A Quantitative Description of Atmospheric Absorption and Radiation at Equilibrium Surface Temperature

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An analytical theory is proposed for the earth-atmosphere system at its equilibrium surface temperature, 289.16 K. A non-linear relation is formulated between atmospheric absorption and atmospheric radiation by modifying Kirchhoff's law on thermal radiation. For the first time, the Global Energy Balance can be realized in a wide range of atmospheric absorptivity, transmittance, and surface emissivity. It is revealed that atmospheric radiation becomes negative once the atmospheric absorptivity is below its threshold value. It is proven that the upward cumulative long-wave atmospheric radiation spontaneously increases from 3.8 W m^{-2} to 199.4 W m^{-2} as the long-wave atmospheric absorptivity increases from 0.4 to 1.0 whilst the long-wave atmospheric transmittance decreases from 0.6 to 0.1.

1 Introduction

For over a century, many attempts have been made to balance the global energy budget, both at the top of the atmosphere (TOA) and at the Earth's surface [1]. It is known that the lack of precise knowledge of the surface energy fluxes profoundly affects the ability to study climate change [2]. In fact, the power equation at the surface remains unbalanced as the uncertainty in the net energy flux between the surface and the atmosphere is over 17 W m^{-2} [3]. To date, many static explanations for the global energy balance have been confined to using one set of fixed parameters to describe atmospheric absorption and radiation [2], whereas the taken-for-granted Kirchhoff's law at the core of the radiative transfer description of atmospheric absorption and radiation seems theoretically invalid [4].

In this paper, several thermodynamic variables of theoretical importance are redefined to formulate the basic equations, including those previously treated as constants. By continuously mapping the surface emissivity and longwave (LW) atmospheric absorptivity, several coupled quadratic equations are derived and simultaneously solved, which are in quantitative agreement with the latest experimental observations. In light of these new findings, implications for some fundamental issues in climate studies are briefly discussed.

2 Theory

In general, the thermodynamic variables in the atmosphere-surface system are dependent and should be described in coupled equations.

2.1 Outgoing longwave radiation and surface radiance

It is known that the total power balance at the TOA can be written as

$$\pi R^2 S\,(1-r) = 4\pi R^2 I_{LW}^{\uparrow} \tag{1}$$

where S is the solar constant, R the radius of the Earth, r the effective reflectivity of the Earth at the TOA, including the SW solar radiation reflected at the surface and then transmitted upward to the TOA, and $I_{LW}^{\uparrow}$ denotes the outgoing LW radiation (OLR) into outer space. From (1),

$$I_{LW}^{\uparrow} = \frac{S\,(1-r)}{4}\,. \tag{2}$$

Notice that OLR is merely determined by the albedo and the solar constant.

By treating the Earth as a graybody, the surface radiation can be obtained from the Stefan-Boltzmann law,

$$I_E = \varepsilon_E \sigma T^4\,, \tag{3}$$

where ε_E is defined as the Earth's mean surface emissivity, and T is the equilibrium mean surface temperature. In general, ε_E is to be treated as a thermodynamic variable in this study, although it has been often approximated as unity so far.

2.2 Modification of Kirchhoff's Law

In theory, the upward *cumulative* atmospheric absorption at any altitude can be calculated using the line-by-line method provided all of the relevant lineshape functions are known. At the TOA, the total LW atmospheric absorption can be expressed as

$$A_{LW} = \iint_0^{\infty} \alpha_\lambda\,(T_A)\,\rho\,(z)\,I_E\,(\lambda, z)\,d\lambda\,dz\,, \tag{4}$$

where α_λ is the spectral absorptivity of the atmosphere, predominately determined by water vapor, T_A is the atmospheric temperature at at a given altitude, ρ is the air density, $I_E\,(\lambda, z)$ represents the attenuated surface LW emission spectra at different altitudes. Naturally, α_λ represents both the resonant and continuum absorption by air molecules detected under continuous excitation [5,7]. Note that α_λ is scaled by the Planck function $B\,(\lambda, T)$ with its maximum at the center of the atmospheric window near 10 μm.

