Predicting Total Angular Momentum in TRAPPIST-1 and Many Other Multi-Planetary Systems Using Quantum Celestial Mechanics

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TRAPPIST-1 harbors at least 7 Earth-mass planets orbiting a 0.089 solar mass dwarf M-star. Numerous other multi-planetary systems have been detected and all obey a quantization of angular momentum per unit mass constraint predicted by quantum celestial mechanics (QCM) as derived from the general theory of relativity (GTR). The universality of this constraint dictates that the TRAPPIST-1 system should obey also. I analyze this recently discovered system with its many mean motion resonances (MMRs) to determine its compliance and make some comparisons to the Solar System and 11 other multi-planetary systems.

1 Introduction

In the past 25 years, more than 3500 exoplanets have been detected, many in multi-planetary systems with 4 or more planets [1]. Extreme examples include HD 10180 with 9 planets and TRAPPIST-1 with 7 planets. In each of the discovered systems the understanding of their formation and stability over tens of millions or even billions of years using Newtonian dynamics remains an interesting challenge.

A prediction of whether additional planets exist beyond those already detected is not an expected outcome of the dynamical studies. However, a different approach [2] called quantum celestial mechanics (QCM) offers the potential ability to predict the existence of additional angular momentum in the planetary system, which could indicate additional planets to be detected or additional mass in the form of rings or spherical shells of mass chunks orbiting the star, such as the Kuiper belt or the Oort Cloud in our Solar System.

The history of the formation of most of these planetary systems remains an active research area, ranging from *in situ* formation from a dust disk to pebble accretion followed by sequential inward migration toward the central star [3]. Their stability may depend upon numerous factors, and many research groups continue to investigate the long-term stability for millions of orbits over tens of millions of years, including in models for the history of our Solar System.

There is a recent paper [4] that considers the total angular momentum deficit (AMD) of multi-planetary systems with the proposal that the AMD is a way to classify their predicted stability. The AMD is defined by the total angular momentum difference

$$AMD = \sum_{k=1}^{n} \mu_k \sqrt{GM\, r_k} \left(1 - \sqrt{1-\epsilon_k^2}\, \cos\, i_k\right) \qquad (1)$$

between the maximum total orbital angular momentum when all the planets orbit in the same plane and the total angular momentum determined from the orbital data. The Solar System and HD 10180 are two examples discussed in which the outer system of planets is AMD-stable, the inner system of planets is AMD-unstable, and the whole system is AMD-unstable.

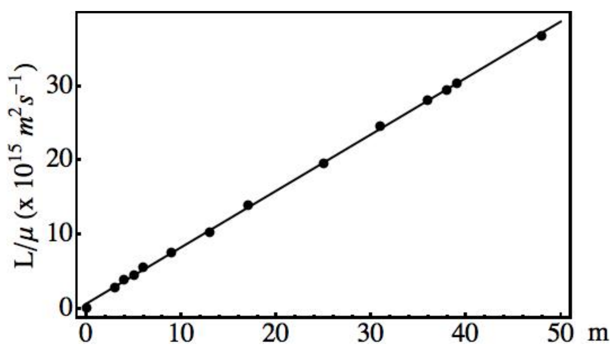

Fig. 1: Solar System fit to QCM total angular momentum constraint. The uncertainties are within the data circles.

In fact, this AMD approach demonstrates that the AMD-unstable systems tend to have orbital period ratios concentrated around the lower integer mean motion resonance ratios such as 3:2 and 2:1, a result perhaps somewhat in conflict with expectations. This unexpected outcome is interesting because many planetary systems exhibit at least one mean motion resonance (MMR), which had been expected to contribute a stabilizing factor in parts of those systems. The AMD research therefore means that not all MMRs are beneficial toward stabilizing the planetary orbits.

The recently discovered TRAPPIST-1 system has 7 Earth-mass planets all within 0.1 au of its dwarf M-star of 0.089 solar masses [5]. Three of the planet pairs exhibit a 3:2 MMR and another pair exhibits the 4:3 MMR, yet studies indicate that this system has been in existence for at least 7 billion years. Perhaps an additional factor contributes to the stability of these multi-planetary systems.

