Deflection of Light Rays and Mass-Bearing Particles in the Field of a Rotating Body

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As proved earlier, the space of a rotating body is Riemannian only if it is filled with a distributed matter (Progr. Phys., 2022, v.18, 31–49). In this paper we consider motion of massless (light-like) and mass-bearing particles in the space of a rotating body filled with an electromagnetic field, where the influence of gravitation is negligible. Solving the equations of motion of a particle that does not have an electric charge, we find that its motion deflects from a straight line due to the space curvature caused by the rotation of space. That is, the trajectories of light rays and mass-bearing particles are deflected near a rotating body due to the curvature of space caused by its rotation. This is one more fundamental effect of the General Theory of Relativity, in addition to the deflection of light rays in the field of a gravitating body.

In this small paper, which is a continuation of the previous one [1], we consider the equations of motion of a massless (light-like) particle and a mass-bearing particle in the space of a rotating body, which is filled with an electromagnetic field, and the influence of gravitation is negligible.

As proved in the previous paper, a rotating space has a significant curvature due to its space-time non-holonomity (non-orthogonality of the time lines to the three-dimensional spatial section). For this reason, we expect to find that the space curvature caused by the rotation of space deflects light rays and mass-bearing particles near a rotating body.

Please note that, as proved earlier using Einstein's equations [1], the space of a rotating body is Riemannian only if it is filled with a distributed matter, say, an electromagnetic field. Therefore, the above problem statement will not be valid and mathematically correct in an empty space or in a space filled only with a gravitational field. In the case we are considering, as in the previous article, the "space filler" is an electromagnetic field.

In this work, as well as in our other works, we use the mathematical apparatus of chronometric invariants, which are physically observable quantities in the General Theory of Relativity. This mathematical apparatus was created in 1944 by our esteemed teacher A. L. Zelmanov (1913–1987). Its basics can be learned from Zelmanov's publications [2–4], of which his 1957 presentation [4] is the most useful and complete, and also from our previous article [1]. For a deeper study of this mathematical apparatus, read the respective chapters of our monographs [5, 6], especially — the chapter Tensor Algebra and the Analysis in [6].

The equations of motion of both mass-bearing and massless (light-like) particles were studied in detail in our two monographs [5, 6]. The first one [5] focused onto free motion of particles, and the second one [6] focused onto non-geodesic motion of particles: the right hand side of the equations of non-geodesic motion is non-zero, and contains the external force deflecting the particles from geodesic (shortest) trajectories.

The chronometrically invariant equations of motion are the physically observable projections of the general covariant four-dimensional equations of motion onto the time line and the three-dimensional spatial section of a particular observer. Such projections are invariant along the spatial section of the observer (his observed space) and are expressed through the properties of his local reference space. Those who are interested in how the equations of motion are derived can refer to the respective chapters of our monographs, where all these equations are explained in detail.

In this paper we consider mass-bearing particles that do not have an electric charge, and massless (light-like) particles are not electrically charged by definition. As a result, the right hand side of the equations of their motion, containing the force acting on electrically charged particles from the electromagnetic field, is equal to zero. Therefore, these are free particles, and the equations of their motion are the equations of motion along geodesic lines.

The chr.inv.-equations of motion of a free mass-bearing particle describe the motion along an ordinary geodesic line

$$\left.\begin{aligned}&\frac{dm}{d\tau}-\frac{m}{c^2}\,F_i\,\mathrm{v}^i+\frac{m}{c^2}\,D_{ik}\,\mathrm{v}^i\mathrm{v}^k=0\\&\frac{d\,(m\mathrm{v}^i)}{d\tau}+2m\left(D^i_k+A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k-mF^i+m\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k=0\end{aligned}\right\},$$

and the chr.inv.-equations of motion of a free massless (light-like) particle describe the motion along an isotropic geodesic line (a.k.a. a null geodesic line)

$$\left.\begin{aligned}&\frac{d\omega}{d\tau}-\frac{\omega}{c^2}\,F_i\,c^i+\frac{\omega}{c^2}\,D_{ik}\,c^ic^k=0\\&\frac{d\,(\omega c^i)}{d\tau}+2\,\omega\left(D^i_k+A^{\cdot i}_{k\cdot}\right)c^k-\omega F^i+\omega\Delta^i_{nk}\,c^nc^k=0\end{aligned}\right\}.$$

