Theory of Anomalous Magnetic Moment and Lamb Shift of Extended Electron in Stochastic Electrodynamics

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The very presence of zero-point field allows us to consider the structure of the electron with center of charge in circular motion around center of mass. Considering extended electron structure in stochastic electrodynamics, mass and charge corrections are derived without any logarithmic divergence terms. Using these corrections, the anomalous magnetic moment of the electron has been expressed in a series as a function of fine-structure constant. The evaluated magnetic moment is found to be accurate up to ninth decimal place with a difference of 90.22×10^{-12} from the experimental value. In the case of an orbital electron, due to its motion, the surrounding zero-point field is modified and the zero-point energy associated with these modifications leads to a shift in the energy level. By imposing a cut-off frequency equal to the de Broglie frequency, the zero-point energy associated with the orbital electron is attributed to the Lamb shift. The estimated Lamb shift in hydrogen atom is found to be in agreement with the experimental value. These theoretical derivations give a new classical approach to both the anomalous magnetic moment of the electron and the Lamb shift.

1 Introduction

An electron is visualized as a point particle in both quantum mechanics and quantum field theories in general. Efforts to find the size of the electron have led to a very small size $\sim 10^{-20}$ m in high energy scattering experiments [1] and in the penning trap experiment, the finite size effect was considered to be of the order of experimental uncertainty in the measurement of the anomalous magnetic moment of the electron. Thus any sub-structure of the electron is ruled out in quantum field theories and the particles are treated as point particles without any size. The point particle limit of the electron, in most of the theoretical approaches is fine and excellent except for the singularity syndrome and any cut-off procedure leads to a finite structure of the electron.

The concept of an extended structure of the electron originates from the zitterbewegung motion (rapid oscillations of Dirac electron) and such random oscillations are invariably attributed to the presence of zero-point field throughout the universe. The extended electron theories were developed over several decades [2–10] and the perception of point particle having charge and mass or rigid sphere with charge distribution was denied and the structure of the electron had been considered with the charge in an average circular motion about the center of mass. While dealing with extended electron models, a natural question arises that why such extended structure is not detected in scattering experiments. The reason being the charge rotation is at the speed of light and therefore, it cannot be detected at all. However, the footprints of such extended electron can be seen from the recent detection of the de Broglie wave of the electron in the scattering of a beam of electrons in thin silicon crystal [11] and from the high resolution scanning tunneling microscopy images [12].

Recently, the role of spin and the internal electron structure in complex vector formalism was studied by the author [13–15] and it had been shown that the mass of the particle may be interpreted to the zero-point field energy associated with the local complex rotation or oscillation confined in a region of space of the order of the Compton wavelength. Further, the logical classical foundations of quantum mechanics were explored from the consideration of extended electron structure [16, 17]. It is of particular interest whether the calculations of the electron magnetic moment and Lamb shift are possible with the extended electron theories.

In the charge shell model of the electron, Puthoff [18] has shown that the zero-point energy of the particle is equal to the Coulomb energy in the limit when the shell radius tends to zero. The zero-point energy within the shell was found to be proportional to the fine-structure constant. Therefore, it may be expected that the zero-point energy associated with an electron in the point particle limit may be attributed to the charge correction rather than any mass correction which was considered earlier in the stochastic electrodynamics theories.

In stochastic electrodynamics (classical electrodynamics along with zero-point field), a charged point particle is considered as an oscillator and its equation of motion is given by the Brafford-Marshall equation which is simply the Abraham-Lorentz equation with zero-point field. In the stochastic electrodynamics approach, the energy of the electron oscillator was estimated by Boyer [19] and without imposing any cut-off frequency the zero-point energy of the oscillator was found to be $\hbar\omega_0/2$ per mode, where ω_0 and $\hbar$ are the oscillator frequency and reduced Planck constant respectively. Though many quantum phenomena were explained in the stochastic electrodynamics approach, the theory was found to be incomplete [20]. However, it has been found that the introduction of spin into the problem leaves the theory to overcome such failures. The detailed discussion of stochastic electrodynamics with spin was given by Cavelleri *et al* [21] and in this theory, the electron has an extended structure. In view of the extended electron structure, one can impose a cut-off frequency ω_0 and in that case, in the absence of radiation damping and binding terms, the energy associated with the electron has been derived in Section 2.