We propose that the additional factor is the quantization of angular momentum per unit mass predicted by quantum celestial mechanics (QCM). The QCM theory [6] dictates that not all planetary orbits about the central star are available as equilibrium orbits but, instead, QCM determined equilibrium orbits exist only at specific radii. Bodies in orbits at all other radial distances will migrate towards these specific QCM equilibrium orbital radii.

In the following sections we review the QCM proposed angular momentum constraint that leads to a select set of orbital radii for all planetary systems and demonstrate its application to the Solar System, the 5 moons of Pluto, the 7 planets of TRAPPIST-1, and to numerous other exoplanetary systems, including HD 10180.

2 The QCM angular momentum constraint

The total angular momentum in a planetary system is an important physical parameter not often discussed. In 2003, H. G. Preston and F. Potter proposed [6] a new gravitational theory called Quantum Celestial Mechanics (QCM), which is derived from the general theory of relativity (GTR), that claims that all gravitationally bound systems in the Schwarzschild metric will exhibit the quantization of angular momentum per unit mass constraint

$$\frac{L}{\mu} = m\,\frac{L_T}{M_T} \qquad (2)$$

with m being the orbit quantization integer, L the angular momentum of each orbiting body of mass μ, and L_T and M_T the total angular momentum and total mass of the planetary system.

In the simplest applications of QCM, one assumes that after tens of millions of years that the orbiting planet is at its equilibrium orbital radius r with a small eccentricity ϵ so that the Newtonian orbital angular momentum value $L = \mu\,\sqrt{GM\,r(1-\epsilon^2)}$, with M being the star mass, can be used. For most multi-planetary systems, including the Solar System, TRAPPIST-1, and HD 10180, the values of ϵ are all less than 0.2 and will be ignored in the QCM analysis fit to the constraint.

Because the QCM quantization of angular momentum per unit mass constraint is derived from the general relativistic Hamilton-Jacobi equation via a simple transformation, one obtains a new gravitational wave equation [6]. In the familiar Schwarzschild metric this gravitational wave equation will apply to all gravitationally-bound systems with orbiting bodies. However, as in GTR, different metrics can be considered, including the static interior metric, for which the QCM analysis of the Universe [7] predicts a new interpretation of the cosmological redshift in agreement with the data, that all distant sources are in a more negative gravitational potential than all observers, i.e. the distant clocks tick slower.

3 Application of QCM to the Solar System

Our first application of QCM in the Schwarzschild metric was to our Solar System using the known masses and present spacings of its 8 planets. If only the orbital angular momentum of the 8 planets and the Sun are considered, so that $L_T \approx 4\times10^{43}$ kg m^2 s^{-1}, then this value of the total angular momentum meant that QCM predicted that all the planetary orbits should be within the radius of the Sun! Obviously, something was wrong.

At first, we suspected that our derivation of the constraint was incorrect. But a detailed check proved that our derivation had been done correctly, including the numerous approximations needed to obtain an equation with the most important factors. Therefore, in order to achieve the present day orbital spacings, we interpreted the QCM equations to be predicting much more angular momentum in the Solar System, about 50 times as much!

Indeed, we subsequently learned that the Solar System does have much more angular momentum in its system than the contributions from just the Sun and its planets. The Solar System has an enormous angular momentum contribution from the Oort Cloud with its approximately 100 Earth masses of ice chunks orbiting at about an average distance of 40,000 au, thereby dominating the total angular momentum of the Solar System by almost a factor of 50.

The new orbital fits of QCM using the constraint then agreed with the present orbital radii of the planets, and we predicted the total angular momentum in the Solar System to be the much higher value $L_T \approx 1.9\times10^{45}$ kg m^2 s^{-1}. Fig. 1 shows our QCM fit to the 8 planets plus the 5 known dwarf planets, with m values 3, 4, 5, 6, 9, 13, 17, 25, 31, 36, 38, 39, 48.

