Dirac 4×1 Wavefunction Recast into a 4×4 Type Wavefunction

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As currently understood, the Dirac theory employs a 4×1 type wavefunction. This 4×1 Dirac wavefunction is acted upon by a 4×4 Dirac Hamiltonian operator, in which process, four independent particle solutions result. Insofar as the real physical meaning and distinction of these four solutions, it is not clear what these solutions really mean. We demonstrate herein that these four independent particle solutions can be brought together under a single roof wherein the Dirac wavefunction takes a new form as a 4×4 wavefunction. In this new formation of the Dirac wavefunction, these four particle solutions precipitate into three distinct and mutuality dependent particles (ψ_L, ψ_N, ψ_R) that are eternally bound in the same region of space. Given that quarks are readily found in a mysterious threesome cohabitation-state eternally bound inside the proton and neutron, we make the suggestion that these Dirac particles (ψ_L, ψ_N, ψ_R) might be quarks. For the avoidance of speculation, we do not herein explore this idea further but merely present it as a very interesting idea worthy of further investigation. We however must say that, in the meantime, we are looking further into this very interesting idea, with the hope of making inroads in the immediate future.

I am among those who think that Science has great beauty.

Marie Skłodowska-Curie (1867-1934)

1 Introduction

As currently understood, the Dirac theory [1, 2] employs a 4×1 type wavefunction, ψ. This 4×1 Dirac wavefunction is acted upon by a 4×4 Dirac Hamiltonian operator, H_D, in which process, four independent particle solutions result, i.e. $\psi[1]$, $\psi[2]$, $\psi[3]$, and $\psi[4]$. To this day, insofar as the real physical meaning and distinction of these four solutions, it remains unclear what these solutions really mean. We demonstrate herein that these four independent particle solutions can be brought together under a single roof wherein the Dirac wavefunction takes a new form as a 4×4 wavefunction. To that end, we shall start by introducing the well-known Dirac equation.

That is to say: for a particle whose rest-mass and wavefunction are m_0 and ψ respectively, the corresponding Dirac equation is given by:

$$\imath\hbar\gamma^\mu\partial_\mu\psi = m_0c_0\psi, \tag{1}$$

where: $\hbar = 1.054571817 \times 10^{-34}$ J s (CODATA 2018) is the normalized Planck constant, $c_0 = 299792458 \times 10^8\,\mathrm{m\,s^{-1}}$ (CODATA 2018) is the speed of light in *vacuo*, $\imath = \sqrt{-1}$, and:

$$\gamma^0 = \begin{pmatrix} \mathcal{I}_2 & \emptyset \\ \emptyset & -\mathcal{I}_2 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} \emptyset & \sigma^i \\ -\sigma^i & \emptyset \end{pmatrix}, \tag{2}$$

are the 4×4 Dirac γ-matrices where $\mathcal{I}_2$ and $\emptyset$ are the 2×2 identity and null matrices respectively, and the four component Dirac wave-function, ψ, is defined as follows:

$$\psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \tag{3}$$

is the 4×1 Dirac four component wavefunction and ψ_L and ψ_R are the Dirac [1, 2] bispinors that are defined such that:

$$\psi_L = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}, \text{ and, } \psi_R = \begin{pmatrix} \psi_2 \\ \psi_3 \end{pmatrix}. \tag{4}$$

Throughout this paper, unless otherwise specified, the Greek indices will be understood to mean $(\mu, \nu, ... = 0, 1, 2, 3)$ and the lower case English alphabet indices $(i, j, k ... = 1, 2, 3)$.

The Dirac equation can be recast into the Schrödinger formalism as follows: $H_\mathrm{D}\psi = E\psi$, where $E = -\imath\hbar\partial/\partial t$ is the usual quantum mechanical energy operator, and:

$$H_\mathrm{D} = \imath\hbar c_0\gamma^j\frac{\partial}{\partial x^j} - \gamma^0 m_0 c_0^2, \tag{5}$$

is the Dirac Hamiltonian operator. In §4, we shall for the purposes of efficiently making our point regarding the 4×4 wavefunction approach, use the Dirac equation in the Schrödinger formalism.