In the point particle limit, the energy associated with the electron is found to be

$$\Delta E_0 = \frac{2\alpha}{3\pi}\hbar\omega_0\,, \tag{1}$$

where $\alpha = (1/4\pi\epsilon_0)(e^2/\hbar c)$ is the fine-structure constant and $-e$ is the electron charge. This energy may be attributed to the charge correction and the ratio $\Delta E_0/\hbar\omega_0$ corresponds to a correction to fine-structure constant due to interaction of random zero-point field fluctuations. In general, the effective or observed fine-structure constant can be expressed by the relation $\alpha_{obs} \rightarrow \alpha_{th} + \Delta\alpha$. Now, the ratio $\Delta\alpha/\alpha$ can be expressed in the following form:

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta E_c}{\hbar\omega_0} = \frac{2\alpha}{3\pi}\,. \tag{2}$$

In quantum electrodynamics such charge correction was calculated considering the vacuum polarization and it may be noted that the above estimate gives a similar result except for the diverging logarithmic term. The incorporation of this charge correction leads to a replacement of fine-structure constant in the theoretical calculations by $\alpha\,(1 - 2\alpha/3\pi)$.

The total energy of the electron immersed in the zero-point field can be expressed by substituting $\left(\mathbf{p} - e\mathbf{A}_{zp}/c\right)$ for momentum in the relation $E^2 = p^2c^2 + m^2c^4$ [15]:

$$E^2 = p^2c^2 - 2ec\mathbf{p}.\mathbf{A}_{zp} + e^2A_{zp}^2 + m^2c^4, \tag{3}$$

where $\mathbf{A}_{zp}$ is the electromagnetic vector potential of zero-point field. The energy in the last two terms in the above equation can be written in the form $E_0 = mc^2 + e^2A_{zp}^2/2mc^2$. Thus under the influence of zero-point field, there appears a correction to mass and such correction to mass must be of the order of fine-structure constant. The derivation of such mass correction of extended electron in stochastic electrodynamics is given in Section 2. We find that the mass correction Δm depends on the reduced particle velocity $\beta = \mathbf{v}/c$ and the ratio $\Delta m/m$ is expressed by the relation

$$\frac{\Delta m}{m} = \frac{\alpha}{2\pi}\left(1 + \beta^2\right). \tag{4}$$

From the knowledge of mass and charge corrections, the anomalous magnetic moment a_e of the electron is estimated in Section 3.

Under the influence of central Coulomb potential, an orbital electron moves with a velocity proportional to the fine structure constant. When the electron moves in the zero-point field, it induces certain modifications in the surrounding zero-point field. Since these zero-point field modifications may be considered at least of the order of the de Broglie wavelength, the energy associated with the shift in the electron energy levels can be obtained by imposing a cut-off frequency equal to the de Broglie frequency ω_B and the derived zero-point energy is attributed to the Lamb shift. The derivation of Lamb shift and its calculation are given in Section 4. The energy shift in the electron circular orbit is found to be

$$\Delta E_L = \frac{4\alpha^5}{3\pi}m_rc^2, \tag{5}$$

where $m_r = mM/(m + M)$ is the reduced mass and M is the nuclear mass. The calculation of the Lamb shift has been performed using charge correction in the Coulomb field and the mass correction for the electron. Finally, the conclusions are presented in Section 5. The derived formulas elucidate a complete classical approach to both the anomalous magnetic moment of the electron and the Lamb shift.

2 Zero-point energy associated with an extended electron

When an electron moves in the zero-point field, we mean that the center of mass moves with velocity $\mathbf{v}$. The particle motion then contains both internal rotational motion and the translational motion. Denoting the center of mass motion by a position vector $\mathbf{x}$ and the radius of internal rotation by a vector ξ, a complex vector connected with both internal and translational motions of an extended electron can be expressed by a complex vector $X = \mathbf{x} + \mathbf{i}\xi$, where $\mathbf{i}$ is a pseudoscalar representing an oriented volume in geometric algebra. A complete account of complex vector algebra was elaborately discussed in the reference [14].