So, for the first time, we were able to use the QCM angular momentum constraint to fit the equilibrium orbital radii of all the planets of the Solar System and to verify that the constraint could be an important factor in predicting additional angular momentum in a planetary system. One should note that the QCM fit does not require the division of the system into the inner planets and the outer planets, a prominent feature of other approaches, including AMD.

The successful application of the QCM angular momentum constraint to the Solar System encouraged us to try to find a definitive test. But the QCM constraint fit to the Solar System and to the orbiting satellites of the Jovian planets could not be considered definitive tests of QCM because their system total angular momentum values were not known to within 10%. So a decade long hunt began to find a multi-bodied system for which the physical parameters are known to be within a few percent.

4 Pluto system as a definitive test of QCM

Fortunately, in 2012, the dwarf planet Pluto was reported to have 5 moons. Their orbital stability was being studied in reference to the Pluto-Charon barycenter, and the moons are nearly in a 1:3:4:5:6 resonance condition!

An early QCM linear regression fit with $R^2 = 0.998$ to the angular momentum constraint for the Pluto system revealed more angular momentum could be present in this sys-

	m	r (au)	P (days)	P2/P1	$(n_2/n_1)^3$	L_{max} 10^{39} kg m^2 s^{-1}	MMR(P)	MMR(n)
b	15	0.0115	1.51087	1.000	1.000	1.103		
c	18	0.0158	2.42182	1.603	1.675	1.802	1.603	1.675
d	21	0.0223	4.04961	2.680	2.600	0.540	1.672	1.552
e	24	0.0293	6.09961	4.037	3.815	1.828	1.506	1.467
f	28	0.0385	9.20669	6.094	5.954	1.651	1.509	1.560
g	31	0.0469	12.35294	8.176	8.000	2.066	1.342	1.344
h	36	0.0619	18.76700	12.421	12.366	0.826	1.519	1.546
						9.815		

Table 1: Fit of the 7 planets of TRAPIST-1 to the QCM angular momentum constraint.

tem, hinting that at least one more moon could exist. This fit used the smallest set of integers possible with m values 2, 6, 9, 10, 11, 12. A set with larger integers was also available beginning with $m = 4$ for a good fit but indicating a lower total angular momentum value for the system.

Then, in 2015, the New Horizons spacecraft sent back precise data about the Pluto system that established 5 tiny moons only. That limitation allowed us to have a definitive test [8] of QCM because the total angular momentum was then known to within 2.4%. With the m values 4, 10, 15, 16, 18, 19, the QCM angular momentum constraint applied to the Pluto system predicted $L_T = 6.28 \times 10^{30}$ kg m^2 s^{-1}, a value commensurate with the value $L_T = 6.26\,(\pm 0.14) \times 10^{30}$ kg m^2 s^{-1} calculated from the known physical parameters.

We therefore consider the Pluto system to be the definitive test of the QCM angular momentum constraint because we know the pertinent physical parameters to within 2.4%, and the predicted QCM total angular momentum determined from the slope of the QCM plot of L/μ vs m agrees with the total value determined in the standard way using Newtonian physics.

5 QCM constraint applied to TRAPPIST-1

There has been great interest in the TRAPPIST-1 system because at least 3 of the planets are in the so-called Habitable Zone where liquid water and perhaps some kind of life form could have evolved over its nearly 9 billion year history [10]. However, being so close-in to their M-star also means that these planets could be experiencing a severe UV radiation flux as well as particle winds emanating from the star. Studies of their atmospheric content are under way by researchers to determine whether water still exists or whether the UV radiation has dissociated any previously existing water vapor with the resulting particles having evaporated away to leave behind an arid surface environment [9, 11].

We know that the planetary system orbiting TRAPPIST-1 harbors at least 7 Earth-mass planets orbiting close-in to the dwarf M-star of 0.089 $M_\odot$ [5]. More planets further out beyond 1 au could exist, a possibility that QCM may suggest by interpreting the constraint fit. The orbital period ratios reveal that planet pairs d/e, e/f and g/h exhibit nearly a 3:2 mean motion resonance (MMR) and the pair f/g has a 4:3 MMR [9]. Planet pairs b/c and c/d do not have a first order MMR although their period ratios are near 5:3.