Now, in closing this introductory section, we shall give the synopsis of the present paper. In §2, we shall for instructive, completeness and self-containment purposes, present the traditional free particle solutions of the Dirac equation. Thereafter in §3, we shall discuss some of the major shortcomings of the Dirac equation – this we do in order to demonstrate that there still is a lot more about the Dirac equation that still needs to be understood. Then, in §4, we present the main task of the present paper – i.e. the Dirac wavefunction is cast into a 4×4 type wavefunction. Thereafter in §5, we proceed to make our suggestion regarding the new formulation of the Dirac wavefunction. Lastly, in §6 a general discussion is given and no conclusion is made.

2 Free particle solutions of the Dirac equation

The free particle solutions of the Dirac equation are obtained by assuming a free particle wavefunction of the form: $\psi = ue^{\imath p_\mu x^\mu/\hbar}$, where: u, is a four component object, i.e. $u^T = (u_0 \; u_1 \; u_2 \; u_3)$, where the superscript-$T$ on u is the transpose operator. This u-function is assumed to have no space and time dependence. With this in mind, one will proceed to substituting this free particle solution: $\psi = ue^{\imath p_\mu x^\mu/\hbar}$, into (1), where-after some elementary algebraic operations – they will be led to the following linear quad-set of simultaneous equations:

$$(E - m_0c_0^2)u_0 - c_0(p_x - ip_y)u_3 - cp_zu_2 = 0, \tag{6a}$$

$$(E - m_0c_0^2)u_1 - c_0(p_x + ip_y)u_2 + cp_zu_3 = 0, \tag{6b}$$

$$(E + m_0c_0^2)u_2 - c_0(p_x - ip_y)u_1 - cp_zu_0 = 0, \tag{6c}$$

$$(E + m_0c_0^2)u_3 - c_0(p_x + ip_y)u_0 + cp_zu_1 = 0. \tag{6d}$$

An important fact to note about the above array or set of simultaneous equations is that the four solutions u_j (where: $j = 0, 1, 2, 3$) are superluminally entangled, that is to say, a change in one of the components affects every other component instantaneously i.e. in zero time interval. What this means is that for linearly dependent solutions of u_j, the Dirac equation – just as it predicts spin as a relativistic quantum phenomenon, it also predicts entanglement as a quantum phenomenon. If they exist as a separate reality in different regions of space, then, the particles, ψ_L, and, ψ_R, are entangled.

Without further ado, we shall now present the four formal solutions of the Dirac equation, these are given by: $\psi[k] = u[k] \exp\left(\imath p_\mu x^\mu/\hbar\right)$, where the $u[k]$'s are such that:

$$u[1] = \begin{pmatrix} 1 \\ 0 \\ \dfrac{c_0p_z}{E + m_0c_0^2} \\ \dfrac{c_0(p_x + ip_y)}{E + m_0c_0^2} \end{pmatrix} \quad u[2] = \begin{pmatrix} 0 \\ 1 \\ \dfrac{c_0(p_x - ip_y)}{E + m_0c_0^2} \\ -\dfrac{c_0p_z}{E + m_0c_0^2} \end{pmatrix} \tag{7}$$

$$u[3] = \begin{pmatrix} \dfrac{c_0p_z}{E - m_0c_0^2} \\ \dfrac{c_0(p_x + ip_y)}{E - m_0c_0^2} \\ 1 \\ 0 \end{pmatrix} \quad u[4] = \begin{pmatrix} \dfrac{c_0(p_x - ip_y)}{E - m_0c_0^2} \\ -\dfrac{c_0p_z}{E - m_0c_0^2} \\ 0 \\ 1 \end{pmatrix}$$

These solutions (2) are obtained as follows:

1. From (6), u_0, and u_1 are fixed so that: $u_0 = 1$, and, $u_1 = 0$, and the resultant set of equations is solved for u_2, and, u_3.
2. Similarly, from (6), u_0, and u_1 are fixed so that: $u_0 = 0$, and, $u_1 = 1$, and the resultant set of equations is solved for u_2, and, u_3.
3. Again, from (6), u_2, and u_3 are fixed so that: $u_2 = 1$, and, $u_3 = 0$, and the resultant set of equations is solved for u_0, and, u_1.
4. Similarly, from (6), u_2, and u_3 are fixed so that: $u_2 = 0$, and, $u_3 = 1$, and the resultant set of equations is solved for u_0, and, u_1.