In stochastic electrodynamics, the expression for the electric field vector of electromagnetic zero-point field can be written in the following form

$$\begin{aligned} E_{zp}(\mathbf{x}, t) = Re\Bigg\{\sum_{\lambda=1}^{2}\int d^3k\,\epsilon(\mathbf{k}, \lambda)\,\frac{H(\omega)}{2}\times \\ \times\left[ae^{i(\mathbf{k}.\mathbf{x}-\omega t)} + a^*e^{-i(\mathbf{k}.\mathbf{x}-\omega t)}\right]\Bigg\}, \end{aligned} \tag{6}$$

where $\epsilon(\mathbf{k}, \lambda)$ is the polarization vector which is a function of wave vector $\mathbf{k}$, polarization index $\lambda = 1, 2$ and $Re\{\}$ represents the real part. We define $a = e^{i\theta(\mathbf{k},\lambda)}$ and $a^* = e^{-i\theta(\mathbf{k},\lambda)}$ and the phase angle is introduced to generate random fluctuations of the zero-point field. The normalization constant in (6) is set equal to unity. The spectral function $H(\omega)$ represents the magnitude of zero-point energy and in stochastic electrodynamics its value is found to be $\left(\hbar\omega/8\pi^3\epsilon_0\right)^{1/2}$. In the complex vector formalism, we replace $\mathbf{x}$ by X in the electric field $E_{zp}(\mathbf{x}, t)$ and expanding in terms of Taylor series yields

$$E_{zp}(X,t) = E_{zp}(\mathbf{x},t) + \mathbf{i}\xi \left.\frac{\partial E_{zp}(\mathbf{x},t)}{\partial x}\right|_{x\to 0} - \frac{\xi^2}{2}\left.\frac{\partial^2 E_{zp}(\mathbf{x},t)}{\partial x^2}\right|_{x\to 0} + O(\xi^3) + \ldots \tag{7}$$

Neglecting higher order terms and denoting

$$E_{zp}(\xi,t) = \xi \left.\frac{\partial E_{zp}(\mathbf{x},t)}{\partial x}\right|_{x\to 0}, \tag{8}$$

one can express the electric field vector in a complex form

$$E_{zp}(X,t) = E_{zp}(\mathbf{x},t) + \mathbf{i}E_{zp}(\xi,t). \tag{9}$$

The random zero-point fluctuations influence both the center of mass and the center of charge and therefore the force acting on the extended particle can be decomposed into force acting on center of charge and force acting on center of mass. The equation of motion of center of mass is then expressed in the form

$$m\ddot{\mathbf{x}} - \Gamma_a m\dddot{\mathbf{x}} + m\omega_0^2\mathbf{x} = eE_zp(\mathbf{x},t), \tag{10}$$

where $\mathbf{v} = \dot{\mathbf{x}}$, $\Gamma_a = 2e^2/3mc^3$ and an over dot represents differentiation with respect to time. The second and third terms on the left are radiation damping and binding terms. It should be noted that for the zero-point field acting on center of mass, both particle charge and mass appear at the center of mass point. On the other hand, for the field acting on the center of charge, the effective mass seen by the zero-point field is the potential equal to $e^2/2\xi \sim m_z c^2$, where m_z is the effective mass at the center of charge and the magnitude of ξ is of the order of the Compton wavelength. In this case both radiation damping and binding forces are absent and the equation of motion of center of charge can be written in the form

$$m_z\ddot{\xi} = eE_{zp}(\xi,t). \tag{11}$$

The average zero-point energy of the electron in its rest frame was previously estimated and it had been shown to be equivalent to the zitterbewegung energy [15]. Further, it was shown that the particle mass arises from the internal complex rotations and a relation between particle spin and mass had been derived previously in the following form [13]:

$$mc^2 = \Omega_s \cdot S \tag{12}$$

In the above equation, S is the spin bivector, Ω_s is the angular frequency bivector and it shows that the mass of an electron is equal to the zero-point energy associated with the local complex rotation in the spin plane.

In the case of center of mass motion of the particle with velocity $\mathbf{v}$, as a result of super position of internal complex rotations on translational motion, the particle is associated with a modulated wave containing internal high frequency ω_0 and envelop frequency ω_B which is the de Broglie frequency of the particle. Differentiating the position complex vector $X = \mathbf{x} + \mathbf{i}\xi$ with respect to time gives the velocity complex vector $U = \mathbf{v} + \mathbf{iu}$ and the complex conjugate of U is obtained by taking reversion operation on it,$\bar{U} = \mathbf{v} - \mathbf{iu}$ and the product $U\bar{U} = v^2 + u^2$ [13]. Dividing this equation throughout by ξ^2 and denoting $\omega_B = |v|/\xi$, $\omega_0 = |u|/\xi$ and $\omega_c = |U|/\xi$, we obtain the effective cut-off frequency ω_c of the modulated wave in the particle frame of reference in the form $\omega_c^2 = \omega_0^2 + \omega_B^2 = \omega_0^2(1+\beta^2)$. In the equation of motion of center of mass (9), the strength of radiation damping and binding terms are much smaller than the force term on the right. Therefore, neglecting radiation damping and binding terms in (10) and integrating the expression with respect to time gives