The formation of this system has been a challenge for modeling, and in a recent study [3] a pebble accretion and inward migration history have been proposed to accommodate its formation, including a process called resonance trapping as planets sequentially move inward and build.

The pertinent data for the 7 known planets and the predicted m values from the system's linear regression fit to the QCM angular momentum constraint are provided in Table 1. This set of m values is the lowest set of integers that achieved a linear regression least squares fit of $R^2 > 0.999$ for both plots: L/μ vs m and P_2/P_1 vs $(n_2/n_1)^3$, with $n = m+1$ for the assumed circular orbits. Of course, other integer sets with larger m values will also fit the constraint as well, but they will have a smaller slope and therefore a smaller system total angular momentum value calculated with (2).

In Fig. 2 is the plot of L/μ vs m with all uncertainties within the small circles around each data point. From the slope 8.77×10^{12} m^2 s^{-1} of this QCM fit, one predicts a system total angular momentum of 1.56×10^{42} kg m^2 s^{-1}. The angular momentum from the star rotation plus the orbital motion of the 7 planets is much less, about 1.2×10^{40} kg m^2 s^{-1}, using the values given in Table 1 and a star rotation period of 3.295 days.

The angular momentum difference could be accommodated in several ways, including a larger integer for the first m value and larger integers overall, thereby reducing the QCM predicted total angular momentum. Or the difference could be due to the presence of at least one additional planet further out beyond a distance of about 1 au. For example, if the additional planet had the mass of Saturn, its orbit at about 3.8 au would be sufficient to account for the discrepancy between the total angular momentum values. And, of course, this system could have the equivalent of the Oort Cloud at a large distance from the star.

The period ratios provided in both columns 5 and 6 are referenced to planet b. For a circular orbit, $n = \ell+1$, and

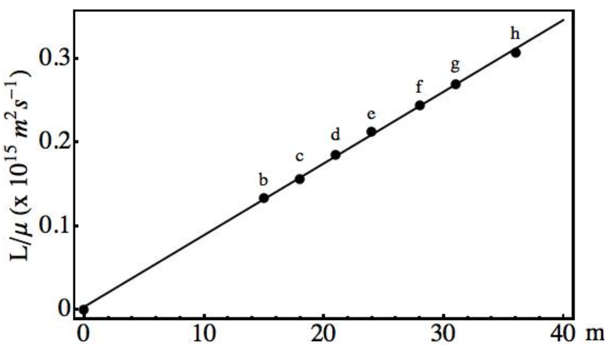

Fig. 2: QCM angular momentum constraint applied to the TRAPPIST-1 system of 7 planets close-in to the dwarf M-star. The uncertainties all lie within the data circles.

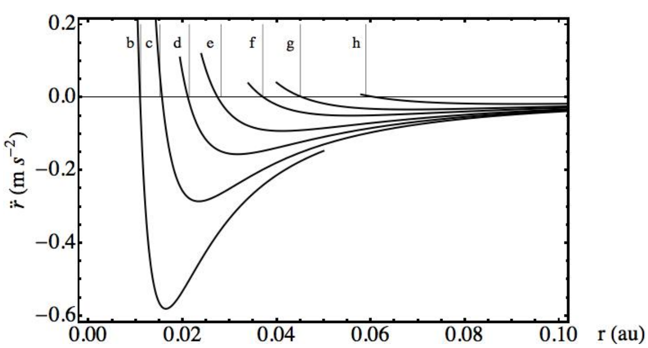

Fig. 3: The QCM predicted radial accelerations $\ddot{r}$ for each of the 7 planets of TRAPPIST-1. Note that some planets should experience corrections to their radial positions over tens of millions of years.

we assume $\ell = m$, its maximum value. QCM predicts period ratios

$$\frac{P_2}{P_1} = \left[\frac{m_2 + 1}{m_1 + 1}\right]^3 . \tag{3}$$

The largest discrepancy of the QCM predicted period ratios in column 6 from the actual values in column 5 is for planet e at 5.5%.

