simulations equations:

$$\begin{array}{lll} (E - \mathrm{m}_0c^2) - c(p_x - ip_y) - cp_z = 0 & \ldots & (a) \\ (E - \mathrm{m}_0c^2) - c(p_x + ip_y) + cp_z = 0 & \ldots & (b) \\ (E + \mathrm{m}_0c^2) - c(p_x - ip_y) - cp_z = 0 & \ldots & (c) \\ (E + \mathrm{m}_0c^2) - c(p_x + ip_y) + cp_z = 0 & \ldots & (d) \end{array} \tag{13}$$

Adding together equations (13a) and (13b), one obtains:

$$E = p_x c + m_0 c^2, \tag{14}$$

and likewise, adding together equations (13c) and (13d), one obtains:

$$E = p_x c - m_0 c^2. \tag{15}$$

Undoubtedly, the solutions (14) and (15), are indeed acceptable solutions — hence, the scalar Dirac [15,16] equation (9), is as a result, an acceptable equation describing this hypothetical Dirac scalar particle. The question now is what do these solutions (14) and (15) mean?

First — we must notice that these solutions (14) and (15) tell us that the energy of the ϕ-scalar particle is determined by this particle's momentum along the x-axis. If this particle did have a non-zero momentum along the other two axis *i.e.*, the y and z-axis, what the equations (14) and (15) are telling us, is that this momentum is of no consequence whatsoever in determining the energy of this particle. This does not make sense. The only reasonable solution to this dilemma is to assume that ($p_y = p_z = 0$) and ($p_x \neq 0$). This means that the ϕ-particle only moves along the x-axis and nothing else. If this is the case that it only moves along the x-axis, then — clearly, this ϕ-particle can not be an extended particle, but a point-particle. If the ϕ-particle is indeed a point-particle, it must be invisible hence non-detectable. This not only a natural conclusion to reach, but a logical one.

Second — we have the two solutions equation (14) and (15) having different energies. What does this mean? One way to look at this is to assume that there exists two such particles with each having different energies. The other would be to assume that there is just one ϕ-particle — *albeit*, with the mass *discretely fluctuating* between the two mass extremums *i.e.*, $(-m_0)$ and $(+m_0)$. That is to say, the ϕ-particle is unstable and its instability is naturally transmitted to the neutrino *via* the $(\phi - \psi)$-coupling. As the unobservable ϕ-particle changes its energy state, it will excite and de-excite the observable neutrino into the energy states of the other two flavours. If the mass only fluctuated between the two mass extremums *i.e.*, $(-m_0)$ and $(+m_0)$, it would mean the neutrino would fluctuate between two states only, without it returning to its natural state. We know that a neutrino of any type will fluctuate between all the three states. In-order for the neutrino to enter its natural state, there is need for ϕ to enter into a third eigenstate of is mass. Naturally, this must be the eigenstate ($m_0 = 0$). Therefore, the ϕ-particle will discretely fluctuate between the three states $(-m_0, 0, +m_0)$ and each of these states corresponds to a particular value of energy which switches the neutrino to the right energy state of a given neutrino state.

5 The neutrino oscillations

How do these oscillations in the particle's state occur in the present model? Just as happens in *quantum gauge transformations* — for an answer to this very important question, we envisage a discrete *gauge-transformation-like* spontaneous and random change in the state of the ϕ-particle occurs in the phase *i.e.*:

$$\phi \longmapsto e^{i\chi_i}\phi, \tag{16}$$

where χ is some continuous and differentiable smooth function of the four position x^μ and or four momentum p^μ. In-order to preserve the composite-state ψ, such a change as that given in equation (16) is to be simultaneously met with a corresponding conjugate change in the phase of the neutrino, *i.e.*:

$$\psi \longmapsto e^{-i\chi_i}\psi, \tag{17}$$

and these two changes, leave the Ψ-state unchanged, *i.e.*:

$$\Psi \longmapsto \left(e^{i\chi_i}\phi\right)\left(e^{-i\chi_i}\psi\right) = \phi\psi = \Psi. \tag{18}$$

We expect that there be three phase changes corresponding to the three mass states $(-m_0, 0, +m_0)$, hence three energy states.