$$\dot{\mathbf{x}} = \frac{e}{m}\int_0^\tau E_{zp}(\mathbf{x},t)\,dt, \tag{13}$$

where the upper limit of integration is chosen to be the characteristic time $\tau = 2\pi/\omega_c$. Substituting the electric field vector $E_{zp}(\mathbf{x}, t)$ given in (6) into (13) and performing the integration gives

$$\dot{\mathbf{x}} = \frac{e}{m}\sum_{\lambda=1}^{2}\int d^3k\epsilon(\mathbf{k},\lambda)\frac{H(\omega)}{2} \times\left\{ae^{i\mathbf{k}.\mathbf{x}}\left(\frac{e^{-i\omega\tau}-1}{-i\omega}\right) + a^*e^{-i\mathbf{k}.\mathbf{x}}\left(\frac{e^{i\omega\tau}-1}{i\omega}\right)\right\}. \tag{14}$$

Now, using $|\dot{\mathbf{x}}|^2 = \dot{\mathbf{x}}\dot{\mathbf{x}}^*$, we find

$$|\dot{\mathbf{x}}|^2 = \frac{e^2}{m^2}\sum_{\lambda,\lambda'=1}^{2}\iint d^3kd^3k'\epsilon(\mathbf{k},\lambda)\epsilon(\mathbf{k}',\lambda')\frac{H^2(\omega)}{2\omega^2} \times(1-\cos\omega\tau)\left\{aa^{*\prime}e^{-i(\mathbf{k}-\mathbf{k}').\mathbf{x}} + a^*a'e^{i(\mathbf{k}-\mathbf{k}').\mathbf{x}}\right\}, \tag{15}$$

where the terms containing aa' and $a^*a^{*\prime}$ are dropped because of their stochastic averages are zero. Taking the stochastic average of (15) on both sides and using the following relations

$$\left\langle aa^{*\prime}e^{-(\mathbf{k}-\mathbf{k}').\mathbf{x}}\right\rangle = \langle a^*a'e^{+(\mathbf{k}-\mathbf{k}').\mathbf{x}}\rangle = \delta^3(k-k')\,\delta(\lambda-\lambda'),$$

$$\left\langle\sum_{\lambda,\lambda'=1}^{2}\epsilon(\mathbf{k},\lambda)\epsilon(\mathbf{k},\lambda')\delta(\lambda-\lambda')\right\rangle = \left\langle\sum_{\lambda=1}^{2}|\epsilon(\mathbf{k},\lambda)|^2\right\rangle = 1-\frac{k_x^2}{k^2} = \frac{2}{3},$$

$$\int d^3k = \int d\Omega\,k^2dk = 4\pi\int k^2dk = \frac{4\pi}{c^3}\int\omega^2d\omega,$$

the average value $\langle|\dot{\mathbf{x}}|^2\rangle$ is found to be

$$\langle|\dot{\mathbf{x}}|^2\rangle = \frac{4\alpha}{3\pi}\frac{\hbar^2}{m^2c^2}\int_0^{\omega_c}\omega(1-\cos\omega\tau)d\omega, \tag{16}$$

where the upper limit of integration is the chosen cut-off frequency ω_c. Because of this cut-off frequency, the zero-point field spectral components of wavelength of the order of $2\pi c/\omega_c$ are only effective and thus there exists an upper bound to the energy available from the electromagnetic zero-point field. For an extended particle of radius R, a convergence form factor can be obtained by finding the upper bound to the energy available from the electromagnetic zero-point field. A detailed calculation of such convergence form factor was calculated by Reuda [22]. This convergence form factor is given by

$$\eta(\omega) = \eta(\delta) = \frac{9}{\delta^4}\left(\frac{\sin\delta}{\delta}\right)^2\left(\frac{\sin\delta}{\delta} - \cos\delta\right)^2, \quad (17)$$