In the last two columns are the calculated MMRs for the adjacent planets when calculated from values in column 4, the MMR(P), and values calculated from column 6, for the MMR(n), revealing the amazing first order resonances d/e, e/f, g/h, and f/g, as well as the possible higher order resonances b/c and c/d. Planet c exhibits the biggest difference in QCM predicted values at about 7.2%.

Recall that QCM in the Schwarzschild metric predicts a specific but limited set of radii for circular equilibrium orbits that have both inward and outward forces acting, in direct contrast to Newtonian orbital dynamics which has an equilibrium orbit at all planetary orbital radii. For QCM the approximate expression for the effective gravitational potential is

$$V_{eff} = -\frac{GM}{r} + \frac{\ell(\ell+1)\, L_T^2}{2r^2\, M_T^2} , \tag{4}$$

where the angular momentum quantization integer ℓ originates in the θ-coordinate. We have taken $\ell = m$ for the expression. Whence, the expected value of the orbital radial acceleration near the equilibrium radius is defined by

$$\ddot{r}_{eq} = -\frac{GM}{r^2} + \frac{\ell(\ell+1)\, L_T^2}{r^3\, M_T^2} . \tag{5}$$

A computer simulation of the TRAPPIST-1 system could use this equation to study its long-term QCM dynamic stability contributions but must also include perturbations by the other planets. The net QCM accelerations are very small, varying from around a hundredth to a few tenths of a meter per second squared.

A plot of the QCM radial accelerations near the equilibrium radii for all 7 planets is shown in Fig. 3, where the vertical lines labelled b to h are the reported present radial orbital distances of the planets. As can be seen from the plot, a small radial movement inward for planet e is predicted to occur because its present radial acceleration is negative with respect to the QCM equilibrium orbital distance.

One would expect that the planets will oscillate about the QCM equilibrium orbital radii throughout their history, never settling at the exact radius at which no further radial acceleration would occur. Perturbations from the other nearby planets as they pass by will be larger than the QCM accelerations, but they last for short time intervals while the small QCM accelerations are acting constantly.

This TRAPPIST-1 system has existed for many billions of years, so some sort of stabilizing influence has been at play. We suspect that the QCM angular momentum constraint is the important additional factor, providing accelerations on both sides of the predicted QCM equilibrium orbital radius. A computer simulation will be needed to determine the outcomes over long time periods.

6 HD 10180 and other exosystems

The QCM quantization of angular momentum per unit mass constraint is expected to apply to all gravitationally bound systems described in the Schwarzschild metric.

In previous articles we analyzed multi-planetary systems with 4 or more planets and found that they all can fit the QCM angular momentum constraint. We list some of those systems for comparison in Table 2 in order of increasing star mass in column 2. Their m values and slope b are derived from the linear regression plots of L/μ versus m. The QCM value of L_T in column 6 is calculated from b and then compared to their known total angular momentum values (sum of columns 7 and 8).

Therefore, from the values in Table 2 we notice:

1. That our Solar System's b value is much larger than all the other multi-planetary system's b values. Why? Because the Solar System has the overwhelming angular momentum contribution from its Oort Cloud, a physi-

System	Star $M_\odot$	N	*m* values	*b* 10^{15} m^2 s^{-1}	QCM L_T 10^{45} kg m^2 s^{-1}	Star L_T 10^{42} kg m^2 s^{-1}	Planets L_T 10^{42} kg m^2 s^{-1}
TRAPPIST-1	0.089	7	15,18,21,24,28,31,36:	0.00877	0.00156	0.0113	0.012
GJ 667 C	0.31	7	16,21,26,29,34,39:55	0.0333	0.0206	0.00971	0.169
GJ 581	0.31	6	8,10,14,20,25:47	0.0456	0.0283	0.00454	0.229
HD 40307	0.75	6	9,12,16,19,22:35	0.0863	0.129	0.179	0.340
Tau Ceti	0.783	7	13,14,18,20:25,31,49	0.0923	0.145	0.0820	0.311
HR 8832	0.794	7	4,6,9,12:15,41,44	0.144	0.229	0.491	4.131
Kepler-20	0.912	6	8,10,12,15:18,24	0.105	0.191		0.846
Kepler-11	0.95	6	11,12,15,17,19:26	0.113	0.215		5.60
55 Cancri	0.95	5	3,8,12:23,62	0.160	0.304	0.118	78
Sun	1.0	8	:3,4,5,6,13,17,25,31	0.762	1.524	0.192	31
HD 10180	1.062	9	3,6,7,8,12,14:17,29,46	0.185	0.393	0.436	5.153
Kepler-90	1.20	8	14,15,17,28:33,36,43,50	0.0949	0.228	0.738	