To proceed further, an effective LW cumulative atmospheric absorptivity, a_{LW}, at the TOA can be introduced

$$A_{LW} = a_{LW} I_E \,. \tag{5}$$

Obviously, the maximum LW atmospheric absorption is I_E when $a_{LW} = 1$.

Similarly, SW atmospheric absorption can be written as

$$A_{SW} = a_{SW} \left[1 + r_{SE} \left(1 - a_{SW}\right)\right] I^{\downarrow}_{SW} \left(TOA\right) \,. \tag{6}$$

where $I^{\downarrow}_{SW}(TOA)$ in the actual downward SW solar radiation at the TOA by subtracting the reflected SW solar radiation at the TOA, r_{SE} is defined the SW surface reflectivity. In this study, the SW atmospheric absorption is fixed.

Using Kirchhoff's law, it would appear that $\alpha = \varepsilon$, where α and ε are the spectral absorptivity and the emissivity of a non-scattering medium, respectively. Nevertheless, it seems unrealistic to expect that atmospheric radiation is equal to atmospheric absorption. So far, many attempts have been made at *ab initio* calculation of atmospheric radiation based on Schwarzschild's equation with the Planck function and an effective emissivity, but the results seem over-simplified. Besides, it has been revealed that Kirchhoff's law is problematic and should not be considered as a basic law [4].

In this paper, it is postulated that the fraction, denoted by β, of upward cumulative atmospheric radiation (UCAR), is proportional to the LW atmospheric absorptivity

$$a_{LW} = \gamma \beta \tag{7}$$

where γ denotes the proportionality factor that is used to parameterize the rest of the unclear dependence during radiative transfer in the atmosphere. In effect, (7) can be considered as a modified Kirchhoff's law for atmospheric radiation. In the absence of internal reflection, it would appear the sum of the LW atmospheric absorptivity and the LW atmospheric transmittance, τ_{LW}, is unity.

$$\tau_{LW} = 1 - a_{LW} \,. \tag{8}$$

Substituting (7) into (8) yields

$$\tau_{LW} = 1 - \gamma \beta \,. \tag{9}$$

It is shown in this study, however, that (8) and (9) are invalid in the presence of atmospheric radiation which is empowered by atmospheric absorption and other non-radiative energy fluxes.

2.3 Formulation for power balance conditions

To derive the power balance equation at the surface, that ensures the net energy flux at surface is exactly zero at thermal equilibrium, the net downward energy flux (NDEF) is denoted as N_0. Thus the power balance equation at the surface can be simply written as

$$N_0 = I_E \,. \tag{10}$$

As the downward SW solar radiation into the surface $I^{\downarrow}_{SW}(0)$ is known, it can be taken away from N_0 and explicitly expressed in the power balance condition,

$$N + I^{\downarrow}_{SW}(0) = I_E \,, \tag{11}$$

where N represents the NDEF when $I^{\downarrow}_{SW}(0)$ is excluded from N_0, *viz*.

$$N = N_0 - I^{\downarrow}_{SW}(0) \,. \tag{12}$$

Note that (11) and (12) are equivalent irrespective of the value of $I^{\downarrow}_{SW}(0)$.

At the TOA, the power balance equation for OLR can be expressed as,

$$I^{\uparrow}_{LW} = \tau_{LW} I_E + I^{\uparrow}_{A} \tag{13}$$

where τ_{LW} is the LW atmospheric transmittance, $I^{\uparrow}_{A}$ is the UCAR that can escape from the atmosphere into space. It is to be shown that the upward LW radiation at the TOA is a constant.

2.4 Formulation for atmospheric radiation

In the absence of the physical surface underneath the atmosphere while the LW radiation were still available, the upward LW atmospheric radiation at the TOA can be obtained by assuming it is proportional to the total atmospheric absorption without invoking Stefan-Boltzmann law.

$$I^{\uparrow}_{A} = \beta \left(A_{LW} + A_{SW}\right) \,. \tag{14}$$

The two absorption terms in (14) belong to, respectively, the one-way cumulative LW atmospheric absorption from the surface radiation A_{LW}, and the two-way cumulative SW atmospheric absorption from the solar radiation A_{SW}. In this hypothetical case, those non-radiative energy exchange processes are absent.

