A Solution to the Flyby Anomaly Riddle

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The Flyby Anomaly is one of the unsolved problems of current physics in that the Doppler-shift determined speeds are inconsistent with expected values assuming the validity of Newtonian gravity. We postulate that the Flyby Anomaly is a consequence of the assumption that the speed of light is isotropic in all frames, and invariant in the method used to measure the velocity of the space probes by means of the Doppler Effect. The inconsistent anomalous values measured: positive, null or negative are simply explained relaxing this assumption. During space probe energy assistance maneuvers the velocity components of the probe in the direction of the observer V_o are derived from the relative displacement Δf of the radiofrequency f transmitted by the probe, multiplied by the local speed of the light c' by the Doppler effect: $V_o = (\Delta f / f)\, c'$. According to the Céspedes-Curé hypothesis, the movement through variable gravitational energy density fields produces slight variations of the refractive index n' of space and therefore of the speed of light c' which leads to unaccounted corrections of the Doppler data that are based on an invariant c. This leads to incorrect estimates of the speed or energy change in the flyby maneuver in the Earth's frame of reference. The simple theory presented is applied to hyperbolic flyby trajectories of Galileo I and the spacecraft NEAR accurately reproducing the NASA measured values and thereby providing additional experimental evidence for a variable speed of light dependence on the gravitational energy density of space with fundamental consequences in astrophysics and cosmology.

1 Introduction

The Flyby Anomaly is an unexpected energy increase or decrease of spacecraft during flybys maneuvers of Earth and other planets employed as gravitational assist techniques for Solar system exploration. The anomalous measurements have been observed as shifts in the S-band and X-band Doppler and ranging telemetry. It has been observed in a number of spacecraft: NEAR, Galileo I and II, Cassini, Rosetta I, II and III, Messenger, Juno, Hayabusa, and EPOXI I and II [1–3]. The Flyby Anomaly has been included in a list of "unsolved problems in physics". We find very significant a comment of Anderson et al. [2], that the same inconsistency in the Doppler residuals which lead to the velocity anomaly are found in the ranging data, as we believe both can be explained by the theory developed here.

A large number of papers have been advanced in attempts to explain the anomalous, and at times inconsistent, measurement results of the very small, but significant, unaccounted speed and energy change experienced by spacecraft during maneuvers to increase or decrease its relative energy.

A comprehensive review of anomalous phenomena observed in the solar system was published by Lämmerzahl et al. (2006) [4] which includes prominently the Flyby Anomaly. It lists numerous possible causes of the anomaly. It reaches the conclusion, in this respect, that none of them can explain the observed measurements. "New physics" has been attempted by postulating variants of gravitational theories [5–9], or modification of inertia [10], and also the possible influence of halos of dark matter [11].

More conventional causes that have been considered include: The effect of Earth oblateness which is known to produce perturbations of orbiting spacecraft. Hence a possible cause of the Flyby Anomaly might be the non spherical mass distribution of the oblate Earth. An unsuccessful attempt has been made by K. Wilhelm and B.N. Dwivedi (2015) [12] to explain the anomalous Earth flybys of several spacecraft on the basis of asymmetry of the mass distribution of the Earth causing an offset of the effective gravitational centre from the geometric centre.

The possibility of electromagnetic forces acting between a charged probe and the Earth's magnetic fields has been examined [13], also the influence of the Earth high atmosphere [14] or the emission of thermal energy from the spacecraft [15]. However, to this date none of the above adequately explains the cause of the anomaly.

A light speed anisotropy hypothesis is used by R.T. Cahill to argue that the Doppler-shift determined speeds are inconsistent with expected speeds, and hence affect the measurement of the probe during flyby [16]. Cahill revisits the Michelson-Morley experiment controversy citing numerous new interferometer results which take into account the effect if the medium that light transverses in these experiments (e. g. gas, coaxial cable or optical fiber). He points out that speed anomalies are not real and are actually the result of using an incorrect isotropic light speed relationship between the observed Doppler shift and the speed of the spacecraft.