Here in the equations of motion and so forth, m is the relativistic mass of the travelling particle, ω is the relativistic frequency of the massless (light-like) particle, $d\tau$ is the physically observable time interval expressed through the linear velocity v_i of the rotation of space

$$d\tau = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i\, dx^i,$$

$$v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}\,, \qquad v^i = -c\, g^{0i}\sqrt{g_{00}}\,,$$

and, respectively the chr.inv.-vector of the physically observable velocity of the travelling particle has the form

$$\mathrm{v}^i = \frac{dx^i}{d\tau}\,, \qquad \mathrm{v}_i\mathrm{v}^i = h_{ik}\mathrm{v}^i\mathrm{v}^k = \mathrm{v}^2,$$

which in the ultimate case transforms into the chr.inv.-vector of the physically observable velocity of light, the square of which is $c_i\, c^i = h_{ik}\, c^i c^k = c^2$.

Please note that, according to the theory of chronometric invariants, the square of any chr.inv.-quantity, and also lifting and lowering indices in chr.inv.-quantities is determined through the chr.inv.-metric tensor

$$h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k\,, \quad h^{ik} = -g^{ik}, \quad h^i_k = -g^i_k = \delta^i_k\,,$$

which is obtained as the spatial chr.inv.-projection of the fundamental metric tensor $g_{\alpha\beta}$ and has all its properties everywhere in the observer's three-dimensional spatial section.

Concerning the physically observable characteristics of space, which are terms in the equations of motion, these are the chr.inv.-vector of the gravitational inertial force F^i (where $\mathrm{w} = c^2(1-\sqrt{g_{00}})$ is the gravitational potential), the antisymmetric chr.inv.-tensor of the angular velocity of rotation of space, A_{ik}, the symmetric chr.inv.-tensor of deformation of space, D_{ik}, and the chr.inv.-Christoffel symbols Δ^i_{jk}, (coherence coefficients of space), i.e.

$$F_i = \frac{1}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right),$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right),$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,,$$

$$\Delta^i_{jk} = h^{im}\Delta_{jk,m} = \frac{1}{2}\, h^{im}\left(\frac{{}^*\partial h_{jm}}{\partial x^k} + \frac{{}^*\partial h_{km}}{\partial x^j} - \frac{{}^*\partial h_{jk}}{\partial x^m}\right),$$

where the chr.inv.-operators of derivation have the form

$$\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}\,, \qquad \frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{g_{0i}}{g_{00}}\frac{\partial}{\partial x^0}\,.$$

In our further calculation of the deflection of light rays and mass-bearing particles in the field of a rotating body we will use the same space metric that we introduced in the previous paper [1]. This is the metric of a space, where the three-dimensional space rotates due to the non-holonomity of the space-time, but there is no field of gravitation (or, to be more exact, the influence of gravitation is negligible).

Assume that the space rotates along the equatorial axis φ, i.e., along the geographical longitudes, with the velocity $v_3 = \omega\, r^2\sin^2\theta$, where $\omega = const$ is the angular velocity of this rotation. Then, according to the definition of v_i,

$$v_3 = \omega\, r^2\sin^2\theta = -\frac{c\, g_{03}}{\sqrt{g_{00}}}\,,$$

we obtain the metric of such a space

$$ds^2 = c^2dt^2 - 2\,\omega\, r^2\sin^2\theta\, dt\, d\varphi - - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

As you can see, the non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of this metric are equal to

$$g_{00} = 1\,, \qquad g_{03} = -\frac{\omega\, r^2\sin^2\theta}{c}\,,$$

$$g_{11} = -1\,, \quad g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\theta\,,$$

and, according to the definition of the chr.inv.-metric tensor h_{ik}, its non-zero components in the metric are equal to

$$h_{11} = 1\,, \quad h_{22} = r^2, \quad h_{33} = r^2\sin^2\theta\left(1 + \frac{\omega^2 r^2\sin^2\theta}{c^2}\right),$$

$$h^{11} = 1\,, \quad h^{22} = \frac{1}{r^2}\,, \quad h^{33} = \frac{1}{r^2\sin^2\theta\left(1 + \frac{\omega^2 r^2\sin^2\theta}{c^2}\right)}\,,$$

where, since the matrix h_{ik} is diagonal, the upper-index components of h_{ik} are obtained as $h^{ik} = (h_{ik})^{-1}$ just like the invertible matrix components to any diagonal matrix.