Similarly, the downward cumulative atmospheric radiation (DCAR) at the bottom of the atmosphere, can be derived

$$I^{\downarrow}_{A} = \left(1 - \beta\right) \left(A_{LW} + A_{SW}\right) \,. \tag{15}$$

Adding (14) and (15) yields,

$$I^{\uparrow}_{A} + I^{\downarrow}_{A} = a_{LW} I_E + A_{SW} \,, \tag{16}$$

which is simply an energy conservation statement.

In reality, however, the bottom of the atmosphere is physically in contact with the Earth's surface, hence the thermal energy exchange, in addition to radiation, is inevitable. As a result, (14)–(16) should be modified accordingly. Specifically, a portion of the total energy absorbed by the atmosphere must be used to achieve and maintain the thermal equilibrium in the atmosphere-surface system, as required by (11), which is exactly equal to N. Thus we have,

$$I^{\uparrow}_{A} = \beta \left(A_{LW} + A_{SW} - N\right) \,, \tag{17}$$

$$I_A^{\downarrow} = (1-\beta)\,(A_{LW} + A_{SW} - N)\,, \tag{18}$$

$$I_A^{\uparrow} + I_A^{\downarrow} = a_{LW} I_E + A_{SW} - N\,. \tag{19}$$

Note that (19) predicts that the total atmospheric radiation can either be zero or negative if the total atmospheric absorption is equal to or less than N, respectively. Here, (19) is to be used as the criterion to quantitatively determine the eventuating total atmospheric radiation, $I_A^{\uparrow} + I_A^{\downarrow}$, which, in turn, allows calculation for other climate variables, such as LW atmospheric transmittance and the net downward energy flux (NDEF).

2.5 Corollary

Substituting (5), (7), and (17) into (13), the power balance condition at the TOA can be rewritten as a quadratic function of the UCAR fraction β,

$$\tau_{LW} = -\gamma\beta^2 - \left(\frac{A_{SW} - I_E + I_{SW}^{\downarrow}(0)}{I_E}\right)\beta + \frac{I_{LW}^{\uparrow}}{I_E}\,, \tag{20}$$

with its y-intercept close to 0.6, which is determined by the ratio of OLR to the surface radiation. Note that (20) indicates that LW atmospheric transmittance is not unity in the absence of UCAR, as derived from (9) and shown in Fig. 1, due to the contribution of SW absorption by the atmosphere.

Substituting (7) into (20), we obtain the dependence of LW atmospheric transmittance on the LW atmospheric absorptivity,

$$\tau_{LW} = -\frac{a_{LW}^2}{\gamma} - \left(\frac{A_{SW} - I_E + I_{SW}^{\downarrow}(0)}{\gamma I_E}\right) a_{LW} + \frac{I_{LW}^{\uparrow}}{I_E}\,, \tag{21}$$

which indicates that the relation between LW transmittance and LW absorptivity is not linear, but quadratic, as shown in Fig. 2. As a result, the well-known linear relation between τ_{LW} and a_{LW}, (9), should be replaced by (21). To obtain the analytical formula for the atmospheric radiation that satisfies energy conservation law, substituting (5) and (7) into (17) yields a quadratic equation for UCAR,

$$I_A^{\uparrow} = \gamma I_E \beta^2 + \left(A_{SW} - I_E + I_{SW}^{\downarrow}(0)\right)\beta\,. \tag{22}$$

Dividing (17) by (18) and then substituting the result into (22) yields,

$$I_A^{\downarrow} = (1-\beta)\left(\gamma I_E \beta + A_{SW} - I_E + I_{SW}^{\downarrow}(0)\right)\,. \tag{23}$$

3 Calculated results

Based on the latest experimental data used in [7] and [8], as shown in Table 1, all of the numerical calculations are based on solving the coupled quadratic equations, (20) to