Table 2: QCM angular momentum constraint applied to selected multi-planetary systems listed in order of star mass. N is the number of known planets which determine the *m* values for a linear regression fit $R^2 \geq 0.999$. The *m* values for planets with orbital radii less than Mercury's are to the left of the colon. The predicted QCM L_T in column 6 is calculated using the QCM slope *b* times the star mass.

cal property that dictates QCM to predict the very large orbital spacings for its planets. We cannot say much more about the Solar System, i.e., predict whether more planets or dwarf planets exist, because the overwhelming but unknown total angular momentum contribution of the Oort Cloud precludes making such a prediction.

2. That for the TRAPPIST-1 system, with its incredibly small QCM *b* value, we expect another planet or more orbiting bodies because the QCM predicted total angular momentum value is much greater than the orbital contribution from its 7 known planets and the rotation of the central star. Perhaps the proposed pebble accretion and inward migration train is the explanation for its formation, but QCD would suggest otherwise, that the planets formed *in situ* by gathering the local dust accumulating at the QCM equilibrium radii, assuming that the total angular momentum in this system did not change significantly during their formation.
3. That even for the HD 10180 system fit, as shown in Fig. 4 with its 9 planets, the total angular momentum from its star rotation plus the known orbiting planets falls far short of the QCM predicted total angular momentum, so more orbiting mass is expected.
4. That all the systems in Table 2 are expected to have additional angular momentum based upon the predicted QCM value of L_T. If more planets in these systems are detected, they should have orbital radii corresponding to the listed QCM *m* values that dictate their allowed equilibrium orbital distances.

Perhaps another exosystem will be discovered in the near future that also has a large angular momentum contribution and very large QCM orbital spacings so that direct comparisons can be made to the Solar System in terms of the total angular momentum parameter.

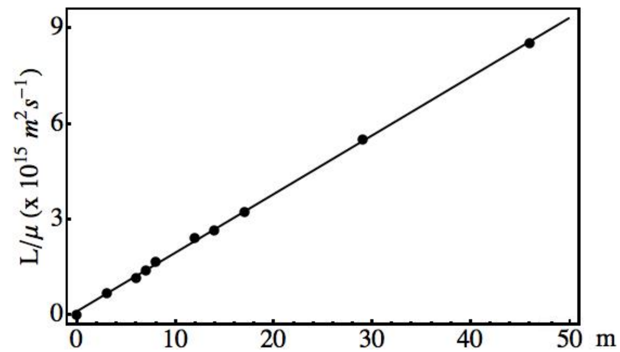

Fig. 4: QCM angular momentum constraint applied to HD 10180. Uncertainties lie within the data circles.

Note that both the 4 inner planets of the Solar System and the 7 planets of the TRAPPIST-1 system have been determined to be unstable by the AMD analysis [4]. Yet both systems have been in existence for more than 4 billion years, i.e., more than 4 billion Earth orbits. Perhaps the small QCM gravitational potential valleys around their QCM orbital equilibrium radii, such as those shown in Fig. 3, are contributing factors to their long-term stability. Or the existence of additional orbital mass further out contributes to their stability also. A computer simulation of these systems and the others that includes the QCM constraint could be done to determine whether this QCM effect is large enough to ensure their long-term stability.