Now, having presented the solutions of the Dirac equation, we shall proceed to present what we feel are some of the important major shortcomings of the Dirac equation.

3 Major shortcomings of the Dirac equation

While the Dirac equation is one of the most successful equations in physics, it is not without its own shortcomings. We here briefly review some of its shortcomings.

3.1 Anomalous gyromagnetic ratios

It is a well-known fact that in its bare and natural form, the Dirac equation predicts a gyromagnetic ratio ($\mathrm{g_D}$) equal to two (i.e. $\mathrm{g_D} = 2$) and this prediction is very close to the gyromagnetic ratio of the electron [$\mathrm{g_E} = 2 + 0.002319304362(2)$], hence, the Dirac equation is said to give a good description of the electron. On the contrary, the spin-1/2 proton ($\mathrm{g_E}$) and neutron ($\mathrm{g_N}$), which – like the electron – are thought to be fundamental particles and thus are naturally expected to readily submit to a successful description by the Dirac equation – these particles have gyromagnetic ratios that are at variance i.e. ($\mathrm{g_P} = 2 + 3.5856947(5)$; $\mathrm{g_N} = 2 - 5.8260855(9)$) with the Dirac prediction. The Dirac equation lacks in its nature infrastructure the devices to correctly predict the g-ratio of any arbitrary spin-1/2 particle. This state of affairs and aspect of the Dirac equation is very disappointing. In a future paper, we will propose a solution to this problem. We must say that, in the existing literature, there exists appropriate amendments that have been made to the Dirac theory in order to solve this problem. However, these solutions lack the much needed universal character.

3.2 Negative energy solutions

Further – as is well-known, one of Dirac's purpose in the formulation of his equation was to eliminate the unwanted negative energy solutions present in the Klein-Gordon equation [3, 4]. However, negative energy solutions are still present in Dirac's equation, and this led Dirac [5] to accept these solutions as physically realistic and to propose the existence of antimatter. Carl Anderson [6] confirmed Dirac's hypothesis and latter, Giuseppe Occhialini and Patrick Blackett [7] did the same.

The existence of antimatter is now commonplace in the scientific literature. What is not clear about this antimatter particles (antiparticles) is whether or not they have negative energy and mass. Do antimatter particles fall up or down in a gravitational field? Experiments [8] are not clear and this question still needs to be answered (see *e.g.* [9–12]). For according to Einstein's [13] mass energy equivalence ($E = mc_0^2$), if the energy of antiparticles is negative, their mass should be negative too. If this is the case, it follows from *Newton's Law of Gravitation* that in a gravitational field, antiparticles aught to fall up and not down!

3.3 Whereabouts of antimatter

Furthermore – apart from the the question of whether antiparticles fall up or down, there is the yet to be an answer to the question of the whereabouts of this antimatter [14–16]. The Dirac equation not only is symmetric under electric *Charge Conjugation* (C), but, symmetric under all the known discrete symmetries of *Time* (P) and *Parity* (P) reversal including the combination of any of these these discrete symmetries – i.e. CP, CT, PT, and, CPT. This symmetric nature of the Dirac equation leads to the prediction that the Universe must contain equal amounts of matter and antimatter. This is at variance with physical experience. Otherwise, due to the annihilation of matter and antimatter into radiation, the Universe would be a radiation bath, which is clearly not the case.

This prediction of the Dirac equation is '*very unfortunate*' because it is at complete variance with physical and natural as we know and experience it. That is to say, given that matter and antimatter will annihilate to form radiation should they ever come into contact, the exist of equal positions of matter and antimatter in the Universe would mean that if the Dirac prediction on the matter-antimatter census is correct, then, the Universe aught to be no more than a radiation bath. Clearly, this is not what we see around us.