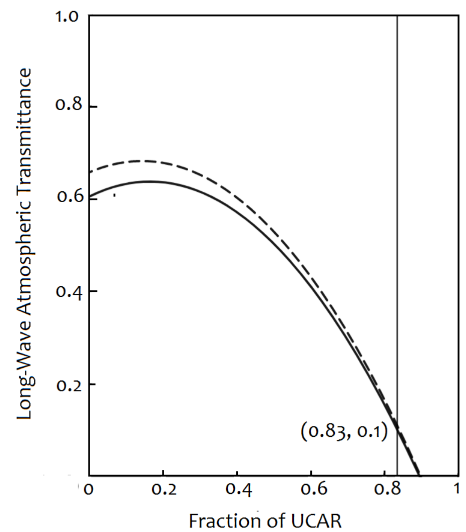

Fig. 1: Dependence of LW atmospheric transmittance τ_{LW} on the fraction of UCAR at the TOA, calculated from (20) assuming the surface emissivity is 1.0 (solid curve) and 0.92 (dashed curve). The coordinate (0.83, 0.1) represents the maximum β at $\tau_{LW} = 0.1$, used in this study.

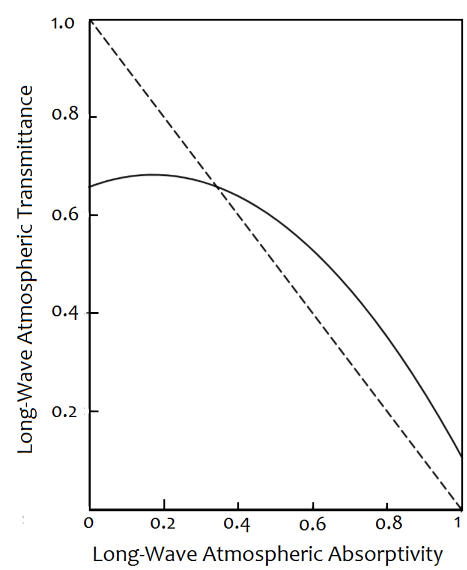

Fig. 2: Dependence of LW atmospheric transmittance on LW atmospheric absorptivity at the TOA, obtained from (21) in this study (solid curve) and from (8) (dashed line).

surface mean temperature	289.16	K
albedo	0.2985	
solar constant	1365.2	$W\,m^{-2}$
reflected solar radiation at TOA	101.9	$W\,m^{-2}$
SW atmospheric absorption	78	$W\,m^{-2}$
surface solar SW radiation	161	$W\,m^{-2}$

Table 1: The observed data used in [7] and this study.

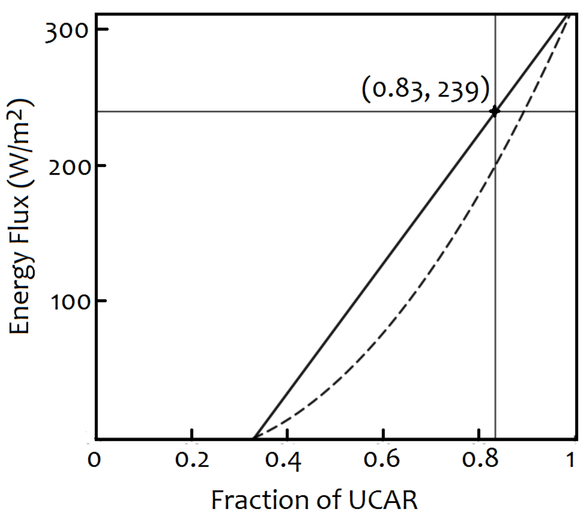

Fig. 3: Dependence of the total LW atmospheric radiation (solid line) and LW UCAR (dashed curve) on the fraction of UCAR the at the TOA, calculated from (19) and (22), respectively.

(24). A wide range of different values for surface emissivity and LW atmospheric absorptivity are considered. Specifically, the proportionality γ-factor in (7) is first determined by using the LW atmospheric transmittance $\tau_{LW} = 0.1009$ at the surface emissivity $\varepsilon_E = 1$ and then by maximizing the LW atmospheric absorptivity to $a_{LW} = 100\%$. This operation is equivalent to first assuming the atmospheric transmittance becomes its minimum whilst the LW atmospheric absorption reaches to its maximum, $A_{LW} \to I_E$.

