Scalar Field Effects on the Space-Time Continuum and the Appearance of the Rest-Mass

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The failure to fulfill Lorentz's condition leads to the emergence of a new scalar field, which in turn should have the meaning of a new physical field. In this study, we prove that the appearance of the scalar field in the theory of the Elastodynamics of the Space-Time Continuum can more clearly explain the emergence of rest-mass and the expression of elementary particles through symmetric and anti-symmetric electromagnetic tensors. The use of the scalar field in the previous theory requires a redefinition of both the Lorentz force and the electrodynamic power, and then a rewrite of the electromagnetic stress tensor.

1 Introduction

In modern physics, we can ask the question *what is the origin of mass?* Einstein's famous equation $E = mc^2$ of special relativity theory can be written in an alternative form as $m = E/c^2$. When expressed in this form, it suggests the possibility of explaining mass in terms of energy. Einstein was aware of this possibility from the beginning. Indeed, his original 1905 paper was titled, "Does the Inertia of a Body Depend on Its Energy Content?". Anyway, when a collision between a high-energy electron and a high-energy positron occurs, we often observe that many particles emerge from this event. The total mass of these particles can be thousands of times the mass of the original electron and positron. Thus, mass has been physically created from energy. So energy and mass are equivalent, but the question remains: how is energy transformed into rest-mass?

Using the theory of the Elastodynamics of the Spacetime Continuum [9, 16] (which is a result of applying mechanical continuum laws (elastic continuum) to the space-time continuum), it can be shown that rest-mass energy density arises from the volume dilatation deformation of the space-time continuum, while distortion deformations correspond to massless shear transverse waves. Applying the previous theory to the electromagnetic waves, we find that there is no volume dilatation, which means that the rest-mass density of the photon is equal to zero. But with the existence of the scalar field Ψ (which requires a generalization of the Maxwell-Heaviside equations), it can be proven that rest-mass is no longer equal to zero.

2 Materials and methods

2.1 Generalize the Lorentz force and the electrodynamics power

The basic laws of classical electrodynamics can be summarized in differential form (Maxwell/Heaviside equations) by these four equations [1, see pp. 24]:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{1}$$

$$\vec{\nabla} \times \vec{B} - \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 J \tag{2}$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \overrightarrow{0} \tag{3}$$

$$\vec{\nabla} \cdot \vec{B} = 0\,. \tag{4}$$

Let $\vec{A}$ and φ be, respectively, the vector and scalar potentials of the classical electromagnetic field; they can be connected via different relations, called gauges or gauge conditions/relations, since they contain some arbitrariness. An important example of this is the Lorentz gauge [2]:

$$\varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0\,. \tag{5}$$

We will now assume that equality in (5) is not satisfied; that is, in addition to the presence of the electric and magnetic fields, there is a scalar field Ψ [3]:

$$\varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\Psi = \varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t} + \vec{\nabla} \cdot \vec{A}. \tag{6}$$

In order to introduce the scalar field into electromagnetic theory, Either new terms must be introduced into the Lagrangian of the electromagnetic field [4], which guarantees the expression of longitudinal waves in the equations of field motion. Or by introducing the invariant scalar field (our case) into Maxwell's equations, which provide a description of the longitudinal waves [5]. By adding derivatives of the field Ψ to Maxwell/Heaviside equations, we get the following [4–7]:

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla}\times\vec{B}-\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}-\vec{\nabla}\cdot\Psi=\mu_0\vec{J}$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\cdot\vec{E}+\frac{\partial\Psi}{\partial t}=\frac{\rho_e}{\varepsilon_0}\,. \tag{7}$$

Using (6)–(7), we can obtain the inhomogeneous potential wave equations (automatically) for both scalar and vector potentials without an extra gauge condition:

$$\varepsilon_0\mu_0\frac{\partial^2\varphi}{\partial t^2}-\nabla^2\varphi=\frac{\rho_e}{\varepsilon_0}\,. \tag{8}$$

$$\varepsilon_0\mu_0\frac{\partial^2\vec{A}}{\partial t^2}-\nabla^2\vec{A}=\mu_0\vec{J}. \tag{9}$$

From (7), we can make sure that the electric field, the magnetic field, and the scalar field all satisfy the following inhomogeneous field wave equations:

$$\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}-\nabla^2\vec{E}=\mu_0\left(-\nabla\frac{\rho_e}{\varepsilon_0}-\frac{\partial\vec{J}}{\partial t}\right) \tag{10}$$

$$\varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}-\nabla^2\vec{B}=\mu_0\vec{\nabla}\times\vec{J} \tag{11}$$

$$\varepsilon_0\mu_0\frac{\partial^2\Psi}{\partial t^2}-\nabla^2\Psi=\mu_0\left(\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho_e}{\partial t}\right). \tag{12}$$

The existence of the longitudinal expansion/contraction waves (12), implies the existence of an elastic continuum (which has volume dilatation) [6–9]. Maxwell's theory does not accept the existence of this type of wave, because Maxwell's theory is described by an antisymmetric tensor

$$F_{\mu\vartheta}=\partial_\mu A_\vartheta-\partial_\vartheta A_\mu$$

the trace of which equals zero, where A_μ is the four-dimensional electromagnetic potential. This tensor $F_{\mu\vartheta}$ can only describe transverse waves, which means that the vacuum used in electromagnetism cannot be compressed. Therefore, there was a need to introduce an elastic continuum by analogy with a continuous elastic medium (mechanical continuum) like the Foka-Podolsky Lagrangian [6]. In order to obtain both the generalized power and the generalized Lorentz force, a source transformation must be defined [7]:

$$\rho'_e=\rho_e-\varepsilon_0\frac{\partial\Psi}{\partial t}\,,\quad \vec{J}\,'=\vec{J}+\frac{1}{\mu_0}\vec{\nabla}\cdot\Psi\,. \tag{13}$$

