LETTERS TO PROGRESS IN PHYSICS

Cosmological Redshift: Which Cosmological Model Best Explains It?

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Here we list three options that General Relativity has proposed over the past decade to explain the non-linear cosmological redshift, observed by astronomers. 1) If the redshift law is linear for nearby galaxies, then turns into exponential for distant galaxies, and triangulation of galaxies reveals non-zero curvature of space, then our Universe is an expanding Friedmann world. 2) If the linear redshift law turns into parabolic for distant galaxies, then our Universe is a static de Sitter world with $\lambda > 0$, in which the physical vacuum has a positive density, the observable curvature of space is positive, and the non-Newtonian gravitational forces acting there are repulsive forces increasing with distance (which cause photons to lose energy as they move). 3) If for distant galaxies the linear redshift law turns into exponential, but triangulation of galaxies does not reveal even the slightest curvature of space, then our Universe has a flat space, where the redshift in the spectra of distant objects is due only to the fact that the light-like sub-space (home of photons) of any metric space-time rotates with the speed of light, thereby creating a repulsive centrifugal force (which causes photons to lose energy as they move). In this case, any particular space metric creates only an addition to the exponential redshift law, which must take place even in a flat unperturbed space.

Cosmological redshift was discovered by Vesto Slipher (Flagstaff Observatory, Arizona), who first registered it on September 17, 1912 in the spectrum of Andromeda Nebula M31 [1], then over subsequent years in the spectra of other galaxies [2, 3]. Slipher's discovery of the cosmological redshift and the key contribution of his measurements into the discovery of the redshift law are explained in detail in the comprehensive 2013 surveys [4–6].

Slipher explained this result by the Doppler effect, saying that most galaxies move away from the observer with high velocities (therefore their spectra become redshifted). A few years after the discovery in the early 1920s, a number of scientists came up with the idea of explaining the cosmological redshift in the framework of one of the cosmological models proposed by the General Theory of Relativity. They all tried to deduce the dependence of the redshift and the corresponding radial velocity of galaxies on their distance from the observer as the Doppler effect in the framework of de Sitter's cosmological model. These were researchers such as Ludwik Silberstein, Knut Lundmark, Carl Wirtz, Edwin Hubble, Willem de Sitter. Their work is discussed in detail in recent historical studies by Michael Way, Harry Nussbaumer, Cormac O'Raifeartaigh and their co-authors (if any), which are referred here in context of the discovery of the redshift law (see References).

Abbé Georges Lemaître was one of the researchers. After his "Docteur en Sciences" graduation from Université catholique de Louvain à Bruxelles and being ordained as a diocesan (secular) priest, he spent 1923–1925 outside Belgium. During 1923 he was a research associate in astronomy with Arthur Eddington at the Cambridge Observatory in England, then during 1924 — with Harlow Shapley at the Harvard Observatory (Massachusetts). Eddington introduced Lemaître to the General Theory of Relativity and relativistic cosmology, and with Shapley he studied the spectra of galaxies.

Returning to Belgium in 1925, Lemaître, like his aforementioned predecessors, tried to explain the observed cosmological redshift in the framework of de Sitter's cosmological model. This is a spherical universe of constant curvature filled with the physical vacuum called the λ-field, which is given by the λ-term in de Sitter's space metrics. Such a universe is usually static ($\lambda = const$), but can also be expanding if the λ-term and the space curvature (it is proportional to λ), having the same numerical value at any point in space, are proportional to the expansion rate, i.e., they depend on time (the case considered by Lemaître and his predecessors). Galaxies in an expanding universe are scattering away from the observer, so their observed spectra must be redshifted due to the Doppler effect. But, following his predecessors, Lemaître had come to an unsatisfactory result. He had deduced the same linear redshift law as Silberstein before him. But the obtained solution becomes invalid at the coordinate origin and even at a small distance from it, and also there the light source and the observer cannot be swapped (the solution depends on the coordinates). This means that, if the λ-term and the space curvature depend on time (the universe is expanding or compressing), then they can have the same numerical values at any point in space only if the space is either inhomogeneous or anisotropic (or both) thereby contradicting the conditions of spherical symmetry and isotropy, which are assumed in de Sitter's metric. In other words, Lemaître had proved that the studies of his predecessors, in which they tried to deduce the Doppler redshift in an expanding de Sitter universe, lead to nonsense. Lemaître explained all of the above in his 1925 paper [7], which was then reprinted in 1926 [8].

The mentioned defeat does not mean that de Sitter's metric itself is bad, but is due to the fact that this metric can only be static. Whereas the Doppler redshift, which Lemaître and his predecessors tried to deduce, is specific to such a space metric that initially depends on time.