Based on (16), the proportionality γ-factor in (7) is calculated, $\gamma = 1.196235$. Meanwhile, the β-factor for UCAR, 0.8354, is obtained simultaneously, which is also the maximum value for the β-factor, as shown in Fig. 3. Furthermore, the calculations are made for the surface emissivity ε_E <1. Note that the proportionality γ-factor is kept as a constant once it is determined in the first place, whilst neither additional parameters nor approximation are applied.

4 Discussion

4.1 Connecting radiation to cumulative absorption

In line with Kirchhoff's law, absorptivity and emissivity is often considered as identical in a non-scattering optical medium. In the case of the atmosphere, this implies that the absorbed radiation energy in each thin layer of an atmospheric model is completely emitted in the form of photons without being transformed into internal thermal energy in the atmosphere. Nevertheless, such an radiative transfer description seems invalid for the real atmospheric radiation where photon-particle scattering and radiation heating cannot be described by using Schwarzschild's equation. Hence, Kirchhoff's law is modified in this study with quantitative agreement with the latest observations.

In history, atmospheric radiation detected near the surface was described by using Stefan-Boltzmann law, such as the empirical equation used by Ångström [6],

$$I_A^{\downarrow} = \varepsilon_A \sigma T_A^4 \,, \tag{24}$$

where ε_A is the atmospheric emissivity, T_A is the air temperature near the surface. As the atmosphere can hardly be treated as a single isothermic layer, ε_A is in fact a random variable. Hence (24) is unfit for formulating atmospheric radiation. It has been recently shown that the atmospheric emissivity ε_A be equal to LW absorptivity a_{LW} only in the absence of clouds, see (78) in [7], but the fundamental link between atmospheric radiation and atmospheric absorption seems obscure. In general, it would appear that the distinction between the spontaneous resonant emission from the water vapor and other LW-radiation absorbers, such as CO_2, and the continuum thermal radiation governed by Planck's law remains to be further explored.

To circumvent such theoretical uncertainties, the fraction of upward *cumulative* atmospheric radiation (UCAR) at TOA, β, is introduced as a new variable in (7). In effect, the proportionality γ-factor is phenomenologically used to link the thermal radiation by the atmosphere to the *cumulative* LW atmospheric absorption based on (7). In this way, LW atmospheric radiation can be formulated. Further, the γ-factor in (7) is theoretically determined as one of the simultaneous solutions, $\gamma = 1.196235$, which appears an intrinsic invariant for the surface-atmosphere system.

4.2 Realization of the global energy balance

Because numerous energy fluxes exist between the Earth's surface and the atmosphere, it seems unlikely to identify and account all of them with absolute uncertainty. In fact, inconsistencies often arise when these different components are brought together to the power balance equation [3]. Specifically, efforts have been made to determine the net LW surface radiation, defined as the difference between the upward and downward radiation intensities,

$$I_N^{(LW)} = I_A^{\downarrow} - I_E \,. \tag{25}$$

Using the optimal estimates for $I_E = 398\,\mathrm{W\,m^{-2}}$ and $I_A^{\downarrow} = 342\,\mathrm{W\,m^{-2}}$, (25) gives $I_N^{(LW)} = -56\,\mathrm{W\,m^{-2}}$, whereas a wide range for the net LW surface radiation , $-49 > I_N^{(LW)} > -65$ $\mathrm{W\,m^{-2}}$, was predicted by individual CMIP5 models [2]. Using the net SW downward radiation, $I_{SW}^{\downarrow}(0) = 161\,\mathrm{W\,m^{-2}}$, the global mean surface net radiation,

$$I_N = I_N^{(LW)} + I_{SW}^{\downarrow}(0)\,, \tag{26}$$

is used to obtain $I_N = 105\,\mathrm{W\,m^{-2}}$, which happens to be about half way between two uncompromising values, $113\,\mathrm{W\,m^{-2}}$