where $\delta = \omega R/c$ and the values of $\eta(\delta)$ lie in the range 0 to 1. For $\omega \sim \omega_0$ and $R \sim 2\hbar/3mc$, we have $\delta \sim 2/3$ and the convergence form factor $\eta\,(2/3) \sim 3/4$. In view of the extended structure of the particle, the convergence form factor is introduced in the energy calculation. In general, the total energy of an oscillator is a sum of both kinetic and potential energies and it is equal to twice the kinetic energy. Now, the zero-point energy associated with the particle is expressed in the form

$$\begin{aligned} \Delta E_c = m\langle|\dot{\mathbf{x}}|^2\rangle = \frac{2\alpha}{3\pi}\frac{\hbar^2\omega_c^2}{mc^2}\eta(\omega_c) \\ \times\left[1 + \frac{2}{\omega_c^2\tau^2}(1 - \cos\omega_c\tau - \omega_c\tau\sin\omega_c\tau)\right]. \end{aligned} \quad (18)$$

Substituting $\omega_c\tau = 2\pi$, $\omega_c^2 = \omega_0^2(1+\beta^2)$, using the Einstein de Broglie relation $\hbar\omega_0 = mc^2$ and approximating $\eta(\omega_c) \sim 3/4$ in (18) gives finally

$$\Delta E_c = \frac{\alpha}{2\pi}\, mc^2(1 + \beta^2)\,. \quad (19)$$

This energy change gives the correction to the mass, $\Delta m = \Delta E_c/c^2$ and we get the relation (4). The result in (19) differs from our previous calculation in reference [15] by the term $(1 + \beta^2)$, where we have assumed $\omega_c = \omega_0$. It may be noted that the energy associated with the particle derived in (19) depends on the particle velocity. However, in the point particle limit, $R \to 0$, $\omega_c \to \omega_0$ and $\eta\,(\omega_0 R/c) \to 1$. Thus in the point particle limit the energy in (18) reduces to the expression (1). It may be noted that both mass correction and charge correction are derived from the common origin zero-point field.

3 Estimation of the anomalous magnetic moment

Dirac theory of the electron predicts the magnetic moment of the electron $g = 2$. However, a small deviation of magnetic moment $a_e = (g - 2)/2$ is known as the anomalous magnetic moment and it was discovered by Kusch and Foley [23]. The quantization of electromagnetic field led to quantum electrodynamics and the first theoretical calculation of a_e in the purview of quantum electrodynamics was due to Schwinger [24] and it was estimated to be $a_e = \alpha/2\pi$. The quantum electrodynamics theoretical calculations of a_e almost over fifty years by several authors showed an excellent agreement between theory and experiment and an extensive review of a_e was given by Kinoshita [25]. High precession penning trap measurements of a_e were done by several authors and a recent measurement of a_e was given by Henneke *et al.* [26], $a_e(exp) = 1.15965218073(28)\times 10^{-3}$. In this section we shall explore an entirely different classical approach for the calculation of a_e.

Any change in the mass of the particle due to particle motion in the fluctuating zero-point field brings a change in the spin angular frequency in (12).

$$(m + \Delta m)c^2 = \Omega \cdot S\,. \quad (20)$$

Combining (12) and (20) gives the ratio

$$\frac{\Delta m}{m} = \left|\frac{\Omega - \Omega_s}{\Omega_s}\right|. \quad (21)$$

The ratio of change in spin frequency to the spin frequency represents the anomalous magnetic moment. In an alternative way, this can be arrived by considering the energy term $(ge\mathbf{B}/2mc)\cdot S$ and identifying m as the theoretical mass and replacing $m_{th} = m_{obs} - \Delta m$. To a first approximation we get $g/2\,(1 + \Delta m/m)$ in place of $g/2$. Now, from (4) the anomalous magnetic moment of the electron can be expressed in the form

$$a_e = \frac{\Delta m}{m} = \frac{\alpha}{2\pi} + \frac{\alpha}{2\pi}\beta^2\,. \quad (22)$$