Using the definition of the antisymmetric chr.inv.-tensor of the angular velocity of rotation of space, A_{ik}, we obtain that its non-zero components in the rotating space we are considering are equal to

$$A_{13} = \omega\, r\sin^2\theta\,, \qquad A_{31} = -A_{13}\,,$$

$$A_{23} = \omega\, r^2\sin\theta\cos\theta\,, \qquad A_{32} = -A_{23}\,,$$

$$A^{13} = \frac{\omega}{r\left(1 + \frac{\omega^2 r^2\sin^2\theta}{c^2}\right)}\,, \qquad A^{31} = -A^{13},$$

$$A^{23} = \frac{\omega\cot\theta}{r^2\left(1 + \frac{\omega^2 r^2\sin^2\theta}{c^2}\right)}\,, \qquad A^{32} = -A^{23}.$$

Using the definition of the chr.inv.-Christoffel symbols Δ^i_{jk} (coherence coefficients of space), after some algebra, we obtain formulae for their non-zero components in the rotating space we are considering. They have the form

$$\Delta^1_{22} = -r\,,$$

$$\Delta^1_{33} = -r\sin^2\theta\left(1+\frac{2\omega^2r^2\sin^2\theta}{c^2}\right),$$

$$\Delta^2_{12} = \Delta^2_{21} = \frac{1}{r}\,,$$

$$\Delta^2_{33} = -\sin\theta\cos\theta\left(1+\frac{2\omega^2r^2\sin^2\theta}{c^2}\right),$$

$$\Delta^3_{13} = \Delta^3_{13} = \frac{1}{r\left(1+\frac{\omega^2r^2\sin^2\theta}{c^2}\right)}\left(1+\frac{2\omega^2r^2\sin^2\theta}{c^2}\right),$$

$$\Delta^3_{23} = \Delta^3_{32} = \frac{\cot\theta}{1+\frac{\omega^2r^2\sin^2\theta}{c^2}}\left(1+\frac{2\omega^2r^2\sin^2\theta}{c^2}\right).$$

Now, using the obtained physically observable characteristics of the rotating space we are considering, we will modify the general formulae of the chr.inv.-equations of free motion (see above) in accordance with the space metric. As a result, we will obtain the chr.inv.-equations of motion of a free mass-bearing particle and a free massless (light-like) particle in the rotating space. The solution of these equations will show the effect of deflection of light rays and mass-bearing particles in the field of a rotating body.

Since $g_{00} = 1$ in the metric, and the rotation of space is stationary ($v_3 = \omega r^2\sin^2\theta$ is not time-dependent), then the gravitational potential $\mathrm{w} = c^2(1-\sqrt{g_{00}})$ is equal to zero and, hence, the gravitational inertial force vanish, $F_i = 0$.

In addition, you can see that neither the fundamental metric tensor $g_{\alpha\beta}$ nor the chr.inv.-metric tensor h_{ik} of the metric are not time-dependent, the rotating space we are considering does not deform and, hence, the tensor of deformation of space vanish, $D_{ik} = 0$.

As a result, since $F_i = 0$ and $D_{ik} = 0$, the chr.inv.-equations of motion of a free mass-bearing particle in the rotating space we are considering take the simplified form

$$\left.\begin{aligned} &\frac{dm}{d\tau} = 0\\ &\frac{d(m\mathrm{v}^i)}{d\tau} + 2mA_{k\cdot}^{\cdot i}\mathrm{v}^k + m\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k = 0 \end{aligned}\right\},$$

and the chr.inv.-equations of motion of a free massless (light-like) particle are simplified to the form

$$\left.\begin{aligned} &\frac{d\omega}{d\tau} = 0\\ &\frac{d(\omega c^i)}{d\tau} + 2\omega A_{k\cdot}^{\cdot i}c^k + \omega\Delta^i_{nk}c^n c^k = 0 \end{aligned}\right\}.$$

The above equations are identical. Therefore they are solved in the same way and have the same solution.

Consider the above equations of motion of a free mass-bearing particle as a sample (the solution for a free massless particle will be the same).