The scalar field **S** used in [7], is associated with Ψ by $\Psi=-\mathbf{S}$. The electrodynamics power theorem is given by:

$$\mu_0(\vec{J}\cdot\vec{E})=-\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0\mu_0\vec{E}^2+\vec{B}^2\right)-\vec{\nabla}(\vec{E}\times\vec{B})\,. \tag{14}$$

Using (13–14), the electrodynamics power is transformed in the following way:

$$\vec{J}\cdot\vec{E}-\Psi\frac{\rho_e}{\mu_0\varepsilon_0}=-\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0\vec{E}^2+\frac{\vec{B}^2}{\mu_0}+\frac{\Psi^2}{\mu_0}\right)-\frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B}+\vec{E}\cdot\Psi) \tag{15}$$

where $\vec{J}\cdot\vec{E}-\Psi\frac{\rho_e}{\mu_0\varepsilon_0}$ represents the volume creation rate of electromagnetic energy (joules per cubic meter per second) or alternatively represents the rate of change of mechanical energy per unit volume, i.e. the rate at which the field does work on the charges per unit volume. The Lorenz force is given by:

$$\mu_0\left(\rho_e\vec{E}+\vec{J}\times\vec{B}\right)=\varepsilon_0\mu_0((\vec{\nabla}\cdot\vec{E})\cdot\vec{E}+(\vec{\nabla}\times\vec{E})\times\vec{E})+ +(\vec{\nabla}\times\vec{B})\times\vec{B}-\varepsilon_0\mu_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B})\,. \tag{16}$$

Using (13), the generalized Lorenz force is transformed into the following form:

$$\rho_e\cdot\vec{E}+\vec{J}\times\vec{B}-\Psi\vec{J}=\varepsilon_0((\vec{\nabla}\cdot\vec{E})\cdot\vec{E}+(\vec{\nabla}\times\vec{E})\times\vec{E})+ +\frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\times\vec{B}-\varepsilon_0\frac{\partial}{\partial t}(\vec{E}\times\vec{B}-\Psi\cdot\vec{E})+ +\frac{1}{2\mu_0}\vec{\nabla}\Psi^2-\frac{1}{\mu_0}\vec{\nabla}\times(\Psi\cdot\vec{B}) \tag{17}$$

where $\left(\rho_e\cdot\vec{E}+\vec{J}\times\vec{B}-\Psi\vec{J}\right)$ represents the rate of change of mechanical momentum per unit volume and time. Note that the scalar field and the electric vector field have different signs indicating that the scalar field decelerates the charge, and that the deceleration is proportional to the current density, which in turn is proportional to the velocity of the charge. Thus, the electric vector field accelerates the charge while the scalar field decelerates it.

3 Elastodynamics of the Space-Time Continuum

Einstein's general theory of relativity is based on the geometry of continuous spacetime, which can be described by the following field equation [8, see pp. 875]:

$$R_{\mu\vartheta}-\frac{1}{2}g_{\mu\vartheta}R+g_{\mu\vartheta}L=\frac{8\pi G}{c}T_{\mu\vartheta} \tag{18}$$

where
$R_{\mu\vartheta}$: Ricci curvature tensor,
$g_{\mu\vartheta}$: metric tensor,
R: curvature scalar,
L: the cosmological constant, which can be neglected for small distances,
$T_{\mu\vartheta}$: the stress energy-momentum tensor.
In (18), everything on the left-hand side refers to the curvature of spacetime, and everything on the right-hand side refers to mass and energy.

According to the theory of the Elastodynamics of the Space-Time Continuum [9, 16], energy propagates in the Space-Time Continuum, which causes deformation of the Space-Time Continuum with longitudinal waves corresponding to mass and transverse waves corresponding to massless field energy. This leads implicitly to the proposition that the space-time continuum must be a deformable continuum. This deformation, which has a physical nature [9], can be expressed through strain that results from stress, so the stress energy-momentum tensor results in strains in the space-time continuum (strained space-time). The presence of strain in the space-time continuum leads to a deformation in the geometry of this space-time continuum. We can say it in the following way: the energy-momentum stress tensor produces a strain in the spacetime continuum, and that strain changes the geometry of the space-time continuum, and leads to the deformations with the longitudinal component being mass. The stress-strain relation for an isotropic and homogeneous space-time continuum can be written as the following [10]:

$$2\Upsilon_0\varepsilon^{\mu\vartheta} + \lambda_0 g^{\mu\vartheta}\varepsilon = T^{\mu\vartheta}. \tag{19}$$