The first term on right of the above equation gives the well known Schwinger's result and to obtain this result we have chosen $\eta(\omega_c) \sim 3/4$ in (18). The velocity of an orbital electron in an atom is proportional to α. For a linear motion of the electron we approximate $\beta^2 = \alpha^2/3$ and to account for two modes of polarization of zero-point field, it is multiplied by 2. The reduced velocity is now written in the form $\beta^2 = 2\alpha^2/3$. Substituting this result in (22) and using charge correction relation $\alpha \to \alpha\,(1 - 2\alpha/3\pi)$ gives finally the anomalous magnetic moment of the electron as a function of fine-structure constant:

$$\begin{aligned} a_e = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) - \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 + \frac{\pi^2}{3}\left(\frac{\alpha}{\pi}\right)^3 - \frac{2\pi^2}{3}\left(\frac{\alpha}{\pi}\right)^4 + \\ + \frac{4\pi^2}{9}\left(\frac{\alpha}{\pi}\right)^5 - \frac{8\pi^2}{81}\left(\frac{\alpha}{\pi}\right)^6. \end{aligned} \quad (23)$$

The calculation of a_e is performed using the CODATA recommended fine-structure constant $\alpha = 7.2973525376(50)\times 10^{-3}$ [27] and from (23) the value is estimated to be $a_e(th) = 1.15965227095\times 10^{-3}$. Though this classical estimate is not at par with the quantum electrodynamics calculations, the difference $a_e(th) - a_e(exp) = 90.22\times 10^{-12}$ shows the result is at least accurate up to ninth decimal place. With proper approximation to the reduced velocity, equation (22) may be used for finding the anomalous magnetic moment of any other lepton.

4 Lamb shift

Relativistic theory of a bound electron predicts that the energy levels $2S_{1/2}$ and $2P_{1/2}$ are degenerate. However, the energy level shift $2S_{1/2} - 2P_{1/2}$ was experimentally found to be 1058.27 + 1.0 MHz in 1947 by Lamb and Rutherford [28]. For the Lamb shift calculation, we consider the average deviation in the path of orbital electron is equal to twice the radius of rotation (diameter) of the extended electron. Thus the orbital radius spreads out over a distance 2ξ and the corresponding change in the Coulomb potential is expressed in the form $V(\mathbf{r} + 2\mathbf{i}\xi)$. Expanding this function in terms of Taylor series gives

$$V(\mathbf{r} + 2\mathbf{i}\xi) - V(\mathbf{r}) = 2\mathbf{i}\xi \,\frac{\partial V(\mathbf{r})}{\partial \mathbf{r}} - 2\xi^2 \left(\frac{\partial^2 V(\mathbf{r})}{\partial r^2}\right) + \ldots \quad (24)$$

In the Welton's approach of Lamb shift calculation [29], considering the symmetric potential, an additional multiplying factor 1/3 was introduced in the second term on right of (24). Since the deviation is considered as a bivector which represents rotation in local space, any such factor is not required in the present calculation. The radius of rotation is a vector in the spin plane and therefore, it can be expressed in the form $\xi = |\xi| \exp(-\mathbf{i}\sigma_s \omega_0 t)$, where $\mathbf{i}\sigma_s$ is a unit bivector in the spin plane [14]. Then, the stochastic average $\langle\xi\rangle = 0$ and the average of square of radius of rotation $\langle\xi^2\rangle = \langle|\xi|^2\rangle/2$. Using the relation $\dot{\xi} = -\mathbf{i}\sigma_s\omega_0\xi$, we find $\langle|\xi^2|\rangle = \langle|\dot{\xi}|^2\rangle/\omega_0^2$. Now, taking the stochastic average on both sides of (24), we obtain the stochastic average of change in the potential energy:

$$\Delta E_L = \langle V(\mathbf{r} + 2\mathbf{i}\xi) - V(\mathbf{r})\rangle = \frac{\langle|\dot{\xi}^2|\rangle}{\omega_0^2} \left|\frac{\partial^2 V(\mathbf{r})}{\partial r^2}\right| \quad (25)$$

where the higher order terms are neglected. The energy in (25) corresponds to the Lamb shift in the energy levels due to the interaction of the electron with the zero-point field. We consider that the zero-point field around the atom is modified due to the extended electron in the orbit and as a consequence the electron orbit spreads out around the Coulomb source. Since the modifications in the zero-point field takes place at the atomic size, we choose the cut-off frequency equal to the de Broglie frequency ω_B. Such low frequency cut-off was not considered previously and this may be one of the reasons for not finding the exact estimate of the Lamb shift in stochastic electrodynamics. Considering the equation of motion of center of charge $\ddot{\xi} = eE_{zp}(\xi, t)/m$ and using the same method of derivation given in Section 3, and imposing the upper cut-off frequency ω_B, we obtain the zero-point energy associated with the orbital electron shift in the form

$$m\langle|\dot{\xi}|^2\rangle = \frac{2\alpha}{3\pi}\,\frac{\hbar^2\omega_B^2}{mc^2}\,\eta(\omega_B) \times \left[1 + \left\{\frac{2}{\omega_B^2\tau^2}(1 - \cos\omega_B\tau + \omega_B\tau\sin\omega_B\tau)\right\}\right]. \quad (26)$$