τ_{LW}	a_{LW}	β	var.($\mathrm{W\,m^{-2}}$)	Zhong	[7]	[8]
0.1	0.899		OLR	–	239.4	239
0.1	1.0	a_{LW}/γ	OLR	239.4		
0.1	0.899	0.39	$I_{LW}^{(N)}$	–	64.4	
any	any	a_{LW}/γ	$I_{LW}^{(N)}$	0		
0.1	0.899		$A_{LW}+A_{SW}$	–		521
0.1	0.899	0.38	$A_{LW}+A_{SW}$	–	521.8	
0.1	1.0	$1/\gamma$	A_{max}	474.4		
0.1	1.0	0.83	$A_{LW}+A_{SW}$	474.4		
0.24	0.899	0.744	$A_{LW}+A_{SW}$	430.8		
0.33	0.8	0.67	$A_{LW}+A_{SW}$	395.1		
0.51	0.6	0.49	$A_{LW}+A_{SW}$	315.8		
0.6	0.4	0.34	$A_{LW}+A_{SW}$	236.6		
0.63	0.2	0.24	$A_{LW}+A_{SW}$	157		
0.1	0.899	–	UCAR $I_A^{\uparrow}$	–		199
0.1	0.899	0.39	UCAR $I_A^{\uparrow}$	–	199.4	
0.1	1.0	0.38	UCAR $I_A^{\uparrow}$	199.4		
0.33	0.8	0.67	UCAR $I_A^{\uparrow}$	106.8		
0.51	0.6	0.49	UCAR $I_A^{\uparrow}$	40.3		
0.6	0.4	0.34	UCAR $I_A^{\uparrow}$	3.8		
0.63	0.2	0.24	UCAR $I_A^{\uparrow}$	-13		
		$1-\beta$				
0.1	1.0	0.17	DCAR $I_A^{\downarrow}$	39.3		
0.1	0.899	–	DCAR $I_A^{\downarrow}$	–		333
0.1	0.826	0.62	DCAR $I_A^{\downarrow}$	–	332	
0.23	0.899	0.35	DCAR $I_A^{\downarrow}$	49.4		
0.33	0.8	0.33	DCAR $I_A^{\downarrow}$	52.9		
0.51	0.6	0.51	DCAR $I_A^{\downarrow}$	40.1		
0.6	0.4	0.66	DCAR $I_A^{\downarrow}$	0.77		

Table 2: Calculated thermodynamic variables (var.).

and 98 W m^{-2}, estimated by Stephens *et al* [3] and Trenberth *et al* [8], respectively. To explain the remaining imbalance, both the global mean sensible heat flux and the latent heat flux were considered, knowing the lack of adequate information from direct observations. Thus, it was recommended that the surface budget estimates not be used as references [2, 8].

By introducing the net downward energy flux (NDEH) at the surface, nevertheless, such statistical estimates become unnecessary. Moreover, a number of climate scenarios previously unconsidered have been quantitatively predicted, under the same Global Energy Balance condition with zero net surface energy flux, as shown in the fourth row in Table 3. In essence, any actual thermal energy transfer between the surface and the atmosphere that appears either undefined or difficult to be measured can be implicitly treated as part of N. Note that (10) implies the net downward energy flux N_0 should be solely determined by the mean surface temperature and the surface emissivity as $I_E = \varepsilon_E \sigma T_E^4$, rather than by LW DCAR as previously taken for granted in other studies [2,3], although LW DCAR may well be part of N_0. In passing, NDEH at the surface is conceptually different from the net downward heat flux introduced by Gregory *et al* [11] to describe a hypothetical vertical radiative transfer process initiated at the TOA.

4.3 The stable range of atmospheric absorption

It is shown that the total atmospheric absorption be limited by the maximum external radiation, both from the Sun and the Earth's surface. To remain at the current equilibrium surface temperature, 289.16 K, it is theoretically predicted that the minimum of the total atmospheric absorption is close to 236 W m^{-2}, being significantly lower than the value that has been assumed so far. In a recent study [7], for example, the total absorption by the atmosphere 521.8 W m^{-2} was assumed. This seems unlikely because the value is 46.6 W m^{-2} higher than the maximum atmospheric absorption, $I_E + A_{SW} = 396.4 + 78 = 474.4\,\mathrm{W\,m^{-2}}$. In another report [2], it was claimed that LW DCAR $I_A^{\downarrow} = 342\,\mathrm{W\,m^{-2}}$ which requires atmospheric absorption even higher than 521 W m^{-2}.