Eq. (19) gives the stress in term of strain for a homogeneous and isotropic space-time continuum, both Υ_0 and λ_0 are Lamé constants, and they are linked together through K_0 the bulk modulus:

$$\frac{1}{2}\Upsilon_0 = K_0 - \lambda_0. \tag{20}$$

Here Y_0 is the shear modulus, which corresponds to the resistance of the space-time continuum to distortions, K_0 represents the resistance of the space-time continuum to dilatation, where distortions describe a change of shape of the space-time continuum without a change in volume, and dilatation describes a change of volume without a change of shape of the space-time continuum [9-10], $T^{\mu\vartheta}$ is the energy-momentum stress tensor, the tensor $\varepsilon^{\mu\vartheta}$ is the strain tensor, the volume dilatation $\varepsilon = \varepsilon^{\alpha}{}_{\alpha}$ is the trace $\varepsilon^{\mu\vartheta}$. If we compare (19) and (18) we find an interesting similarity [9] (if we neglect the cosmological constant). The trace $T^{\alpha}{}_{\alpha}$ of (19) takes the following relation:

$$2\left(\Upsilon_0 + 2\lambda_0\right)\varepsilon = T^{\alpha}{}_{\alpha}. \tag{21}$$

The total rest-mass energy density of the system is related to the trace $T^{\alpha}{}_{\alpha}$, by the following [11-12]:

$$T^{\alpha}{}_{\alpha}\left(x^k\right) = \rho c^2. \tag{22}$$

Using the last formula in (21), we get the relation between the invariant volume dilatation and the invariant rest-mass:

$$2\left(\Upsilon_0 + 2\lambda_0\right)\varepsilon = \rho c^2. \tag{23}$$

By using (20), (23) takes the following expression:

$$4K_0\varepsilon = \rho c^2. \tag{24}$$

Eq. (24) shows that the rest-mass is the result of the dilatation of the spacetime continuum; the volume dilatation is an invariant, as is the rest-mass energy density. The strain energy density of the space-time continuum is a scalar given by [9]:

$$\mathcal{E} = \frac{1}{2}T^{\mu\vartheta}\varepsilon_{\mu\vartheta}. \tag{25}$$

In order to get the dilatation energy density and distortion energy density, we first need to write the tensor decomposition of $\varepsilon^{\mu\vartheta}$ as a sum of a strain deviator (distortion) tensor $e^{\mu\vartheta}$ and a scalar (dilatation) tensor e_s [9]:

$$\varepsilon^{\mu\vartheta} = e^{\mu\vartheta} + e_s g^{\mu\vartheta} \tag{26}$$

where:

$$e^{\mu}_{\vartheta} = \varepsilon^{\mu}{}_{\vartheta} - e_s\,\delta^{\mu}_{\vartheta},$$

$$e_s = \frac{1}{4}e^{\alpha}_{\alpha} = \frac{1}{4}\varepsilon. \tag{27}$$

In the same way, the energy-momentum stress tensor is decomposed into a stress deviator tensor $t^{\mu\vartheta}$ and a scalar t_s [9]:

$$T^{\mu\vartheta} = t^{\mu\vartheta} + t_s g^{\mu\vartheta} \tag{28}$$

where:

$$t^{\mu}_{\vartheta} = T^{\mu}_{\vartheta} - t_s\,\delta^{\mu}_{\vartheta},$$

$$t_s = \frac{1}{4}T^{\alpha}{}_{\alpha}. \tag{29}$$

Using (26–29), one can get the following expression for the scalar $\mathcal{E}$ [13]:

$$\mathcal{E} = \frac{1}{2}K_0\varepsilon^2 + \Upsilon_0 e^{\mu\vartheta}e_{\mu\vartheta} = \mathcal{E}_{\parallel} + \mathcal{E}_{\perp} \tag{30}$$

where:

$$\mathcal{E}_{\parallel} = \frac{1}{32K_0}\left(\rho c^2\right)^2 = \frac{1}{2}K_0\varepsilon^2, \quad \mathcal{E}_{\perp} = \Upsilon_0 e^{\mu\vartheta}e_{\mu\vartheta}. \tag{31}$$

The strain energy density of the space-time continuum can also be written in the following way [13]:

$$\mathcal{E} = \frac{1}{2K_0}t_s^2 + \frac{1}{4\Upsilon_0}t^{\mu\vartheta}t_{\mu\vartheta}. \tag{32}$$

From (30) or (32), we can see that the strain energy density is separated into two terms: the first term corresponds to the rest-mass longitudinal density (the dilatation energy density), while the second is the massless transverse term (the distortion energy density). Now we need to calculate the strain energy density in two cases:

$$\tilde{\sigma}^{\mu\vartheta} = \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0\vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2 + \frac{1}{\mu_0}\Psi^2\right) & \mathrm{S_x/c} - \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_x}\Psi & \mathrm{S_y/c} - \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_y}\Psi & \mathrm{S_z/c} - \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_z}\Psi \\ \mathrm{S_x/c} + \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_x}\Psi & -\mathrm{T_{xx}} - \frac{1}{2\mu_0}\Psi^2 & -\mathrm{T_{xy}} - \frac{1}{\mu_0}\Psi_{B_z} & -\mathrm{T_{xz}} + \frac{1}{\mu_0}\Psi_{B_y} \\ \mathrm{S_y/c} + \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_y}\Psi & -\mathrm{T_{yx}} + \frac{1}{\mu_0}\Psi_{B_z} & -\mathrm{T_{yy}} - \frac{1}{2\mu_0}\Psi^2 & -\mathrm{T_{yz}} - \frac{1}{\mu_0}\Psi_{B_x} \\ \mathrm{S_z/c} + \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_z}\Psi & -\mathrm{T_{zx}} - \frac{1}{\mu_0}\Psi_{B_y} & -\mathrm{T_{zy}} + \frac{1}{\mu_0}\Psi_{B_x} & -\mathrm{T_{zz}} - \frac{1}{2\mu_0}\Psi^2 \end{pmatrix}$$