The scalar equation of motion solves as $m = const$. With this solution taken into account, we substitute here the obtained formulae for the tensor of the angular velocity of rotation of space, A_{ik}, and the Christoffel symbols Δ^i_{jk}. As a result, neglecting higher order terms (otherwise the equations are unsolvable), we obtain the vectorial equations of motion in the component form suitable for their further analysis

$$\left.\begin{aligned} &\frac{d\mathrm{v}^1}{d\tau} - 2\omega r\sin^2\theta\,\mathrm{v}^3 - r\mathrm{v}^2\mathrm{v}^2 - r\sin^2\theta\,\mathrm{v}^3\mathrm{v}^3 = 0\\ &\frac{d\mathrm{v}^2}{d\tau} - 2\omega\sin\theta\cos\theta\,\mathrm{v}^3 + \frac{2}{r}\,\mathrm{v}^1\mathrm{v}^2 - \sin\theta\cos\theta\,\mathrm{v}^3\mathrm{v}^3 = 0\\ &\frac{d\mathrm{v}^3}{d\tau} + \frac{2\omega}{r}\,\mathrm{v}^1 + 2\omega\cot\theta\,\mathrm{v}^2 + \frac{2}{r}\,\mathrm{v}^1\mathrm{v}^3 + 2\cot\theta\,\mathrm{v}^2\mathrm{v}^3 = 0 \end{aligned}\right\}.$$

Even a brief look at the obtained equations of motion shows that the three possible effects are conceivable:

1. The deflection of a travelling free particle along the geographic longitudes (the third equation in the above system);
2. The deflection of a travelling free particle along the geographic latitudes (the second equation);
3. The acceleration or braking of a travelling free particle in the radial direction (the first equation).

The problem is that the above system of differential equations is unsolvable in the general form. Therefore, we will consider a simplified particular case of the equations, and calculate all three of the above effects just for this case.

Consider a particle travelling at a very high radial velocity v^1 in the equatorial plane exactly along the radial axis to the origin of the coordinates. Say, a particle from the near-Earth space travels freely in the equatorial plane directly to the Earth's surface. In this case, the velocities of its deflection along the geographical latitudes and longitudes, v^2 and v^3, are negligible compared to v^1, and the above equations take the simplified form

$$\left.\begin{aligned} &\frac{d\mathrm{v}^1}{d\tau} - 2\omega r\,\mathrm{v}^3 - r\mathrm{v}^2\mathrm{v}^2 - r\mathrm{v}^3\mathrm{v}^3 = 0\\ &\frac{d\mathrm{v}^2}{d\tau} + \frac{2}{r}\,\mathrm{v}^1\mathrm{v}^2 = 0\\ &\frac{d\mathrm{v}^3}{d\tau} + \frac{2\omega}{r}\,\mathrm{v}^1 + \frac{2}{r}\,\mathrm{v}^1\mathrm{v}^3 = 0 \end{aligned}\right\}.$$

In addition, we assume that the particle's velocity in the radial direction gains only a very small increment or decrement α' compared to its numerical value v^1, which, according to our initial assumption, is very large. As a result, we set $\mathrm{v}^1 = const$ in the equations of motion along the equatorially longitudinal axis φ (third equation) and the latitudinal axis θ (second equation), but solve the equation of motion along the radial axis r (first equation) with respect to $\mathrm{v}^1 + \alpha'$, i.e., with respect to the small parameter α. Otherwise, the above system of differential equations is unsolvable.

1. Consider the third equation of motion (along the equatorial axis φ). With the above assumptions, this equation takes the form, respectively,

$$y' + a\,y + b = 0\,, \qquad \varphi'' + a\,\varphi' + b = 0\,,$$

where we used the following notations

$$y = \mathrm{v}^3 = \frac{d\varphi}{d\tau}\,, \quad a = \frac{2}{r}\,\mathrm{v}^1 = const, \quad b = \frac{2\,\omega}{r}\,\mathrm{v}^1 = const.$$

The above differential equations for the velocity $y = \mathrm{v}^3$ and the coordinate φ with respect to the physically observable time $\tau = x$ are solved as

$$y = \frac{C}{\mathrm{e}^{ax}} - \frac{b}{a}\,, \qquad \varphi = \frac{C_1}{\mathrm{e}^{ax}} - \frac{bx}{a} + C_2\,,$$

where the constants of integration found using the initial conditions $x = x_0 = 0$ and $y = y_0 = 0$, are equal to

$$C = \frac{b}{a} = \omega\,, \quad C_1 = -\frac{b}{a^2} = -\frac{\omega\,r}{2\,\mathrm{v}^1}\,, \quad C_2 = -C_1 = \frac{\omega\,r}{2\,\mathrm{v}^1}\,.$$

As a result, we obtain solutions for the particle's velocity $y = \mathrm{v}^3$ along the equatorial axis (along the geographic longitudes), as well as for the equatorial coordinate φ (geographical longitude) of the arrival point of this particle.