An empirical formula that adequately predicts the flybys measured up to 2005 was published by Anderson et al. [1, 2] using all likely variables in the problem. The empirical formula developed by Anderson et al. did not fit later anomalous flybys. However, a modification by Jouannic et al. (2015) [3] was able to predict the new data. From the conclusions of this work we read that "This could signify that it (*the anomaly*) is caused by a force related either to mass, altitude, or both". In this paper we show that indeed, planet mass and distance from the planet, which are some of the important variables in determining the gravitational energy density of space and hence of the local index of refraction of quasi-empty space [17, 18] produces minute variations in the local speed of light c' due to the Céspedes-Curé hypothesis [19], explained below. These unaccounted variations of the local index of refraction lead to small erroneous measurements of spacecraft velocity and derived energy, based on a constant c, and is shown here to be the cause of the Flyby Anomaly. Hence we coincide with Cahill in that speed anomalies are not real but rather an artifact of how the speeds are measured with the Doppler effect. In this paper the fundamentals of the proposed Flyby Anomaly explanation are presented with analytical relations showing how the anomalous behavior can be accurately predicted. Numerical calculations are presented for the Galileo I (December, 1990) Earth flyby and NEAR (January, 1998) Earth flyby. We also show how the anomaly can be simply predicted for any other spacecraft provided detailed information of the measurement of entry and exit points are available. Additionally we briefly discuss some of the fundamental consequences of the Céspedes-Curé hypothesis for astrophysics and cosmology.

2 Speed and energy measurement of spacecraft and the Doppler effect

All remote velocity estimations of astronomical bodies use the first order Doppler effect of light [20]. In spacecraft the procedure employs a locally produced radio or light frequency f of accurately known value, or it could be a retransmitted signal such as the case of Pioneer spacecraft [21]. The speed component in the direction of the observer V_o is deduced from the shift Δf of the radio or light frequency f, times the local speed of light c' by means of $V_o = (\Delta f/f)\, c'$. At the present time (year 2020) it is conventionally assumed that the local speed of light c' at any point in the universe is isotropic and identical to the speed of light $c = 299792458\ \mathrm{ms}^{-1}$ measured in vacuum to high accuracy on the surface of the Earth. Clearly, if there are small variations of c' as a result of changing locations with differing gravitational energy density ρ, as occurs during flyby maneuvers, the measured speed component in the direction of the observer V_o, calculated with the Doppler effect, assuming a constant c, will lead to erroneous estimations of the spacecraft speed and resulting energy change during the maneuver. Presently the speed of light c is considered a fundamental constant being the base of the definition of the meter, the length unit in the SI system of units. However, a variable speed of light has been considered by a number of authors, notably including A. Einstein in 1907 [22] and in 1911 [23] and also by R. Dicke in 1957 [24]. In Einstein's early work the speed of light was influenced by the gravitational potential and a constant speed could not be conceived in a gravitational field with variable strength. In Dicke's work he assumes a refractive index n of empty space, different from 1, given by an expression where the value increases with the gravitational field:

$$n = 1 + \frac{GM}{r\,c^2}\,.$$

This proposal provides an alternative to the lensing phenomenon predicted by General Relativity Theory (GRT). There are other more modern variable speed of light theories as reviewed by Magueijo J. in 2003 [25]. The Céspedes-Curé hypothesis [19] is reminiscent of the early proposals of Einstein and Dicke. It predicts that the speed of light is a function of the local total energy density of space ρ according to (1), so that if this hypothesis is correct, it could explain the spacecraft anomalous behavior derived by the Doppler effect.

$$c = \frac{k}{\sqrt{\rho}}\,, \tag{1}$$

where k is a proportionality constant and ρ is the sum of all the sources of energy density including gravitational, ρ_G, electric, ρ_E, magnetic, ρ_M, and any other that may be acting at the site. Calculations [26] show that gravitational energy density is much larger than electric or magnetic. And that the most important source of energy density by several orders of magnitude is the "Cosmic energy density" due to the far away stars and galaxies which has a value of $\rho^* = 1.094291 \times 10^{15}\ \mathrm{Jm}^{-3}$ deduced by Céspedes-Curé [19], see Appendix A, and by Greaves E.D. [18, 26, 27], see Appendix B. Compared to ρ^*, the Sun's ρ_S, the planet about which the flyby maneuver is being done, ρ_p, and all other massive bodies in the vicinity contribute in a very minor amount to the variable total energy density at points along the trajectory of the spacecraft. Hence, this is the cause of the minute amount found for the anomalous values of velocity and energy of spacecraft performing the flyby maneuver. The gravitational energy density ρ due to a mass M at a distance r from its center is given by [19, see page 163],

$$\rho = \frac{1}{2}\,\frac{GM^2}{4\pi r^4} = \frac{GM^2}{8\pi r^4}\,, \tag{2}$$

where G is the universal constant of gravitation. Using this relation the gravitational energy density of any astronomical mass can be calculated at any point in space located a distance r from the mass center. The energy density of space ρ_B and ρ_E associated with the presence of static magnetic B and electric