It could be argued that such an unrealistically high atmospheric absorption is merely fabricated for invoking an imaginary greenhouse effect, bearing in mind that the average solar radiation at the TOA is 342 W m^{-2}. Moreover, it is revealed that (8) and (9) are incorrect in studying the earth-atmosphere system due to the limitation associated with Kirchhoff's law in formulating thermal radiation. From those radiation and energy budget diagrams, e.g. [7–9], it is clear that (8) was used to obtain the LW atmospheric absorption, 356 W m^{-2}, based on that the assumed transmitted surface radiation at the TOA is 40 W m^{-2}, which yields the LW atmospheric absorptivity and the LW atmospheric transmittance equal to 89.91% and 10.01%, respectively.

By using (19), by way of contrast, the predicted LW atmospheric transmittance is close to 0.24 given the LW atmospheric absorptivity is 89.91%, as shown in Table 2, in order to satisfy the power balance condition, determined by (11). As a result, the sum of the LW and SW atmospheric absorption is 430.4 W m^{-2}, instead of 521.8 W m^{-2} as previously assumed in [7,8].

Further, it is shown that the proposed theory is self-consistent as the calculated OLR at TOA from (13) is indeed a constant, independent of the LW atmospheric absorption, as indicated in (2). This implies that a previous calculation of radiation forcing by assuming a change in OLR due to CO_2-doubling [7] appears inconsistent with the definition of OLR in (1). In essence, any increase in LW atmospheric absorption will spontaneously increase in UCAR to exactly keep OLR a constant, as shown in Fig. 4, consistent with Le Chatelier's principle of thermodynamics.

4.4 Characterization of atmospheric radiation

It is found that the fraction for UCAR, β, is always larger than the portion for DCAR whenever the LW atmospheric absorptivity $a_{LW} > 60\%$. This can be explained as the fact that UCAR can easily reach outer space whereas DCAR would be increasingly attenuated towards the Earth's surface. Since DCAR is treated as part of NDEF, the difference $N - I_A^{\downarrow}$ actually represents the contribution to NDEF from other thermal energy transfer processes, both radiative or non-radiative. In fact, it is found that the cumulative downward atmospheric radiation at the surface $I_A^{\downarrow}$ is about one-fourth of NDEF, which implies that DCAR would be more effectively converted into the thermal energy towards the lower-altitude atmospheric layers as it approaches towards the surface where both the air density and the air temperature are the highest, whilst the collisions are the most frequent. Hence, the relatively low range of DCAR found in this study seems consistent with the observed stable surface temperature.

It is noted that whenever LW atmospheric absorptivity decreases to a critical value, ~40%, the total atmospheric radiation, the sum of UCAR and DCAR, becomes zero, as shown in Fig. 4, which implies that no cumulative atmospheric radiation can be detected at the TOA and the surface under this condition. This can be explained in terms of total internal absorption in the atmosphere when its internal thermal energy is insufficient to maintain its equilibrium with the surface. Under this critical condition, the atmospheric radiation is completely absorbed by the atmosphere itself. This explanation is consistent with the definitions of UCAR and DCAR whose sum become negative whenever the total atmospheric absorption is less than the net downward energy flux N in (19), required for preventing the radiation cooling at the surface. Note that once the atmosphere reaches its thermal equilibrium with the surface, the surplus LW atmospheric radiation is primarily utilized by the atmosphere to cool down itself and hence increase its entropy, rather than to warm up the surface.

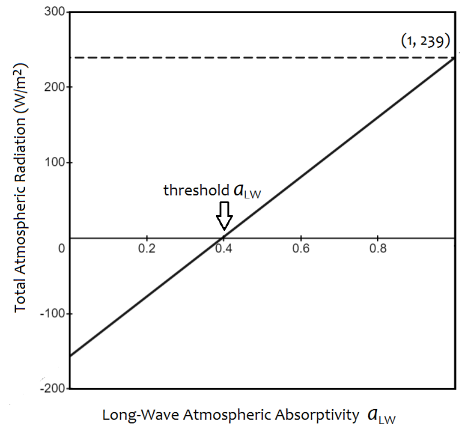

Fig. 4: Dependence of total atmospheric radiation on LW atmospheric absorptivity. Notice that the net atmospheric radiation is negative if LW atmospheric absorptivity a_{LW} is less than 0.4. The coordinate (1, 239) represents the maximum total cumulative atmospheric radiation at the TOA and the surface, 239 W m^{-2}, at the maximum LW atmospheric absorptivity $a_{LW} = 100\%$.