7 Conclusions

Many multi-planetary systems have been discovered and they all had been determined previously to obey the QCM quantization of angular momentum per unit mass constraint. For most of those systems if not all of them, additional angular momentum is predicted by QCM, angular momentum which could be contributed by additional planets or spherical shells of ice.

Now the interesting TRAPPIST-1 system of 7 Earth-like planets has been shown to obey the angular momentum constraint for each known planet in the system. The QCM predicted total angular momentum of its planetary system is 1.56 $\times 10^{42}$ kg m^2 s^{-1} versus the estimated value of 1.2×10^{40} kg m^2 s^{-1} for the 7 planets plus the star rotation contribution. This large total angular momentum discrepancy could indicate that either at least one more planet could exist beyond several 1 au or that a set of m values with larger integers would be a better fit to decrease the predicted total angular momentum.

Also, for the TRAPPIST-1 system, from the determined radial acceleration values near to the QCM predicted orbital equilibrium radii, several planets could migrate slightly. For example, planet e has a present radial distance that should decrease slightly over several thousand years in order to reach its nearby predicted QCM orbital equilibrium radius. Perturbations from the other planets will be important to consider in a computer simulation of its behavior as the planet migrates to its true QCM equilibrium orbital radius.

We also provide a list of 12 multi-planetary systems so that a direct comparison of our Solar System QCM parameters can be made to other systems. The major difference is that our Solar System contains significantly more angular momentum than any other known planetary system discovered. Our QCM theory uses this information to predict the allowed equilibrium orbital distances, an approach that explains why almost all other multi-planetary systems with smaller total angular momentum values can have so many planets within the orbital radius of Mercury. Dynamically, a larger repulsive orbital angular momentum term in the QCM radial acceleration equation will result in the planets forming at larger orbital equilibrium radii.

Finally, the long-term stability of these multi-planetary systems remains a challenge for the traditional modeling using Newtonian universal gravitation without additional constraints. The consideration of the total angular momentum deficit (AMD) has introduced a method to classify their stability but is incomplete. Perhaps the QCM quantization of angular momentum per unit mass approach will be the additional constraint needed in order to better understand the formation and stability of multi-planetary systems.

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References

1. Han E., et al. Exoplanet Orbit Database. II. Updates to Exoplanets.org. *PASP*, 2014, v. 126 (943), 827.
2. Potter F. Multi-planet Exosystems All Obey Orbital Angular Momentum Quantization per Unit Mass predicted by Quantum Celestial Mechanics (QCM). *Progress in Physics*, 2013, v. 9 (3), 60–61.
3. Ormel C., Liu B., Schoonenberg D. Formation of TRAPPIST-1 and other compact systems. arXiv: astro-ph/1703.06924.
4. Laskar J., Petit A. AMD-stability and the classification of planetary systems. arXiv: astro-ph/1703.07125.
5. Gillon M., Triaud A. H. M. J., Demoy B.-O., et al. Seven temperate terestrial planets around a nearby untracool dwarf star TRAPPIST-1. *Nature*, 2017, v. 542, 456–460.
6. Preston H. G., Potter F. Exploring large-scale gravitational quantization without $\hbar$ in planetary systems, galaxies, and the Universe. arXiv: gr-qc/0303112.
7. Potter F., Preston H. G. Cosmological Redshift Interpreted as Gravitational Redshift. *Progress in Physics*, 2007, v. 3 (2), 31–33.
8. Potter F. Update on Pluto and its 5 Moons Obeying the Quantization of Angular momentum per Unit Mass Constraint of Quantum Celestial Mechanics. *Progress in Physics*, 2016, v. 12 (1), 56–58.
9. Luger R., Sestovic M., et al. A seven-planet resonant chain in TRAPPIST-1. *Nature Astronomy*, 2017, v. 1, (Article 0129).
10. Burgasser A. J., Mamejek E. E. On the Age of the TRAPPIST-1 System. arXiv: astro-ph/1706.02018.
11. Borrier V., De Wit J., Bolmont E., et al. Temporal Evolution of the High-energy Irradiation and the Water Content of TRAPPIST-1 Exoplanets. arXiv: astro-ph/1708.09484v1.