E fields are given by [28]:

$$\rho_B = \frac{1}{2\mu_0} B^2 , \tag{2a}$$

and

$$\rho_E = \frac{1}{2}\varepsilon_0 E^2 , \tag{2b}$$

where μ_0 is the magnetic permeability and ε_0 is the electric permittivity of free space. With the usual definition of the index of refraction at a point in space, n', as the ratio of the speed of light of vacuum c on the surface of Earth to the speed of light c' at the point considered (conventionally inside a transparent material) $n' = c/c'$ it is possible with the use of (1) to obtain a relation for n' which is only dependent on values of the energy density of space at the point in question and at the surface of the Earth:

$$n' = \frac{c}{c'} = \frac{\sqrt{\rho'}}{\sqrt{\rho}} = \frac{\sqrt{\rho'}}{\sqrt{\rho^* + \rho_S + \rho_E}} . \tag{3}$$

Here $\rho^* + \rho_S + \rho_E$ is the gravitational energy density at the surface of the Earth. The terms in the sum are: the energy density due to the far away stars and galaxies ρ^*, the Sun, ρ_S and Earth, ρ_E. The values shown in Table 1 and Fig. 1 indicate that the contributions to the local gravitational energy density due to nearby planets is small and negligible compared to the all-pervading energy density ρ^* due to the far away stars and galaxies. Hence for a spacecraft in a flyby maneuver the local value of the index of refraction n' and the local value of the speed of light c' is very nearly equal to the values on the surface of Earth. This leads to the fact that the observed anomalous variations of the speed of spacecraft deduced by the Doppler effect are very small indeed. It also shows that the anomalies are dependent on the mass of the planet and on the distance to the planet as mentioned in the conclusions of the work of Jouannic et al. in [3].

3 Calculation of the anomaly

In order to predict quantitatively the measured energy change that shows an anomalous value it is necessary to have very detailed information of the particular flyby event considered. The information required is data that refers to the spacecraft such as the radio frequencies used for transmission which are used for determining the relative radial velocity via the Doppler effect. The information related to the planet, about which the maneuver takes place, is information that defines the orbit of the spacecraft: the hyperbolic orbit parameters of the flyby: a (semi-mayor axis) and e (eccentricity) and the entry and exit velocity of the probe: V_∞^- and V_∞^+, the measured anomalous velocity V_{anom} and, most important, the points of entry and exit where the velocities were measured. NASA determines the Flyby Anomaly with the Orbit Determination Program (ODP) of the Jet Propulsion Laboratory (JPL) as well as other software at the Goddard Space Flight Center and at the University of Texas [2]. These programs incorporate all the physics mentioned above and the information gathered by the Deep Space Network (DSN) during the flyby. According to the hypothesis presented in this paper the anomaly is due to errors committed due to sub-estimation or over-estimation of the velocity calculated by the use of the Doppler effect formula as explained previously. Below we show how the anomaly can be calculated in reference to Earth flybys. The same considerations apply to flybys about other planets. From (3) we derive

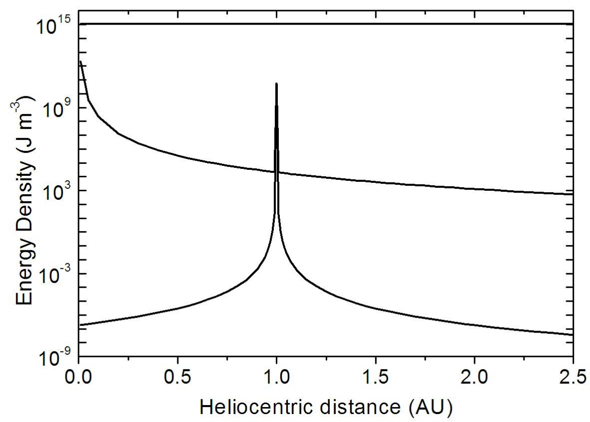

Fig. 1: Gravitational energy density (Jm^{-3}) as a function of distance from the center of the Sun in AU (0 to 2.5 AU) due to the far away stars and galaxies (top line $\rho^* = 1.094291\times10^{15}$ Jm^{-3}), ρ_S due to the Sun (middle line) and ρ_E due to Earth (Line centered at 1 AU) [26].