ε_E	1.0	0.99	0.95	0.92	unit
I_E	396.4	392.44	376.58	364.69	W m^{-2}
$I_{LW}^{\uparrow}$	239.41	239.43	239.44	239.43	W m^{-2}
$I_{LW}^{(N)}$	0.0	0.0	0.0	0.0	W m^{-2}
N	235.4	231.44	215.58	203.69	W m^{-2}
a_{LW}^{th}	0.3971	0.391	0.3653	0.3446	
β	0.8357	0.8359	0.8354	0.8354	

Table 3: Calculated dependence on the surface emissivity.

4.5 The role of surface emissivity

The Earth's surface emissivity is explicitly treated as a thermodynamic variable in this study, whilst in the previous studies the surface emissivity was larger than 0.99 [10]. Note that the surface radiation decreases noticeably from 396.4 W m^{-2} to 364.69 W m^{-2} as the surface emissivity changes from 1.0 to 0.92 and the so-called best estimate for the surface radiance [2], 398 W m^{-2}, is 1.6 W m^{-2} higher than the calculated value at $\varepsilon_E = 1$ in this study. It is of interest to find that atmospheric radiation, both UCAR and DCAR, is independent of the surface emissivity at the maximum LW atmospheric absorptivity a_{LW} =1, although atmospheric radiation decreases non-linearly with the decrease of a_{LW}. This implies that the β-factor in (7) belongs to the intrinsic compositional properties of the atmosphere and hence independent of the intensity of the surface radiation. It is also found that LW atmospheric transmittance increases noticeably as surface emissivity changes from 1.0 to 0.92, as shown in Fig. 1, corresponding to the equilibrium NDEF decreases from 235.4 W m^{-2} to 203.69 W m^{-2}, as shown in Table 3. This indicates the atmosphere can spontaneously adjust its LW transmittance in response to the change in the surface radiance. However, such an spontaneous capability seems incapable of fully maintaining the transmitted surface radiation in the range $a_{LW} < 0.4$ unless atmospheric radiation completely ceases below each threshold value of a_{LW} for a given surface emissivity. Such detailed effects seem unexpected because the surface emissivity was often assumed as unity after Houghton [12]. Thus, the LW surface reflectivity, $r_{LW} = 1 - \varepsilon_E$, can be treated as a key variable in climate modeling. Further studies in this direction are certainly worthwhile.

5 Conclusion

In conclusion, it is shown that Kirchhoff's law on thermal radiation is oversimplified and must be modified in connecting atmospheric radiation with atmospheric absorption. Due to complicated thermal mixing processes associated with atmospheric absorption and emission, the equation for atmospheric transmittance and the atmospheric absorptivity is far from linear. Further, it is revealed that the long-wave atmospheric radiation can be completely absorbed by the atmosphere itself before it reaches to a thermal equilibrium between the surface. For the first time, both the upward cumulative atmospheric radiation and the downward cumulative atmospheric radiation can be theoretically calculated without uncertainty. It is also shown that upward cumulative atmospheric radiation at the top of the atmosphere is in general stronger than downward cumulative atmospheric radiation at the Earth's surface. It is explained that the atmospheric absorption only plays a passive role in achieving its thermal equilibrium with the Earth's surface whilst atmospheric radiation plays a proactive role in enabling the atmosphere to adapt to a wide range of variation in the atmospheric absorptivity values. In essence, only a small fraction of the atmospheric radiation, less than 55 W m^{-2}, can be absorbed by the surface, whereas the larger portion of the atmospheric radiation, up to 199 W m^{-2}, can spontaneously escape into the outer space, providing a unique mechanism for radiation cooling to maximize the entropy of the atmosphere. It is shown that the Global Energy Balance can be realized in a number of climate scenarios without any estimates. It is expected that the proposed theory can be applied in elucidating commonly concerned climate issues without invoking Kirchhoff's law and the greenhouse effect.

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