3.1 Case number (1)

Electromagnetic stress tensor $\sigma^{\mu\vartheta}$ as strain energy density (in case $\Psi = 0$). Using $\sigma_{\alpha\beta} = \eta_{\alpha\mu}\eta_{\beta\vartheta}\sigma^{\mu\vartheta}$, we obtain the following [9]:

$$\sigma_{\alpha\beta} = \begin{pmatrix} \frac{\varepsilon_0}{2}\overrightarrow{\mathrm{E}}^2 + \frac{1}{2\mu_0}\overrightarrow{\mathrm{B}}^2 & -\mathrm{S_x/c} & -\mathrm{S_y/c} & -\mathrm{S_z/c} \\ -\mathrm{S_x/c} & -\mathrm{T_{xx}} & -\mathrm{T_{xy}} & -\mathrm{T_{xz}} \\ -\mathrm{S_y/c} & -\mathrm{T_{yx}} & -\mathrm{T_{yy}} & -\mathrm{T_{yz}} \\ -\mathrm{S_z/c} & -\mathrm{T_{zx}} & -\mathrm{T_{zy}} & -\mathrm{T_{zz}} \end{pmatrix} \qquad (33)$$

where $\mathrm{T_{ij}} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{\mathrm{ij}}\mathrm{E}^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{\mathrm{ij}}\mathrm{B}^2\right)$ is the Maxwell stress tensor. The dilatation energy density (the "mass" longitudinal term) is given by [9]:

$$\mathcal{E}_{\parallel} = \frac{1}{2}K_0\varepsilon^2 = \frac{1}{2K_0}t_s^2 = \frac{1}{32K_0}(\sigma^\alpha_\alpha)^2 \qquad (34)$$

where:

$$\sigma^\alpha_\alpha = \eta_{00}\sigma^{00} + \eta_{11}\sigma^{11} + \eta_{22}\sigma^{22} + \eta_{33}\sigma^{33} \qquad (35)$$

with the metric $\eta^{\vartheta\mu}$ of signature $(+1, -1, -1, -1)$.

The tensor ${\sigma^\alpha}_\alpha$ can be calculated [9,13]:

$${\sigma^\alpha}_\alpha = \frac{1}{2}\left(\varepsilon_0\overrightarrow{\mathrm{E}}^2 + \frac{1}{\mu_0}\overrightarrow{\mathrm{B}}^2\right) + \mathrm{T_{xx}} + \mathrm{T_{yy}} + \mathrm{T_{zz}} = 0 \qquad (36)$$

giving ${\sigma^\alpha}_\alpha = 0$, which means the longitudinal term (the rest-mass term) is equal to zero:

$$\mathcal{E}_{\parallel} = \frac{1}{32K_0}\left(\rho c^2\right)^2 = \frac{1}{32K_0}(\sigma^\alpha_\alpha)^2 = 0\,. \qquad (37)$$

In another sense, the rest-mass of the photon is zero. The term $\mathcal{E}_\perp$ is given by (31) and takes the final expression [9,13]:

$$\mathcal{E}_\perp = \frac{1}{4\Upsilon_0}\sigma^{\mu\vartheta}\sigma_{\mu\vartheta} = \frac{1}{\Upsilon_0}\left(U_{em}^2 - \frac{1}{c^2}S^2\right) \qquad (38)$$

where: $U_{em} = \frac{1}{2}\varepsilon_0\left(\overrightarrow{\mathrm{E}}^2 + c^2\overrightarrow{\mathrm{B}}^2\right)$ is the electromagnetic field energy density.

3.2 Case Number (2)

Electromagnetic stress tensor as strain energy density (in case $\Psi \neq 0$). We found that when $\Psi = 0$, the rest mass density is zero. Now, we need to repeat the previous procedure of Case (1) with the existence of the scalar field ($\Psi \neq 0$). To achieve this we should calculate the tensor $\sigma_{\alpha\beta}$ with the existence of the scalar field Ψ: when $\Psi \neq \mathbf{0}$, the tensor $\sigma^{\mu\vartheta}$ changes to the tensor $\tilde{\sigma}^{\mu\vartheta}$ and this new tensor must fulfill the relations (15-17):

$$\partial_\mu\tilde{\sigma}^{\mu\vartheta} = \begin{pmatrix} -\frac{1}{c}\left(\vec{J}\cdot\vec{E} - \Psi\,\frac{\rho_\mathrm{e}}{\mu_0\varepsilon_0}\right) \\ -\left(\rho_e\cdot\vec{E} + \vec{J}\times\vec{B} - \Psi\vec{J}\right) \end{pmatrix}. \qquad (39)$$

The tensor $\tilde{\sigma}^{\mu\vartheta}$ that achieves the relation (39) is written in the following Eq. (40) shown at the top of the page.