$$c' = c \, \frac{\sqrt{\rho}}{\sqrt{\rho'}} . \tag{4}$$

The radial velocity of the spacecraft during the flyby is obtained by the use of $V_r = \Delta f/f \, c'$ which with (4) gives

$$V_r = c' \, \frac{\Delta f}{f} = c \, \frac{\Delta f}{f} \, \frac{\sqrt{\rho}}{\sqrt{\rho'}} , \tag{5}$$

where the gravitational energy density ρ' is a function of the position of the spacecraft in its orbit and ρ is the gravitational energy density on the surface of the Earth whose value is $\rho = \rho^* + \rho_S + \rho_E$ with ρ_S and ρ_E calculated on the surface of Earth. As the spacecraft nears the planet it moves into varying values of ρ' which according to (5) results in a sub-estimation or over-estimation of the velocity. Likewise, as the spacecraft leaves the vicinity of Earth and gets further away, it travels into different values of the gravitational energy density ρ' which according to (5) results in differing values of the velocity. Important factors determining the value of ρ' are the radial distance to the center of the planet producing the energy

Table 1: Values of the energy density of space at the surface of Earth produced by: the far away stars and galaxies, the mass of the Sun, Earth, the Moon and other planets.

Source of energy density	Symbol	Energy density due to source at	Magnitude (Joules/m^3)‡	Reference
Far away Stars and Galaxies	ρ^*	Earth	1.094291×10^{15}	Céspedes-Curé [19, p. 279]
Sun	ρ_S	Earth 1 AU	2.097×10^4	Greaves [17, 18]
Sun	$\rho_{S@AU-}$	1 AU$-E_{\mathrm{SI}}$†	2.150250×10^4	This work
Sun	$\rho_{S@AU+}$	1 AU$+E_{\mathrm{SI}}$†	2.046034×10^4	This work
Earth	ρ_E	Earth surface	5.726×10^{10}	Greaves [18]
Moon	ρ_{Moon}	Earth	6.57×10^{-1}	Greaves [18]
Jupiter	ρ_{Jup}	Earth	1.91×10^{-2}	Greaves [18]
Venus	ρ_{Ven}	Earth	2.14×10^{-5}	Greaves [18]
Mars	ρ_{Mar}	Earth	2.91×10^{-8}	Greaves [18]

† E_{SI} is the radius of Earth's Gravitational Sphere of Influence: (929000 km) [29, 30].
‡ These values are deceptive due to the $1/r^4$ dependence of the gravitational energy density (2). The energy density of the Earth at its surface is 6 orders of magnitude greater than the Sun's. However, it decreases abruptly so that at a distance greater than 41 earth radii the energy density due to the Sun is higher.

assistance and the radial distance to the Sun. Hence, in order to calculate exactly the anomalous energy change reported, it is necessary to know the exact position of the spacecraft at the point or points where its velocity was calculated in order to establish the initial spacecraft energy and the point or points where the velocity was finally calculated to establish the final spacecraft energy. Also needed are the methods used for the speed measurements such as the frequency used by the spacecraft in its transmission to the Earth tracking stations, and whether it is a spacecraft transmission or an Earth sent-signal retransmitted by the spacecraft. Such detailed information is ordinarily not included in papers publicly available.

Examination of (5) shows that the anomaly is caused by the square root term (SQR)

$$\mathrm{SQR} = \sqrt{\frac{\rho}{\rho'}} = \sqrt{\frac{\rho}{\rho^* + \rho_S + \rho_E}}\,. \tag{6}$$

Here ρ and ρ^* are constants while ρ_S and ρ_E are functions of position, ρ_S is dependent on the radial distance to the center of the Sun and ρ_E is dependent on the radial distance to the center of Earth.

Let us consider ρ_S first, which is given by

$$\rho_S = \frac{GM_S^2}{8\pi r_S^4}\,. \tag{7}$$

Here M_S is the mass of the Sun and r_S the radial distance from the center of the Sun. In order to estimate the influence of this term we calculate the value of ρ_S over the Earth's gravitational Sphere of Influence, E_{SI}, that is at a distance of one AU from the Sun in the range of 1 AU $\pm E_{\mathrm{SI}}$ (plus or minus the radius of the Earth's Sphere of Influence). The values obtained range from $\rho_S = 2.150250{\times}10^4$ to $2.046034{\times}10^4$ Jm^{-3} as shown in Table 1. The variation over the Earth's sphere of influence is of the order of 5%. However, the values of the variation of the gravitational energy density due to the Sun are 5 orders of magnitude less than the energy density due to Earth at its surface. But, as shown by calculations, they become more important than the Earth's energy density due to the $1/r^4$ term in (2) as discussed below.

In (6), the value of ρ_E is given by

$$\rho_E = \frac{GM_E^2}{8\pi r_E^4} \tag{8}$$

with M_E the mass of the Earth and r_E the radial distance from the center of Earth.