The phase change given in equation (16) leads the scalar Dirac equation (9), to transform and become:

$$-\imath\hbar\gamma^\mu\partial_\mu\phi = \left(m_0 + m_j^*\right)c\phi, \tag{19}$$

while the phase change given in equation (17) leads to the Dirac equation (11) for the neutrino, to transform and become:

$$\imath\hbar\gamma^\mu\partial_\mu\psi = \left(m_0 + m_j^*\right)c\psi, \tag{20}$$

where the three-state fluctuating mass m_j^* is such that:

$$m_j^* = \frac{\hbar\gamma^\mu\partial_\mu\chi_i}{c}. \tag{21}$$

In the following subsections, we discuss the possible oscillations of the neutrino for all the three neutrino flavours.

5.1 Oscillations of the Electron-neutrino state

Presented in the self-explanatory diagram in Figure (1) is a graphic visual of the six possible transitions of the natural Electron-neutrino. That is, when the ϕ-particle's mass is zero ($m_0 = 0$), the Electron-neutrino is in its natural state of being an Electron-neutrino. Further, when the mass of the ϕ-particle is negative $(-m_0)$, the Electron-neutrino is in enters the μ-neutrino state and likewise, when mass of the ϕ-particle is positive $(+m_0)$, Electron-neutrino enters the τ-neutrino state.

5.2 Oscillations of the Muon-neutrino state

Just as in Figure (1), we have in Figure (2) a graphic visual of the four possible transitions of natural Muon-neutrino. When the ϕ-particle's mass is zero ($m_0 = 0$), the Muon-neutrino is in its natural state of being an Muon-neutrino. When the mass of the ϕ-particle is negative $(-m_0)$, the Muon-neutrino is in enters the Electron-neutrino state and likewise, when mass of the ϕ-particle is positive $(+m_0)$, Muon-neutrino enters the τ-neutrino state.

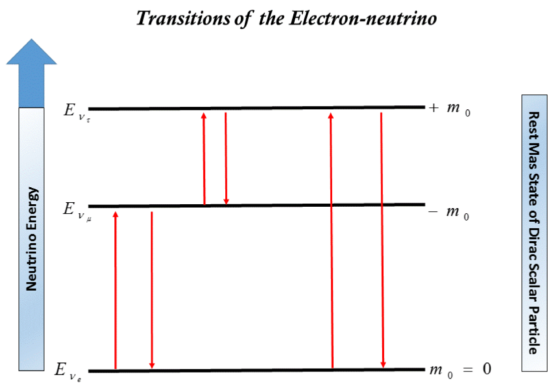

Fig. 1: The six possible transitions of the Electron-neutrino.

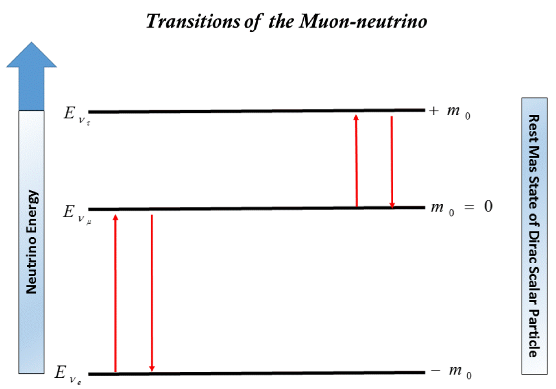

Fig. 2: The four possible transitions of the Muon-neutrino.

5.3 Oscillations of the Tau-neutrino state

Again, just as is the case in the previous cases, Figure (3) is a graphic presentation of the six possible transitions of natural Tau-neutrino. When the ϕ-particle's mass is zero ($m_0 = 0$), the Tau-neutrino is in its natural state of being an Tau-neutrino and when the mass of the ϕ-particle is negative ($-m_0$), the Tau-neutrino enters the Electron-neutrino state and likewise, when mass of the ϕ-particle is positive ($+m_0$), Muon-neutrino enters the μ-neutrino state.