Shown at the top of the page. (40)

Note that when $\Psi \to 0$, then $\tilde{\sigma}^{\mu\vartheta} \to \sigma^{\mu\vartheta}$, and quantity $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is the inverse of the impedance of free space ${\mathrm{z}_0}^{-1}$. The next step is to calculate the longitudinal mass term:

$$\begin{aligned} \tilde{\sigma}^\alpha_\alpha &= \eta_{00}\tilde{\sigma}^{00} + \eta_{11}\tilde{\sigma}^{11} + \eta_{22}\tilde{\sigma}^{22} + \eta_{33}\tilde{\sigma}^{33} = \\ &= \frac{1}{2}\left(\varepsilon_0\overrightarrow{\mathrm{E}}^2 + \frac{1}{\mu_0}\overrightarrow{\mathrm{B}}^2 + \frac{1}{\mu_0}\Psi^2\right) + \mathrm{T_{xx}} + \frac{1}{2\mu_0}\Psi^2 + \\ &+ \mathrm{T_{yy}} + \frac{1}{2\mu_0}\Psi^2 + \mathrm{T_{zz}} + \frac{1}{2\mu_0}\Psi^2\,. \end{aligned} \qquad (41)$$

Taking into account the properties of tensor $\mathrm{T_{ij}}$ and (35–37), we find the following:

$$\tilde{\sigma}^\alpha_\alpha = \frac{2}{\mu_0}\Psi^2\,. \qquad (42)$$

Thus, the mass term is no longer equal to zero:

$$\tilde{\mathcal{E}}_\parallel = \frac{1}{32K_0}\left(\rho c^2\right)^2 = \frac{1}{32K_0}(\tilde{\sigma}^\alpha_\alpha)^2 = \frac{1}{32K_0}\frac{4}{\mu_0^2}\Psi^4\,. \qquad (43)$$

The rest-mass term takes the following expression:

$$\rho = \pm 2\varepsilon_0|\Psi|^2\,. \qquad (44)$$

The massless transverse terms (the distortion energy density) can be calculated as follows:

$$\tilde{\mathcal{E}}_\perp = \frac{1}{4\Upsilon_0}\tilde{t}^{\mu\vartheta}\tilde{t}_{\mu\vartheta}, \qquad \text{where } \tilde{t}^{\mu\vartheta} = \tilde{\sigma}^{\mu\vartheta} \text{ and } \tilde{t}_{\mu\vartheta} = \tilde{\sigma}_{\mu\vartheta}.$$

$$\tilde{\sigma}_{\alpha\beta} = \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 \vec{\mathrm{E}}^2 + \frac{1}{\mu_0}\vec{\mathrm{B}}^2 + \frac{1}{\mu_0}\Psi^2\right) & -\mathrm{S_x}/c + \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_X}\Psi & -\mathrm{S_y}/c + \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_y}\Psi & -\mathrm{S_z}/c + \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_z}\Psi \\ -\mathrm{S_x}/c - \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_X}\Psi & -\mathrm{T_{xx}} - \frac{1}{2\mu_0}\Psi^2 & -\mathrm{T_{xy}} - \frac{1}{\mu_0}\Psi B_z & -\mathrm{T_{xz}} + \frac{1}{\mu_0}\Psi B_y \\ -\mathrm{S_y}/c - \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_y}\Psi & -\mathrm{T_{yx}} + \frac{1}{\mu_0}\Psi B_z & -\mathrm{T_{yy}} - \frac{1}{2\mu_0}\Psi^2 & -\mathrm{T_{yz}} - \frac{1}{\mu_0}\Psi B_x \\ -\mathrm{S_z}c - \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_z}\Psi & -\mathrm{T_{zx}} - \frac{1}{\mu_0}\Psi B_y & -\mathrm{T_{zy}} + \frac{1}{\mu_0}\Psi B_x & -\mathrm{T_{zz}} - \frac{1}{2\mu_0}\Psi^2 \end{pmatrix}. \qquad (45)$$

By using $\tilde{\sigma}_{\alpha\beta} = \eta_{\alpha\mu}\eta_{\beta\vartheta}\tilde{\sigma}^{\mu\vartheta}$, the tensor $\tilde{\sigma}_{\mu\vartheta}$ can be written as in (45) above at the top of the page. The term $\tilde{\sigma}^{\mu\vartheta}\tilde{\sigma}_{\mu\vartheta}$ can now be calculated as in (46) below. The formula in (46) is simplified as in (47) below.

$$\begin{aligned} \tilde{\sigma}^{\mu\vartheta}\tilde{\sigma}_{\mu\vartheta} &= \frac{1}{4}\left(\varepsilon_0 \vec{\mathrm{E}}^2 + \frac{1}{\mu_0}\vec{\mathrm{B}}^2 + \frac{1}{\mu_0}\Psi^2\right)^2 + \mathrm{T_{xx}}^2 + \\ &+ \frac{1}{\mu_0}\mathrm{T_{xx}}\Psi^2 + \frac{1}{4\mu_0^2}\Psi^4 + \mathrm{T_{yy}}^2 + \frac{1}{\mu_0}\mathrm{T_{yy}}\Psi^2 + \\ &+ \frac{1}{4\mu_0^2}\Psi^4 + \mathrm{T_{zz}}^2 + \frac{1}{\mu_0}\mathrm{T_{zz}}\Psi^2 + \frac{1}{4\mu_0^2}\Psi^4 - \\ &- 2\left(\frac{\mathrm{S_x}}{c}\right)^2 - 2\left(\sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_x}\Psi\right)^2 - 2\left(\frac{\mathrm{S_y}}{c}\right)^2 - \\ &- 2\left(\sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_y}\Psi\right)^2 - 2\left(\frac{\mathrm{S_z}}{c}\right)^2 - 2\left(\sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{E_z}\Psi\right)^2 + \\ &+ 2\left(\mathrm{T_{xy}}\right)^2 + 2\left(\frac{1}{\mu_0}\Psi_{B_x}\right)^2 + 2\left(\mathrm{T_{xz}}\right)^2 + \\ &+ 2\left(\frac{1}{\mu_0}\Psi_{B_y}\right)^2 + 2\left(\mathrm{T_{zy}}\right)^2 + 2\left(\frac{1}{\mu_0}\Psi_{B_z}\right)^2. \end{aligned} \qquad (46)$$