Taking these considerations into account in (5) we can write an expression for the corrected speed of the spacecraft which takes into account the change of the index of refraction of space due to the variation of the space gravitational energy density along the spacecraft trajectory:

$$\begin{aligned} V_r &= c\,\frac{\Delta f}{f}\sqrt{\frac{\rho}{\rho^* + \rho_S + \rho_E}} \\ &= c\,\frac{\Delta f}{f}\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi r_S^4} + \frac{GM_E^2}{8\pi r_E^4}}}\,. \end{aligned} \tag{9}$$

Numerical calculations show that the influence of the third term of the denominator, namely the variation of the Earth's gravitational energy density is important only at small distances above the surface of the Earth and it becomes very small at distances where a spacecraft is beginning its approach to the surface of the planet during a flyby.

4 Calculation of the Flyby Anomaly in three cases

To calculate the anomaly, we suppose that the speed of the spacecraft is measured at two points: a point of entry into the Earth's sphere of influence where the speed is V_∞^- and a point of exit from the Earth's sphere of influence where the speed is V_∞^+. If we ignore the change of c, the measured velocities are given by:

$$V_\infty^+ = c\,\frac{\Delta f^+}{f} \quad \text{and} \quad V_\infty^- = c\,\frac{\Delta f^-}{f}.$$

Hence the anomaly measured by NASA is given by

$$\text{An} = V_\infty^+ - V_\infty^- = \frac{c}{f}\left(\Delta f^+ - \Delta f^-\right). \tag{10}$$

At each of these points a correct measurement, one that takes into account the change of the index of refraction, as we propose in this paper, must be done with (9), with V_∞^- the observed Doppler shift at the point of entry, and with V_∞^+ the observed Doppler shift at the point of exit as shown below:

$$V_\infty^+ = c\,\frac{\Delta f^+}{f}\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi\left(r_S^+\right)^4} + \frac{GM_E^2}{8\pi\left(r_E^+\right)^4}}} \tag{11a}$$

$$V_\infty^- = c\,\frac{\Delta f^-}{f}\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi\left(r_S^-\right)^4} + \frac{GM_E^2}{8\pi\left(r_E^-\right)^4}}} \tag{11b}$$

In the Earth's coordinate system, energy is conserved, so that if the correct equations (11a) and (11b) are used, then measurements *should* give: $V_\infty^+ - V_\infty^- = 0$ that is:

$$\begin{aligned} 0 = c\,\frac{\Delta f^+}{f}\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi\left(r_S^+\right)^4} + \frac{GM_E^2}{8\pi\left(r_E^+\right)^4}}} \; - \\ - c\,\frac{\Delta f^-}{f}\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi\left(r_S^-\right)^4} + \frac{GM_E^2}{8\pi\left(r_E^-\right)^4}}}\,. \end{aligned} \tag{12}$$

However, if the SQR terms are different, for (12) to be true it requires that $\Delta f^+ \neq \Delta f^-$, and hence measurements done by NASA with (10) will show an anomaly. The anomaly is contained in the difference of the SQR terms in (12). Since

$$V_\infty^+ = c\,\frac{\Delta f^+}{f} \quad \text{and} \quad V_\infty^- = c\,\frac{\Delta f^-}{f}$$

are almost the same, both of the order of km/s differing by an amount 6 orders of magnitude smaller, of the order of mm/s, we can write the following relation to calculate the measured anomaly:

$$\begin{aligned} V_{\text{anom}} = V_\infty\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi\left(r_S^+\right)^4} + \frac{GM_E^2}{8\pi\left(r_E^+\right)^4}}} \; - \\ - V_\infty\sqrt{\frac{\rho}{\rho^* + \frac{GM_S^2}{8\pi\left(r_S^-\right)^4} + \frac{GM_E^2}{8\pi\left(r_E^-\right)^4}}}\,. \end{aligned} \tag{13}$$

Numerical analysis of (13) shows it is possible to identify three cases.

4.1 First case

The distances from the point of entry and the point of exit to the Sun and to Earth are the same. ($r_S^+ = r_S^-$ and $r_E^+ = r_E^-$). In this case the two terms in the parenthesis of (13) are the same and no anomaly will be detected (incoming and outgoing points are symmetric with respect to the Sun and Earth).

4.2 Second case

In this second case entry point and the exit point are at different distances from the Sun but at the same distance from Earth. It means that $r_S^+ \neq r_S^-$, hence:

$$\frac{GM_S^2}{8\pi(r_S^+)^4} \neq \frac{GM_S^2}{8\pi(r_S^-)^4}\,,$$

so that the SQR terms in (12) are different. For this relation to be correct it requires that $\Delta f^+ \neq \Delta f^-$. Hence if the speeds are being measured with relations

$$V_\infty^+ = c\,\frac{\Delta f^+}{f} \quad \text{and} \quad V_\infty^- = c\,\frac{\Delta f^-}{f}$$

as in (10) the flyby will certainly show an anomaly: $V_\infty^+ \neq V_\infty^-$. However, numerical calculations show that the anomalous values in this case are very small and non measurable.