3.4 Lack of a universal character

Additionally – every fermion particle (electron, proton, neutron, neutrinos, quarks *etc*) in particle physics is described by the Dirac equation. This gives the superficial impression that the Dirac equation is a universal equation for all spin-1/2 particles. A closer look will reveal that, while this equation is used to describe fermion, it needs to be supplemented in order to match-up with experimental data. As already pointed out in §3.1, the g-ratios of every other particle save for that of the electron are not in conformity with the natural Dirac equation. If the Dirac equation were indeed a universal equation for all fermions, it must contain within its natural infrastructure the necessary adjustable parameters that would make it fit with the experimental data of a given particle. These post-experimental adjustments that are made in order that the Dirac equation fits to experimental data are of *ad hoc* nature.

Apart from the inability to explain in a smooth manner the g-ratios of different fermions, we have the issue of the universality of spin. That is to say, the Dirac equation is an equation only capable of explaining spin-1/2 particles, an not any general spin particle. For example, in order to explain spin-3/2, we need to find another equation for this – the Rarita-Schwinger equation [17] in this case. In general, fermions have spin $\pm n/2$ with: $n = 1, 3, 5, 7, ..., 2r + 1, etc$. This implies that a new equation is required for every spin particle.

3.5 Fundamental origins of the Dirac equation

Lastly – another very important and yet largely ignored reality is that of the fundamental origins of the Dirac equation. That is to say, despite success, it remains that Dirac guessed his equation – albeit in a very educated manner. All he sought was an equation linear in both the space and time derivatives such that when this equation is "squared" it would yield the Klein-Gordon equation. It can be said that this issue of the origins of the Dirac equation is not unique to the Dirac equation, but all quantum mechanical equations.

The Klein-Gordon equation is derived from the well-known Einstein [18] energy-momentum dispersion relation: $E^2 = p^2c_0^2 + m_0^2c_0^4$, *via* the successful method of canonical quantization that was used by Schrödinger to arrive at his successful equation that describes the atomic world. Dirac's method of arriving at his equation is not fundamental at all, and to this day, no real progress on this has been made. Where does the Dirac equation really come from? This is yet another question that also needs an answer.

4 4×4 Dirac wavefunction

The very fact that the Dirac Hamiltonian, H_D, is a 4×4 component object acting on, ψ, this readily implies that, ψ, can be a $4 \times k$ component object where: $k = 1, 2, 3, 4, 5, etc$. If: $1 \leq k < 4$, the resulting system of equations is overdetermined and will thus have more than one solution, and if: $k > 4$, the resultant system of equations is under-determined and is unable to yield a solution. If: $k = 4$, the system has one and only solution, and this is the case of the 4×4 Dirac wavefunction that we would like to have a look at.

In the event of a 4×4 Dirac wavefunction where as usual:

$\psi = ue^{\imath p_\mu x^\mu/\hbar}$, the u-function is such that:

$$u = \begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ u_{10} & u_{11} & u_{12} & u_{13} \\ u_{20} & u_{21} & u_{22} & u_{23} \\ u_{30} & u_{31} & u_{32} & u_{33} \end{pmatrix} = \begin{pmatrix} u_a & u_b \\ u_c & u_d \end{pmatrix}, \tag{8}$$

where likewise:

$$\begin{matrix} u_a = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix} & u_b = \begin{pmatrix} u_{02} & u_{03} \\ u_{12} & u_{13} \end{pmatrix} \\ u_c = \begin{pmatrix} u_{20} & u_{21} \\ u_{30} & u_{31} \end{pmatrix} & u_d = \begin{pmatrix} u_{22} & u_{23} \\ u_{32} & u_{33} \end{pmatrix} \end{matrix}. \tag{9}$$

Of this 4×4 component wavefunction, ψ, we shall require of it to observe the following constraint:

$$\psi^\dagger \psi = \psi \psi^\dagger = \varrho \mathcal{I}_4, \tag{10}$$

where: $\mathcal{I}_4$, is the 4×4 identity matrix, and, $\varrho \in \mathbb{R}$, is a real zero-rank object – it is the quantum mechanical probability density amplitude. This constraint i.e. (10) is required by the unified theory of gravitation and electromagnetism [19] that we are currently working on.