$$\begin{aligned} \tilde{\sigma}^{\mu\vartheta}\tilde{\sigma}_{\mu\vartheta} &= \left\{\frac{1}{4}\left(\varepsilon_0 \vec{\mathrm{E}}^2 + \frac{1}{\mu_0}\vec{\mathrm{B}}^2\right)^2 + \mathrm{T_{xx}}^2 + \mathrm{T_{yy}}^2 + \right. \\ &+ \mathrm{T_{zz}}^2 - 2\left(\frac{\mathrm{S_x}}{c}\right)^2 - 2\left(\frac{\mathrm{S_y}}{c}\right)^2 - 2\left(\frac{\mathrm{S_z}}{c}\right)^2 \\ &\left. + 2\left(\mathrm{T_{xy}}\right)^2 + 2\left(\mathrm{T_{xz}}\right)^2 + 2\left(\mathrm{T_{zy}}\right)^2\right\} + \\ &+ \left\{\frac{1}{2}\left(\varepsilon_0 \vec{\mathrm{E}}^2 + \frac{1}{\mu_0}\vec{\mathrm{B}}^2\right) + \mathrm{T_{xx}} + \mathrm{T_{yy}} + \mathrm{T_{zz}}\right\} \cdot \frac{1}{\mu_0}\Psi^2 + \\ &+ \frac{2}{\mu_0}\Psi^2\left\{\frac{\vec{\mathrm{B}}^2}{\mu_0} - \varepsilon_0 \vec{\mathrm{E}}^2\right\} + \frac{1}{\mu_0^2}\Psi^4. \end{aligned} \qquad (47)$$

By making use of (36–38), we find the following [9]:

$$\begin{aligned} &\frac{1}{4}\left(\varepsilon_0 \vec{\mathrm{E}}^2 + \frac{1}{\mu_0}\vec{\mathrm{B}}^2\right)^2 + \mathrm{T_{xx}}^2 + \mathrm{T_{yy}}^2 + \mathrm{T_{zz}}^2 - \\ &- 2\left(\frac{\mathrm{S_x}}{c}\right)^2 - 2\left(\frac{\mathrm{S_y}}{c}\right)^2 - 2\left(\frac{\mathrm{S_z}}{c}\right)^2 + \\ &+ 2\left(\mathrm{T_{xy}}\right)^2 + 2\left(\mathrm{T_{xz}}\right)^2 + 2\left(\mathrm{T_{zy}}\right)^2 = \\ &= \varepsilon_0^2\left(\vec{\mathrm{E}}^2 + c^2\vec{\mathrm{B}}^2\right)^2 - \frac{4}{c^2}\left(\mathrm{S_x^2} + \mathrm{S_y^2} + \mathrm{S_z^2}\right) = \sigma^{\mu\vartheta}\sigma_{\mu\vartheta} \end{aligned} \qquad (48)$$

$$\frac{1}{2}\left(\varepsilon_0 \vec{\mathrm{E}}^2 + \frac{1}{\mu_0}\vec{\mathrm{B}}^2\right) + \mathrm{T_{xx}} + \mathrm{T_{yy}} + \mathrm{T_{zz}} = \sigma^{\alpha}{}_{\alpha} = 0. \qquad (49)$$

Finally:

$$\tilde{\sigma}^{\mu\vartheta}\tilde{\sigma}_{\mu\vartheta} = \sigma^{\mu\vartheta}\sigma_{\mu\vartheta} + \frac{2}{\mu_0}\Psi^2\left\{\frac{\vec{\mathrm{B}}^2}{\mu_0} - \varepsilon_0 \vec{\mathrm{E}}^2\right\} + \frac{1}{\mu_0^2}\Psi^4 \qquad (50)$$

which means that the massless transverse terms (the distortion energy density) take the following expression:

$$\tilde{\mathcal{E}}_{\perp} = \mathcal{E}_{\perp} + \frac{1}{2\Upsilon_0\mu_0}\Psi^2\left\{\frac{\vec{B}^2}{\mu_0} - \varepsilon_0 \vec{\mathrm{E}}^2\right\} + \frac{1}{4\Upsilon_0\mu_0^2}\Psi^4. \qquad (51)$$

4 Results and discussions

Because of the continuity equation (when $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho_e}{\partial t} = 0$), the discovery of the scalar field Ψ is not as easy as the discovery of the electromagnetic fields. This means that the left-hand side of (12) can be zero for a scalar field that is not equal to zero. Then (12) can be written in the form of two equations:

$$\varepsilon_0\mu_0\frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi = 0\,, \quad \vec{\nabla}\cdot\vec{J} + \frac{\partial\rho_e}{\partial t} = 0\,.$$