4.3 Third case

In this third case entry point and the exit point are at different distances from the Sun and at different distance from Earth. It means that, $r_S^+ \neq r_S^-$ and $r_E^+ \neq r_E^-$. In this case the two terms in the parenthesis of (13) are different. Hence if the speeds are being measured with relations

$$V_\infty^+ = c\,\frac{\Delta f^+}{f} \quad \text{and} \quad V_\infty^- = c\,\frac{\Delta f^-}{f}$$

as in (10) the flyby will certainly show an anomaly: $V_\infty^+ \neq V_\infty^-$. Numerical calculations show that an anomaly will be measured in the range of values reported, negative or positive, with a value and sign that depends on the entry and exit points used for measurement. We conclude that the anomaly is due to neglect of the SQR terms in the calculation of the entry and exit velocities derived from the Doppler flyby data.

Table 2: Distances to the Sun and to Earth with calculated entry and exit points that predict, with (13), the measured Flyby Anomaly of the Galileo 1 (December 1990) flyby and the NEAR (January 1998) flyby.

	Galileo 1		NEAR	
	Entry point	Exit point	Entry point	Exit point
Distance from Sun (m)	1.502803×10^{11}	1.502831×10^{11}	1.495630×10^{11}	1.495950×10^{11}
Distance from Earth (m)	1.7651×10^{7}	1.4864×10^{7}	7.2000×10^{7}	1.2200×10^{7}
Spacecraft Velocity (m/s)	8949		6851	
Measured Flyby Anomaly (mm/s)	3.930		13.46	
Calculated Flyby Anomaly (mm/s)	3.944		13.38	
Difference (%)	+0.40		−0.57	

5 Results

In order to apply the theory described above to predict the anomaly measured for any given spacecraft flyby it is necessary to introduce into (13) the values of the parameters of the spacecraft maneuver, namely the spacecraft speed at the entry point and the distances to the Sun and to Earth of the incoming and outgoing points. The spacecraft speed is available, however, the required information of entry and exit points has not been possible to obtain. Only the right ascension and declination of these vector directions are given by Anderson et al. [2]. With these angular parameters we have defined vectors, from the Earth, for incoming and outgoing directions as well as from the Earth to the Sun's direction along its right ascension and declination on the day of the Flyby. Then with calculated tables of numerical values of the SQR terms of (13) for varying entry and exit points along the incoming and outgoing vectors (i.e. values of r_S^+, r_S^- and of r_E^+, r_E^-) excluding the immediate distances (1h 40min before and after the closest approach location) we have arrived at likely entry and exit points that closely predict the observed NEAR (January 23, 1998) flyby. For Galileo I (December 8, 1990) flyby the incoming and outgoing points were calculated along likely in and out points not specifically along the actual incoming and outgoing vectors. Results of these calculations are shown in Table 2.

6 Possible measurement of ρ^* with the Flyby Anomaly

Based on the Flyby Anomaly explanation given above, it is possible to use the experimental results of measured flyby anomalies in spacecraft to calculate, in an independent way, the gravitational energy density values that lead to the measured anomalies. Since the gravitational energy density is composed of the contribution due to the planets and the Sun, which can be accurately calculated with (8), the contribution due to the far away stars and galaxies, ρ^*, could be solved as a single adjustable parameter, and calculated. This could be done by programming the theory presented here in the Orbit Determination Program of the JPL, or by an accurate knowledge of the points of entry and exit in the hyperbolic trajectory where the measurements were made that produced a Flyby Anomaly. This measurement of ρ^*, the gravitational energy density of the far away stars and galaxies, would provide an additional estimation of its value besides that given by Jorge Céspedes-Curé [19, page 279], $\rho^* = 1.094291 \times 10^{15}$ Jm^{-3}, obtained using starlight deflection measurements during total sun eclipses, see Appendix A, or that given by Greaves [26]: $\rho^* = 1.0838{\times}10^{15}$ Jm^{-3}, obtained using NASA accurate measurement of the Pioneer Anomaly when Pioneer 10 was at 20 AU, see Appendix B.

7 Discussion

Eq. (2) assumes a spherical mass distribution for the mass of the Earth or Sun in the calculation of the gravitational energy density. It does not consider the possible influence of the Earth's oblate shape, which is known to affect orbiting spacecraft and could affect hyperbolic orbits.