The obtained solution for the particle's velocity along the equatorial axis φ has the form

$$\mathrm{v}^3 = -\,\omega + \omega\,\mathrm{e}^{-\frac{2}{r}\,\mathrm{v}^1\tau}.$$

Here the first term $-\,\omega$ is the particle's basics equatorial velocity, the origin of which is the banally shift of the equatorial coordinate φ to its negative numerical values due to the Earth's turn over the particle's travel to the Earth.

The second, additional term means that a particle freely travelling to the surface of a rotating body gains an additional velocity directed along the equator (geographical longitudes) opposite to the rotation of the body.

The obtained solution for the equatorial coordinate φ of the arrival point of this particle has the form

$$\varphi = \varphi_0 - \omega\,\tau + \frac{\omega\,r}{2\,\mathrm{v}^1}\left(1 - \mathrm{e}^{-\frac{2}{r}\,\mathrm{v}^1\tau}\right).$$

The third, additional term of this solution means that a particle freely travelling to the surface of a rotating body is deflected along the equator (geographical longitudes) opposite to the rotation of the body.

All this is because the rotation of any body gets space curved near it, thereby creating a "slope of the hill" slowing "down" along the equator towards the rotation of this body. In other words, space is curved by a rotating body in the direction of its rotation. As a result, a particle freely travelling to a rotating body "rolls down the curvature hill" of space along the equator in the direction in which the body rotates.

The same effect is expected for light rays, since the equations of motion for a massless (light-like) particle and a mass-bearing particle are identical, and, hence, their solutions coincide (see above). Only the mass-bearing particle's velocity is replaced with the physically observable velocity of light.

Please note that, as Zelmanov showed in 1944 using the mathematical apparatus of chronometric invariants, the vectorial components of the physically observable velocity of light depend on the geometric properties of space, as well as on the physical properties of distributed matter, despite the fact that the square of the velocity remains invariant.

As a result, the solution for the equatorial coordinate φ of the arrival point of a light ray falling down from space onto the Earth's surface in the equatorial plane has the form

$$\varphi = \varphi_0 - \omega\,\tau + \frac{\omega\,r}{2\,c^1}\left(1 - \mathrm{e}^{-\frac{2}{r}\,c^1\tau}\right),$$

where c^1 is the physically observable velocity of light in the radial direction.

Since the Earth, as well as any other planet or star, has its own gravitational field, a mass-bearing particle freely travelling to its surface gains a substantial acceleration. In this case, the particle's radial velocity cannot be assumed to be constant even in the first order approximation. For this reason, we will calculate the numerical value of the above effect, which we theoretically discovered, for a light ray.

Consider a light ray travelling, say, from the Moon to the Earth's surface along the radial axis r in the equatorial plane of the Earth. In this case, the physically observable velocity of light is equal to $c^1 = -\,3\times10^{10}$ cm/sec, since the vector of the velocity of light is directed opposite to the reading of the radial coordinates, the origin of which is the centre of the Earth. The Earth rotates around its axis with the angular velocity $\omega = 1$ rev/day $= 1.16\times10^{-5}$ rev/sec, and the Earth's radius is equal to $r = 6.4\times10^{8}$ cm. As a result, we obtain that the curvature of space caused by the Earth's rotation around its axis deflects a light ray coming to the Earth's surface from the Moon ($\tau = 1$ sec) in the longitudinal direction in which the Earth rotates by the angle equal to

$$\Delta\varphi = \frac{\omega\,r}{2\,c^1}\left(1 - \mathrm{e}^{-\frac{2}{r}\,c^1\tau}\right) = 1.2\times10^{-7}\ \text{rev} = 0.16'',$$

where the main goal into the effect is made due to the first term, and the second term is equal to 1.5×10^{-41} and, therefore, can be neglected.

The effect calculated for the Earth is small. Meanwhile, this effect increases with the radius and rotation velocity of the cosmic body. For example, the Sun has the radius equal to $r = 7.0\times10^{10}$ cm, and rotates around its axis with the angular velocity $\omega = 4.5\times10^{-7}$ rev/sec. Therefore, the curvature of space caused by the Sun's rotation around its axis deflects a light ray coming to the Sun's surface in the longitudinal direction in which the Sun rotates by the angle equal to

$$\Delta\varphi = 5.3\times10^{-7}\ \text{rev} = 0.68''.$$

Obviously, this effect has a much larger numerical value near a rapidly rotating star, such as Wolf-Rayet stars or neutron stars.