From the last two equations, we can note that the wave equation $\varepsilon_0\mu_0\frac{\partial^2\Psi}{\partial t^2} - \nabla^2\Psi = 0$, is as fundamental an equation as the continuity equation $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho_e}{\partial t} = 0$ [6]. Because the existence of the scalar field is linked to the appearance of the rest mass in the electromagnetic field, the motion of charges in accordance with the equation $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho_e}{\partial t} = 0$, always conjugates the longitudinal waves and happens with volume dilatation. We can write Maxwell's equations (1–4) through the electromagnetic tensor $F_{\mu\vartheta}$:

$$\partial^{\mu}\left[F_{\mu\vartheta}\right] = J_{\vartheta}\,. \qquad (52)$$

The previous tensor is an antisymmetric tensor, which can be written in the following formula:

$$\left[F_{\mu\vartheta}\right] = \frac{1}{2}\left(\left[a_{\mu\vartheta}\right] - \left[a_{\vartheta\mu}\right]\right) \qquad (53)$$

where $a_{\mu\vartheta}$ is an asymmetric tensor, which takes the following

$$\left[S_{\mu\vartheta}\right]=\frac{1}{2}\begin{pmatrix} \frac{2}{c^2}\frac{\partial\varphi}{\partial t} & -\frac{\partial A^x}{\partial t}+\frac{1}{c}\frac{\partial\varphi}{\partial x} & -\frac{\partial A^y}{\partial t}+\frac{1}{c}\frac{\partial\varphi}{\partial y} & -\frac{\partial A^z}{\partial t}+\frac{1}{c}\frac{\partial\varphi}{\partial z} \\ -\frac{\partial A^x}{\partial t}+\frac{1}{c}\frac{\partial\varphi}{\partial x} & -2\frac{\partial A^x}{\partial x} & -\frac{\partial A^x}{\partial y}-\frac{\partial A^y}{\partial x} & -\frac{\partial A^x}{\partial z}-\frac{\partial A^z}{\partial x} \\ -\frac{\partial A^y}{\partial t}+\frac{1}{c}\frac{\partial\varphi}{\partial y} & -\frac{\partial A^y}{\partial x}-\frac{\partial A^x}{\partial y} & -2\frac{\partial A^y}{\partial y} & -\frac{\partial A^y}{\partial z}-\frac{\partial A^z}{\partial y} \\ -\frac{\partial A^z}{\partial t}+\frac{1}{c}\frac{\partial\varphi}{\partial z} & -\frac{\partial A^z}{\partial x}-\frac{\partial A^x}{\partial z} & -\frac{\partial A^z}{\partial y}-\frac{\partial A^y}{\partial z} & -2\frac{\partial A^z}{\partial z} \end{pmatrix}$$

expression:

$$\left[a_{\mu\vartheta}\right]=\begin{pmatrix} \frac{1}{c^2}\frac{\partial\varphi}{\partial t} & -\frac{\partial A^x}{\partial t} & -\frac{\partial A^y}{\partial t} & -\frac{\partial A^z}{\partial t} \\ \frac{1}{c}\frac{\partial\varphi}{\partial x} & -\frac{\partial A^x}{\partial x} & -\frac{\partial A^y}{\partial x} & -\frac{\partial A^z}{\partial x} \\ \frac{1}{c}\frac{\partial\varphi}{\partial y} & -\frac{\partial A^x}{\partial y} & -\frac{\partial A^y}{\partial y} & -\frac{\partial A^z}{\partial y} \\ \frac{1}{c}\frac{\partial\varphi}{\partial z} & -\frac{\partial A^x}{\partial z} & -\frac{\partial A^y}{\partial z} & -\frac{\partial A^z}{\partial z} \end{pmatrix}. \tag{54}$$

We can write another tensor, which is a symmetric tensor $S_{\mu\vartheta}$:

$$\left[S_{\mu\vartheta}\right]=\frac{1}{2}\left(\left[a_{\mu\vartheta}\right]+\left[a_{\vartheta\mu}\right]\right), \tag{55}$$

which is given explicitly at the top of this page.

Using the formula $S^{\alpha}{}_{\alpha}=\eta^{\alpha\beta}S_{\alpha\beta}$, we can get the diagonal components of this tensor to describe the electromagnetic potential $\partial^{\vartheta}A_{\vartheta}$:

$$S^{\alpha}{}_{\alpha}=\Psi=\varepsilon_0\mu_0\frac{\partial\varphi}{\partial t}+\vec{\nabla}\cdot\vec{A}. \tag{56}$$

Therefore, the Lorentz condition is a cancellation of four-dimensional volume dilatation from the space-time continuum. According to [14-15], the pair $\left(S_{\mu\vartheta},F_{\mu\vartheta}\right)$ of tensors can explain the matter-field duality, $F_{\mu\vartheta}$ describes the field properties $\left(F_{\mu\vartheta}\right.$ as the field tensor.$\left.\right)$ and $S_{\mu\vartheta}$ contains matter waves (matter tensor with $S^{\alpha}{}_{\alpha}\neq 0$), which corresponds to (44), and also to (17), which confirms that the scalar field hinders the movement and therefore plays a role similar to inertia. Both tensors $\left(S_{\mu\vartheta},F_{\mu\vartheta}\right)$ can display fundamental properties such as energies or electric charge or rest-mass. Tensor $a_{\mu\vartheta}$ is equivalent to the formula $\left\{a_{\mu\vartheta}\sim\partial_\mu A_\vartheta\right\}$, and the tensor $F_{\mu\vartheta}$ is equivalent to formula $\left\{\partial_\mu A_\vartheta-\partial_\vartheta A_\mu\right\}$, finally the tensor $S_{\mu\vartheta}$ is $\left\{\partial_\mu A_\vartheta+\partial_\vartheta A_\mu\right\}$.