6 General discussion

Clearly, without casting away any of the existing theories (*e.g.*, [17–19]) whose endeavour is to explain the mystery behind the neutrino oscillations, we here have provided an alternative explanation *via* what appears to us to be a mathematically permissible mechanism whereby the massless neutrino is coupled to an unobservable and unstable scalar Dirac point-particle. The resulting mathematics thereof requires that this hypothetical Dirac scalar must be a point-particle. From a physics standpoint, this point-particle nature of the ϕ-scalar implies that this particle can not be observed in nature because it is not an extended particle like the Electron, Proton, Neutrino *etc*. So, we should not expect to observe this particle

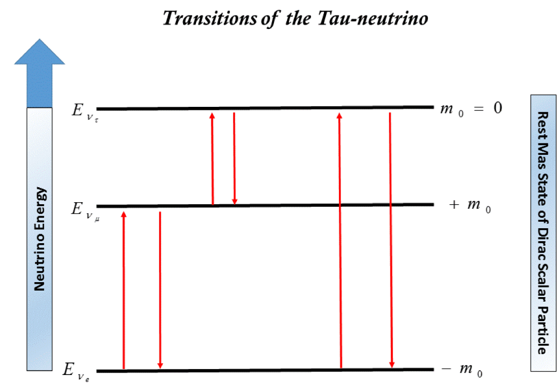

Fig. 3: The six possible transitions of the Tau-neutrino.

at all. We can only assign it to be a property of the neutrino particle — with it, being the "*culprit*" behind the observed phenomenon of neutrino oscillation.

Interestingly, within the context of the present model, one can answer the paramount question of whether of not neutrinos are Majorana or Dirac in nature. Majorana neutrinos satisfy the Majorana [20] equation while Dirac neutrinos satisfy the usual massive Dirac equation (11). In the present model, for these neutrinos to be Majorana, the Dirac scalar must be Majorana too, that is to say, the scalar Dirac equation (9), will have to be such that:

$$-\imath\hbar\gamma^{\mu}\partial_{\mu}\phi = m_0 c\gamma^2\phi. \tag{22}$$

With equation (22) in place, equation (11) will as a consequence thereof, reduce to the [20] equation, *i.e.*:

$$\imath\hbar\gamma^{\mu}\partial_{\mu}\psi = m_0 c\gamma^2\psi, \tag{23}$$

Now, substituting the free particle solution of the ϕ-scalar given in equation (12) into equation (22), just as in equation (13), one obtains the following set of four simulations equations:

$$\begin{aligned}
(E + \imath m_0 c^2) - c(p_x - ip_y) - cp_z &= 0 \quad \ldots \quad (a)\\
(E + \imath m_0 c^2) - c(p_x + ip_y) + cp_z &= 0 \quad \ldots \quad (b)\\
(E - \imath m_0 c^2) - c(p_x - ip_y) - cp_z &= 0 \quad \ldots \quad (c)\\
(E - \imath m_0 c^2) - c(p_x + ip_y) + cp_z &= 0 \quad \ldots \quad (d)
\end{aligned} \tag{24}$$

Adding together equations (24a) and (24b), corresponding to equation (14), one obtains:

$$E = p_x c - \imath m_0 c^2, \tag{25}$$

and likewise, adding together equations (24c) and (24d), corresponding to equation (15), one obtains:

$$E = p_x c + \imath m_0 c^2. \tag{26}$$

In contrast to the solutions given in equations (14) & (15), these solutions equation (25) & (26), are complex. As a rule of quantum mechanics, energy eigenvalues must be real. What this means is that we must reject these solutions [*i.e.*, equations (25) & (26)], and with them, the premise on which they are founded, namely that neutrinos are Majorana. One can try and save the Majorana model by invoking an imaginary mass so that the energy is real, but this will sure not work for so long as mass is a quantum mechanical observable because quantum mechanics will require that the mass be real thus leaving us exactly where we started off *i.e.*, with complex energy states, hence, in-accordance with the present model, neutrinos can not be Majorana, but can only be Dirac in nature.

7 Conclusion

Assuming that what has been presented in the present letter is acceptable, one can put forward the following as the conclusion to be drawn thereof:

1. In addition to the existing theories on neutrino oscillations, the present model is an alternative explanation, where these neutrino oscillations are explained by assuming that the massless neutrino is intrinsically coupled to a hypothetical, massive three-state unstable, invisible, unobservable point-particle which is a Dirac zero-rank scalar. The three-state unstableness of this Dirac scalar is what leads to the observed neutrino oscillations.
2. If complex energy states are physically non-permissible and/or forbidden — be they for the case of observable or non-observable particle(s) — then, according to the present model, neutrinos can not be Majorana in nature as this directly leads to complex energy eigenvalues for the Dirac ϕ-particle. On this basis and this alone, one is to reject this and with it, the idea of Majorana neutrinos.

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