2. Now consider the second equation of motion (along the geographical latitudes, where the polar angle θ is read from the North pole). With the same assumptions as those we used in the third equation above, neglecting higher order terms and taking the obtained solution $\mathrm{v}^3 = -\omega$ into account, this equation takes the form, respectively,

$$y' + ay = 0\,, \qquad \theta'' + a\theta' = 0\,,$$

where

$$y = \mathrm{v}^2 = \frac{d\theta}{d\tau}\,, \qquad a = \frac{2}{r}\,\mathrm{v}^1 = const.$$

The above differential equations are solved as

$$y = \frac{C}{\mathrm{e}^{ax}}\,, \qquad \theta = \frac{C_1}{\mathrm{e}^{ax}} + C_2\,,$$

where the constants of integration found using the initial conditions $x = x_0 = 0$ and $y = y_0 = 0$, are equal to $C = 0$, $C_1 = 0$ and $C_2 = \theta_0$. As a result, the solutions take the final form

$$\mathrm{v}^2 = 0\,, \qquad \theta = \theta_0\,,$$

i.e., a particle freely travelling to the surface of a rotating body is not deflected up or down the geographical latitudes.

3. Finally, consider the first equation of motion (along the radial coordinates). As is explained in the beginning, we assume that the particle's velocity in the radial direction gains a very small increment or decrement α' compared to its numerical value v^1, which, according to our initial assumption, is very large. Thus, we assume $\mathrm{v}^1 = const$ and solve the first equation of motion with respect to $\mathrm{v}^1 + \alpha'$, i.e., with respect to the small parameter α. Neglecting higher order terms and taking the obtained solutions $\mathrm{v}^3 = -\omega$ and $\mathrm{v}^2 = 0$ into account, the first equation of motion takes the form, respectively,

$$y' + b = 0\,, \qquad \alpha'' + b = 0\,,$$

where $y = \alpha'$ and $b = \omega^2 r = const$ (here r is the radius of the rotating body). These simplest equations are solved as

$$y = C - bx\,, \qquad \alpha = -\frac{bx^2}{2} + C_2\,x + C_1\,,$$

where, using the initial conditions $x = x_0 = 0$, $\alpha = \alpha_0 = 0$ and $y = y_0 = 0$, we find that the constants of integration are equal to zero. As a result, we obtain

$$\alpha' = -\omega^2 r\tau\,, \qquad \alpha = -\frac{\omega^2 r\tau^2}{2}\,.$$

This solution means that a particle freely travelling to a rotating body gains an additional speed, and the length of its path is physically "stretched" due to the curvature of space caused by the body's rotation. As a result, the particle reaches the body later (with a delay in time) compared if the body did not rotate.

Thus, according to the obtained solution, the increment of the path length of a light ray that travelled, say, from the Moon to the Earth, and also the delay in time of its arrival are equal to

$$\alpha = -1.7\,\text{cm}, \quad \Delta\tau = \frac{\alpha}{c^1} = 5.7\times10^{-11}\ \text{sec},$$

while such corrections for a light ray that travelled from the Earth to the Sun are equal to

$$\alpha = -6.6\times10^{4}\ \text{cm}, \quad \Delta\tau = \frac{\alpha}{c^1} = 2.2\times10^{-6}\ \text{sec}.$$

So, we theoretically found that a particle travelling freely to a rotating body is deflected slightly from its radial trajectory in the equatorial direction, in which the body rotates, i.e., along the geographical longitudes. In addition, during the travel, the particle gains a small increase of its velocity, and its path is physically "stretched" for a little, as a result of which the particle reaches the body with a delay in time compared to if the body did not rotate.

These two effects take place both for mass-bearing particles and for light rays (massless light-like particles).

The origin of these effects is the space curvature caused by the rotation of space. When any body rotates, the space around it curves towards the direction of its rotation and the centre of the body (the centre of rotation), thereby creating a "slope of the hill" descending "down" along the equator in the direction, in which the body rotates, and also to the centre of the body. When a particle travels freely to a rotating body, it "rolls down" the slope of the space curvature along the equator in the direction, in which the body rotates, as well as to the centre of the body.

These are two new fundamental effects of the General Theory of Relativity, we have discovered "au bout d'un stylo" in addition to the Einstein effect of the deflection of light rays in the field of a gravitating body.

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