According to the theory of the Elastodynamics of the Space-Time Continuum, the antisymmetric rotation tensor $\omega^{\mu\vartheta}$ can be written in the following [9-11]:

$$\omega^{\mu\vartheta}=\frac{1}{2}\left(u^{\mu;\vartheta}-u^{\vartheta;\mu}\right) \tag{57}$$

where u^μ is the displacement of an infinitesimal element of the spacetime continuum from its unstrained position x_μ. The tensor in (57) corresponds to tensor $F^{\mu\vartheta}$ [9, 16, see pp. 64]:

$$F^{\mu\vartheta}=\varphi_0\omega^{\mu\vartheta}. \tag{58}$$

In order to fulfill Lorentz's condition, the electromagnetic potential four-vector A^μ satisfies the following relationship [9, 16]:

$$A^\mu=-\frac{1}{2}\varphi_0 u^\mu_\perp \tag{59}$$

where the constant φ_0 is referred to as the "space-time continuum electromagnetic shearing potential constant" [9, 16, see pp. 64] and $u^\mu_\perp$ indicates that the relation holds for a transverse displacement. From the last equation, we get the Lorentz condition directly $\partial_\mu A^\mu=0$. The previous case corresponds to antisymmetric tensor $F^{\mu\vartheta}$. However, in our case, Lorentz's condition is not satisfied, and therefore we need to generalize the previous relationship (59) to include symmetric tensor $S_{\mu\vartheta}$. According to the theory of the Elastodynamics of the Space-Time Continuum, the symmetric strain tensor $\varepsilon^{\mu\vartheta}$, which is equivalent to a tensor $S^{\mu\vartheta}$, can be written as the following [9. 16, see pp. 53]:

$$\varepsilon^{\mu\vartheta}=\frac{1}{2}\left(u^{\mu;\vartheta}+u^{\vartheta;\mu}\right). \tag{60}$$

The displacements in expressions derived from (60) are written as $u_\parallel$, which means that symmetric displacements are along the direction of motion (longitudinal). We can now write (59) in the following general form:

$$A^\mu=f\left(u^\mu\right). \tag{61}$$

From (57), we can write the following:

$$\partial_\vartheta A^\mu=\partial_\vartheta f\left(u^\mu\right)=\frac{\partial f\left(u^\mu\right)}{\partial u^\mu}\frac{\partial u^\mu}{\partial x_\vartheta}=\frac{\partial f\left(u^\mu\right)}{\partial u^\mu}\left\{\varepsilon^\mu_\vartheta+\omega^\mu_\vartheta\right\}. \tag{62}$$

Eq. (62) also comes automatically from [9, 16], therefore, we can consider the field A^μ as a real physical vacuum in which both electromagnetic waves and elementary particles can propagate and arise due to the dynamic distortion and dilatation of this medium. The mass that appeared in (44) is real rest-mass density, but there are two options: positive rest-mass density and negative rest-mass density $\left(\rho \sim \pm|\Psi|^2\right)$. By multiplying (32) by (32 K_0) and taking into account (31) and the scalar function, we get the following [9, 16]:

$$32K_0\varepsilon = \left(\rho c^2\right)^2 + \frac{8K_0}{\Upsilon_0}\,\tilde{t}^{\mu\vartheta}\tilde{t}_{\mu\vartheta}\,. \tag{63}$$

The last expression is similar to the energy relation of Special Relativity, which can be written after taking the square root as follows:

$$\mathrm{E} = \pm\hbar\omega = \pm c\sqrt{(\rho c)^2 + \frac{8K_0}{c^2\Upsilon_0}\,\tilde{t}^{\mu\vartheta}\tilde{t}_{\mu\vartheta}} \tag{64}$$

where E is the total energy density, noting that $\tilde{t}^{\mu\vartheta}\tilde{t}_{\mu\vartheta}$ is quadratic in structure [13], and equivalent to the momentum density. As we see in (64), the energy equation accepts negative solutions. Generally, (63) is the Klein-Gordon equation. Eq. (44) is reminiscent of the wave function in quantum mechanics, which means the volume density of the particles; thus, we can say that the wave function in quantum mechanics describes the propagation of longitudinal waves in the space-time continuum [9]. Finally, note that the tensor $\sigma^{\mu\vartheta}$ is symmetric, but the tensor $\tilde{\sigma}^{\mu\vartheta}$ is not; the symmetry was broken after the mass appeared. We can confirm that the equations that describe the behavior of elementary particles become fundamentally simpler and more symmetric when the mass of the particles is zero.

5 Conclusion

We found that the addition of the scalar field to the Maxwell-Heaviside equations requires a generalization of both the Lorentz force and power. Using the Elastodynamics of the Space-time Continuum theorem, and after calculating the electromagnetic stress tensor which includes the previous generalizations, the positive rest-mass and the negative rest-mass appear, meaning that the photon acquires mass, which in turn corresponds to the volume dilatation of the space-time continuum.

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