Estimation has been done of the magnitude of the mass of Earth that deviates from spherical shape in order to calculate to what extent this can affect the gravitational energy density along the Flyby Anomaly trajectory. The calculation gives that the non spherical mass is of the order of less than 0.337% of the Earth mass. This amount influences the third term of the denominator in (9) and quantities derived from it. However, the subtraction or addition of this mass to the mass of Earth on the SQR term of (9) affects this term in less than the tenth significant figure. This estimate implies that the mass of Earth causing the gravitational quadrupole does not affect the calculations based on the Céspedes-Curé hypothesis.

The hypothesis also predicts that ranging measurements based on a constant value of c will be affected in the same manner as the anomalous speed measurements based on the Doppler data. Anomalous ranging is briefly mentioned by Anderson et al. [2]. However, no numerical data of this anomaly has been provided. Perhaps due to the small signal-to-noise ratio on the incoming ranging signal and a long integration time (typically minutes) that must be used for correlation purposes [21, page 7].

We calculate the speed of light at the International Space

Station to be

$$c' = 299798845.6\,\mathrm{ms}^{-1},$$

that is 6387.6 ms^{-1} higher than c on the Earth's surface, about 0.002% [31]. Ranging measurements based on a constant c that is lower than is predicted by this theory will be in slight error. And the error will be in the same manner as the anomalous speed measurements. The Cépedes-Curé hypothesis predicts the anomalous measurements of the Pioneer spacecraft without any adjustable parameter [27]. There are reports that that the Pioneer Anomaly was resolved as a thermal effect on papers by Rievers and Lammerzahl [15], Turyshev et al. [32] and Francisco et al. [33]. These reports do complex parameterized models of the thermal recoil to explain the anomaly.

We have reasons to doubt this explanation:

First. A detailed paper about the Pioneer Anomaly (55 pages in Phys. Rev. by Anderson et al. 2002) [21] clearly argues (see sections VIII. B, C and D, pages 32–35) that thermal recoil cannot account for the anomaly,

Second. Rievers and Lämmerzahl [15] do a very complex computational model of the spacecraft constructing all parts of the spacecraft internal and external in finite elements; assigning thermal, and radiative properties for each component, (absorption, reflection and emittance coefficients) in order to arrive at their resulting thermal radiation pressure.

Turyshev et al. [32] do a complex parameterized model for the thermal recoil force of the Pioneer spacecraft with several adjustable parameters. In particular the two adjustable parameters of Eq. (1) on page 2 predict the anomaly. However, any other parameters would negate the thermal origin of the anomaly.

Francisco et al. [33] use different modeling scenarios resulting in different acceleration values and choosing the 4th one with which a Monte Carlo modeling procedure is used to arrive at a value of the reported acceleration of the Pioneer 10 at an instant 26 years after launch.

All of these reports imply models with numerous adjustable parameters which could disprove the thermal origin of the anomaly.

Third. If the anomalous acceleration towards the sun depended on the thermal emission of heat from the RTG, Plutonium ^{238}Pu power sources, with a half life time of 87.74 years, the anomalous acceleration should decrease in time at the same rate, however, this is contrary to the almost flat long term behavior observed [21].

Forth. An anomaly similar to the Pioneer spacecraft was detected in Galileo spacecraft (see Section V. C, page 21) with a value of (acceleration) of $(8 \pm 3) \times 10^{-8}$ cm/s^2, a value similar to that from Pioneer 10, with additional evidence based on ranging data, and in the Ulysses spacecraft (see Section V. D, page 21) Ulysses was subjected to an unmodelled acceleration towards the Sun of $(12 \pm 3) \times 10^{-8}$ cm/s^2, in Anderson et al. [21]. Both spacecraft have completely different geometries and the thermal recoil theory is not applicable to them.

There are some unexplored fundamental aspects to the Céspedes-Curé hypothesis. The elementary relation (4) that is deduced for the relative speed of light c' measured on a space site relative to c on Earth, coupled to Einstein's relation for the rest mass $E = mc^2$ leads to an analytical relation that predicts Mach's principle, i.e. that mass and inertia depend on the far away stars and galaxies. Likewise, the Céspedes-Curé Hypothesis coupled to the electromagnetic expression for the speed of light, $c = 1/\sqrt{\varepsilon_0 \mu_0}$ leads to a direct relationship between the electromagnetic and gravitational forces.