Now, substituting the new 4×4 component wavefunction into the Dirac equation (1), we will have:

$$\begin{pmatrix} (E - m_0 c_0^2)\mathcal{I}_2 & -c_0 \vec{\sigma} \cdot \vec{p} \\ c_0 \vec{\sigma} \cdot \vec{p} & -(E + m_0 c_0^2)\mathcal{I}_2 \end{pmatrix} \begin{pmatrix} u_a & u_b \\ u_c & u_d \end{pmatrix} = 0. \tag{11}$$

As we proceed, the reader must take note of the fact that the object, $\vec{\sigma} \cdot \vec{p}$, is a 2×2 matrix, i.e.:

$$\vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - \imath p_y \\ p_x + \imath p_y & -p_z \end{pmatrix}. \tag{12}$$

This matrix, $\vec{\sigma} \cdot \vec{p}$, is hermitian.

Now, from (11), four equations will result and these are:

$$u_a = \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E - m_0 c_0^2} \right) u_c, \tag{13a}$$

$$u_b = \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E - m_0 c_0^2} \right) u_d, \tag{13b}$$

$$u_c = \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E + m_0 c_0^2} \right) u_a, \tag{13c}$$

$$u_d = \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E + m_0 c_0^2} \right) u_b. \tag{13d}$$

For a solution to this set of simultaneous equation, we shall set as a constraint the following:

$$u_a = u_d = \mathcal{I}_2 \sqrt{\varrho/2}. \tag{14}$$

This naturally leads to the following for, u_b, and, u_c, i.e.:

$$u_b = \sqrt{\varrho/2} \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E - m_0 c_0^2} \right). \tag{15a}$$

$$u_c = \sqrt{\varrho/2} \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E + m_0 c_0^2} \right). \tag{15b}$$

Hence:

$$u = \sqrt{\varrho/2} \begin{pmatrix} \mathcal{I}_2 & \dfrac{c_0 \vec{\sigma} \cdot \vec{p}}{E - m_0 c_0^2} \\ \dfrac{c_0 \vec{\sigma} \cdot \vec{p}}{E + m_0 c_0^2} & \mathcal{I}_2 \end{pmatrix}. \tag{16}$$

Writing this 4×4 matrix (16) in full, it will be as it appears in (17). Immediately, one will be quick to notice that columns (1), (2), (3), and (4) of this matrix (17) are in-fact the traditional solutions ($u[1], u[2], u[3], u[4]$) to the Dirac equation given in (2). What this means is that the 4×4 wavefunction is a grand synthesis of these four traditional solutions into one giant set of mutually dependent quadruplet system of particles.

5 Quarks

Apart from the simplification of bringing four independent particle solutions into a single particle solution, we suggest that this recasting of the Dirac wavefunction into a 4×4 wavefunction provides additional physical simplification in the analysis of the solution. To that end, let us start-off by writing down the full 4×4 Dirac wavefunction: $\psi = ue^{\imath p_\mu x^\mu/\hbar}$. For the 4×4 Dirac wavefunction, the u-function has been defined in (17) and from that definition, it follows that:

$$\psi = \begin{pmatrix} \psi_N & \psi_R \\ \psi_L & \psi_N \end{pmatrix}, \tag{18}$$

where – accordingly:

$$\psi_N = \mathcal{I}_2 \sqrt{\varrho/2} \exp\left(\frac{\imath p_\mu x^\mu}{\hbar} \right), \tag{19a}$$

$$\psi_R = \sqrt{\varrho/2} \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E - m_0 c_0^2} \right) \exp\left(\frac{\imath p_\mu x^\mu}{\hbar} \right), \tag{19b}$$

$$\psi_L = \sqrt{\varrho/2} \left(\frac{c_0 \vec{\sigma} \cdot \vec{p}}{E + m_0 c_0^2} \right) \exp\left(\frac{\imath p_\mu x^\mu}{\hbar} \right). \tag{19c}$$