Since $\omega_B\tau \ll 1$,we neglect the terms in curly brackets and the converging form factor $\eta(\omega_B) = 1$. Now, (26) can be expressed in the form

$$\langle|\dot{\xi}|^2\rangle = \frac{2\alpha}{3\pi}\,\frac{\hbar^2\omega_B^2}{m^2c^2}\,. \quad (27)$$

Substituting this result in (25) and using the relation $\hbar\omega_0 = mc^2$ gives

$$\Delta E_L = \frac{2\alpha}{3\pi}\,\frac{\omega_B^2 c^2}{\omega_0^4}\left|\frac{\partial^2 V(\mathbf{r})}{\partial r^2}\right|. \quad (28)$$

For an orbital electron in a circular orbit, the magnitude of Coulomb potential is equal to twice the kinetic energy of the electron:

$$V(r) = \frac{Ze^2}{r} = m_r v^2 = m_r\omega_B^2 r^2\,. \quad (29)$$

Differentiating (29) twice with respect to r yields

$$\left|\frac{\partial^2 V(\mathbf{r})}{\partial r^2}\right| = m_r\omega_B^2\,. \quad (30)$$

Substituting the above result in (28) gives

$$\Delta E_L = \frac{4\alpha}{3\pi}\,\beta^4 m_r c^2\,. \quad (31)$$

Considering $\beta^2 = \alpha^2$, we finally arrive at the required energy shift given in (5). The charge correction of a free electron is given in (2) and in the case of an atomic electron it may be expected that it is three times that of the free electron. Then the correction for the fine-structure constant is $2\alpha/\pi$. Further, one may consider the mass correction of the reduced mass, same as $\alpha/2\pi$. Using these corrections in (5) and substituting the CODATA values of the electron mass, proton mass and other fundamental constants [27], the calculated Lamb shift in hydrogen spectra is found to be 1058.3696 MHz. Thus the present calculation is considerably in agreement with the standard value of Lamb shift 1057.8439 MHz [27] and the difference 0.5257 MHz may be attributed to the finite size of the proton.

In the quantum electrodynamics treatment, normally the expectation value of $|\nabla^2 V(r)|$ is found to be $\langle|\nabla^2 V(r)|\rangle \propto \alpha^4$ [30], the upper bound of integration is chosen to be ω_0 and the integration yields a logarithmic term. Comparing (5) with the Welton's result given by [31]

$$\Delta E_n = \frac{4\alpha^5}{3\pi}\,\frac{Z^4}{n^3}\,\ln\left(\frac{2}{16.55\alpha^2}\right) m_r c^2\,, \quad (32)$$

we get the correct order of fine-structure constant. The logarithmic term $\ln\left(2/16.55\alpha^2\right) \sim 8$ and one can approximate $\beta^4 = 8(Z\alpha)^4/n^3$. Then, if one wishes to include the principal quantum number, (5) may be rewritten in the form

$$\Delta E_L = \frac{4\alpha}{3\pi}\,\frac{8(Z\alpha)^4}{n^3}\,m_r c^2\,. \quad (33)$$

It may be noted that a complete relativistic quantum electrodynamics evaluation is free from high energy cut-off. However, the above calculation of Lamb shift is entirely different from the quantum electrodynamics treatment, where we consider radiative corrections, and the present calculation is purely based on classical considerations along with the extended structure of the electron.

5 Conclusions

Consideration of extended structure of the electron in zero-point field yields a classical, straightforward and simple approach to find mass and charge corrections. We find the mass correction depends on the particle velocity. The orbital electron reduced velocity is assumed to be proportional to the fine-structure constant. The anomalous magnetic moment of the electron has been expressed as a function of fine-structure constant and the estimated $a_e(th)$ value is found to be correct up to ninth decimal place. Using a low frequency cut-off equal to the de Broglie frequency, the Lamb shift of an extended electron in stochastic electrodynamics is derived and the estimated result deviates from the experimental value by 0.5257 MHz. The theory presented elucidates a classical approach to both anomalous magnetic moment of the electron and Lamb shift and paves the way for further research.

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