8 Conclusions

The values shown in Table 2 indicate that the Flyby Anomaly can be accurately predicted by the theory presented in this work. This theory is capable of explaining qualitatively and quantitatively the anomaly, both, the measured positive, null and negative values. To calculate exact values of the anomaly of a spacecraft it is necessary to know the incoming and outgoing points where the spacecraft velocity was measured. The precise calculation of the Flyby Anomaly provides additional confirmation of the Céspedes-Curé hypothesis, that c the speed of light depends on the gravitational energy density of space as defined by (1) namely:

$$c' = \frac{k}{\sqrt{\rho'}}\,.$$

The evidence presented in this work for the Céspedes-Curé hypothesis has profound consequences in the current cosmology theories since it implies a revision of all astronomical measurements of velocity based on the Doppler, blue and red shifts, of stars and galaxies. These have importance in determination of matters such as the Hubble constant, the expansion of the universe, the flat rotation curve of galaxies (which gave birth to the theory of dark matter) and the extreme values of the redshifts of very far away galaxies (so called inflation) which gave birth to the theory of dark energy. These redshifts do not follow the linear relation proposed by Hubble but rather seem to imply an accelerated rate of expansion. The theories that follows from this hypothesis, the evidence and attempts to gather evidence for it and some of its consequences on current physics are explored in [18] and in the unpublished work mentioned above in [31].

Appendix A. Supporting data (Céspedes-Curé)

See Table 3: Data of starlight deflection measurements, reported by P. Merat [34] (δ in seconds of arc) at different distances from the Sun during total eclipses, used by J. Céspedes-Curé [19, see page 279], to calculate $\rho^* = 1.094291 \times 10^{15}$ Jm^{-3}, the energy density of space due to far-away stars and galaxies.

Appendix B. Supporting data (Greaves)

Data used by E. D. Greaves in [26] for the arithmetic to calculate $\rho^* = 1.0838 \times 10^{15}$ Jm^{-3}, the energy density of space

Table 3: Data of starlight deflection measurements, reported by P. Merat [34] (δ in seconds of arc) at different distances from the Sun during total eclipses, used by J. Céspedes-Curé [19, see page 279], to calculate $\rho^* = 1.094291 \times 10^{15}$ Jm^{-3}, the energy density of space due to far-away stars and galaxies.

Row	r (R_o Units)	$\delta \pm \Delta\delta$ (Merat)
1	2.09	1.02 ± 0.11
2	3.12	0.67 ± 0.08
3	4.02	0.58 ± 0.04
4	5.10	0.40 ± 0.07
5	6.06	0.41 ± 0.04
6	7.11	0.31 ± 0.04
7	7.84	0.24 ± 0.04
8	9.51	0.20 ± 0.06
9	11.60	0.16 ± 0.03

due to far-away stars and galaxies.

The calculation uses the following equations from [26]:

$$\text{Eq. (8)} \quad \rho^* = \frac{\rho_{\text{Sfar}} + \rho_{\text{Efar}} - n'^2(\rho_{S\,1AU} + \rho_E)}{n'^2 - 1}, \text{ and}$$

$$\text{Eq. (19)} \quad n' = 1 - \frac{E_D\ c}{2 f_e\, G\left(\frac{M_S}{r_S^2} + \frac{M_E}{r_E^2}\right)},$$

where: (numerical values in SI units)

n', index of refraction of space at 20 AU (comes out to 0.999973567943846),

ρ^*, energy density of space due to far-away stars and galaxies,

E_D, a steady frequency drift of 5.99×10^{-9} Hz/s from the Pioneer 10 spacecraft [21, page 20],

f_e = 2295 MHz, the frequency used in the transmission to the pioneer spacecraft [21, page 15],

c = 299792458.0 m/s. Speed of light on Earth at surface,

$G = 6.67300 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}, Newton's universal constant of gravitation,

$M_S = 1.98892 \times 10^{30}$ kg, mass of the Sun,

$M_E = 5.976 \times 10^{24}$ kg, mass of the Earth,

1 Astronomical Unit (AU) = 149 598 000 000 m.

The distances r_S and r_E are the distances from the spacecraft at 20 AU (20 AU from the Sun, 19 from Earth) to the center of the Sun and Earth respectively. To calculate Eq. (8) of [26] use is made of the energy density ρ_i given by Eq. (4) also of [26]:

$$\rho_i = \frac{G M_i^2}{8\pi r^4},$$

where r is the distance from the centre of the Sun or Earth to the point where the energy density is being calculated as follows:

For the Earth's surface: r_E = 63781.40 m, radius of Earth,

For the Sun at 1 AU: r_S = 149598000000 m,

For the Sun at 20 AU: Twenty times the previous value used to calculate ρ_{Sfar},

For the Earth at 20 AU: radius of earth + 19 times 149 598 000 000 m used to calculate ρ_{Efar}.

All values were calculated with Microsoft Office Excel 2003 which uses 15 significant digits of precision.

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