In-comparison, i.e. between ψ as defined in (3) and the resultant definition of it in (18), we see that the initially four particles: ψ_a, ψ_b, ψ_c, and, ψ_d, have been reduced to three because, ψ_a, and, ψ_d, are identical – i.e. $\psi_a = \psi_d = \psi_N$. In (18), we have according to the parlance of the Dirac formalism identified ψ_b, and, ψ_c, with the right and the left-handed Dirac components. In terms of handedness, we have in the same parlance defined a new form of handedness in the ψ_N-particle,

$$u = \sqrt{\varrho/2}\begin{pmatrix} 1 & 0 & \dfrac{c_0 p_z}{E - m_0 c_0^2} & \dfrac{c_0(p_x - \imath p_y)}{E - m_0 c_0^2} \\ 0 & 1 & \dfrac{c_0(p_x + \imath p_y)}{E - m_0 c_0^2} & -\dfrac{c_0 p_z}{E - m_0 c_0^2} \\ \dfrac{c_0 p_z}{E + m_0 c_0^2} & \dfrac{c_0(p_x - \imath p_y)}{E + m_0 c_0^2} & 1 & 0 \\ \dfrac{c_0(p_x + \imath p_y)}{E + m_0 c_0^2} & -\dfrac{c_0 p_z}{E + m_0 c_0^2} & 0 & 1 \end{pmatrix} \tag{17}$$

a handedness that we shall call – *neutral-handedness*, hence, ψ_N, is a neutral-handed particle, this particle is neither left nor right-handed, hence our calling it neutral-handed particle and hence the subscript-N in its denotation.

Now, in the set: ψ_N, ψ_R, and, ψ_L, we have a trio of particles that are not only mutually dependent but entangled, and in addition to this, they are confined in the same region of space. Each of these particles do not exist independently of the other, they can never be free of each other far-away from the region defined by the ψ-particle system. The boundary in spacetime of the ψ-particle system is defined by the normalization of conditions of this particle system, i.e. $\langle\psi\,||\,\psi\rangle = \mathcal{I}_4$.

Now, given the following there facts:

1. The proton and neutron are each known to contain three quarks living inside them.
2. Further, the quarks strongly appear to be unable to exist independent of each other.
3. Furthermore, these quarks strongly appear to be eternal prisoners inside the proton and neutron. They are unable to exist beyond the radius demarcating the proton and neutron particle systems.

From these facts – i.e. the obvious similarity in the nature of quarks and the trio ψ_N, ψ_R, ψ_L, it is natural to wonder whether or not these three particles ψ_N, ψ_R, ψ_L are the quarks whose origins we have thus far elusively sought to understand? From this viewpoint, the present recasting of the Dirac wavefunction surely opens up a new avenue of thinking regarding the Dirac equation and quarks, hence justifying the need to seriously consider the 4×4 Dirac wavefunction. With that having been said, we must at this very juncture say that – it is not our intention to explore this idea that the set (ψ_N, ψ_R, ψ_L) might explain quarks and the reason for this is simple that we feel it is too early for us to do so, otherwise all that we would do is to speculate.

6 Discussion

As currently accepted and understood, the Dirac theory [1, 2] employs a 4×1 type wavefunction. This 4×1 Dirac wavefunction is acted upon by a 4×4 Dirac Hamiltonian, in which process, four independent particle solutions result and insofar as the real physical meaning and distinction of these four solutions, it is not clear what these solutions really mean. It is this that this paper has made an endeavour to provoke a thought process were a physical meaning can be attached to these four independent particle solutions of the Dirac equation and this is *via* the recasting of the Dirac wavefunction into a 4×4 type wavefunction.

We first presented this idea of a 4×4 Dirac wavefunction in [20, 21]. Prior to the said presentation [20, 21], we had never seen or heard of it anywhere in the literature. Therein [20], this idea was presented as no more than a *mathematical curiosity*, with no physical meaning attached to it. We had to come back to this idea now because we realised that it is necessary for the theory that we are currently working on [19], that is, a unified field theory of the gravitational and electromagnetic phenomenon.

What we have herein done with Dirac's four independent particle solutions, is to demonstrate that these can be represented as a quadruplet particle system wherein the Dirac wavefunction takes a new form as a 4×4 wavefunction. In this new formation, these four particle solutions precipitate into three distinct and mutuality dependent particles (ψ_L, ψ_N, ψ_R) that are permanently bound in the same region of space.

Realizing that the proton and neutron are composite particles each comprising three quarks that are in (color) confinement, we proceeded logically to make the natural suggestion to the effect that these Dirac particles (ψ_L, ψ_N, ψ_R) might be quarks. Whether or not these particles are quarks, this surely is something that further investigations will have to be establish.

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