Therefore in 1926, Lemaître immediately turned to Friedmann's cosmological model of an expanding (or compressing) universe [9, 10], since the Doppler redshift naturally accompanies the expansion of space. This model describes an approximately empty spherical universe (with no islands of mass or distributed substance), which is expanding or contracting on its own. Success awaited Lemaître on this path. He assumed that the Friedmann universe is expanding with a constant radial velocity, then easily expressed the expansion velocity from Friedmann's space metric and substituted it into the Doppler law known from classical physics. As a result, Lemaître had obtained a linear relationship between the cosmological redshift in the spectra of galaxies and the distance from them to the observer, which means a linear redshift law according to which the redshift for distant galaxies is greater than for nearby ones and increases proportionally to the distance. Then, using Slipher's measurements, he had estimated the numerical value of the constant coefficient in this linear relationship, which is now known as the *Hubble constant*. Lemaître reported these results, including the discovery of the redshift law and the estimation of the redshift law constant, in his fundamental 1927 paper published in *Annales de la Societe Scientifique de Bruxelles* [11]. But this publication in the obscure French-language journal was not noticed in the scientific community.

Two years later, Edwin Hubble published his 1929 paper [12] that brought him worldwide fame. In this paper, with a number of omissions because he was never fluent in General Relativity, Hubble repeated the results obtained by Lemaître, including the linear redshift law and the redshift law constant estimated using Slipher's measurement data. Hubble did not refer to his use of Slipher's measurements and Lemaître's 1927 paper in which Lemaître reported his discovery of the redshift law. Therefore, the redshift law later became known as *Hubble's law* or the *Hubble redshift*.

When in 1931 an English translation of Lemaître's 1927 paper was submitted through Eddington to the *Monthly Notices of the Royal Astronomical Society*, the passages about his discovery of the redshift law and his estimate of the redshift law constant were removed by the editor because these results had already been attributed to Hubble. Finally, the English translation of Lemaître's 1927 paper was published [13], but with significant censorship. In the same issue of the journal, Lemaître also published another paper [14], in which he outlined the details of his theory of the expanding Universe; a short version of the second article was then reprinted in French [15]. Lemaître did not discuss the above editorial decision: as a truly good Catholic, he always believed that "God hath commanded so" and never tried to defend his authorship of the redshift law and the redshift constant.

This story was known to a narrow circle of scientists back in the 1980s [16]. Then in the early 2000s, Hubble's authorship of the redshift law was publicly questioned in favour of Lemaître in the 2003 article [17] and the detailed 2009 book [18] on this subject. This drama was revealed in full power in 2011 by two historians of science [19, 20], which caused widespread resonance in the scientific community in the same 2011 thanks to the science news reports on this subject, which were first published in *Forbes* [21], then — in *Nature News* [22, 23] and *Nature* [24]. All this in 2011–2013 led to a revision of Hubble's rôle in this discovery and the recognition of Lemaître's authorship of the redshift law with the key contribution of Slipher's measurements; see [25–29] for example.

In the century passed since Slipher's measurements, observational astronomy techniques and observational equipment have made significant progress. Astronomers now have a vast amount of data on the redshifts and radial velocities of galaxies (instead of only a few dozens known in the 1920s). As a result, in the last two decades, astronomers claim about the possible existence of a deviation from the linear redshift law, which needs a theoretical explanation: see, for example, the surveys [30–32] on this subject and the original research results referred therein.

However, if following the same way of theoretical explanation as Lemaître and his predecessors did, we arrive at a problem. The essence of the problem is that neither Lemaître nor his predecessors deduced the cosmological redshift law directly from the specific space metric that they chose. In essence, they merely postulated that the redshift occurs in the spectra of galaxies due to their scattering away from the observer, i.e., due to the Doppler effect. They followed the "two-step path" of mathematical deduction. At the first stage of their deduction, they somehow extracted the expansion rate of the Universe from the specific space metric that they chose (as the change rate of the curvature radius of space). Then they merely substituted this speed into the Doppler effect formula known from classical physics, and thus obtained the cosmological redshift law. This is what Lemaître did, and this is what his predecessors did. It cannot be said that such a method is very consistent with theoretical physics, since the origin of the cosmological redshift is initially postulated as a result of the Doppler effect in scattering galaxies in an expanding universe, and also a "mixture" of classical physics and General Relativity is used in the derivation.

If, in solving a physics problem, we decide to solve, say, the forced oscillation equation, we are essentially postulating that the cause of this effect lies in forced oscillations, and then we obtain a solution that automatically "confirms" the initially postulated forced oscillations. In other words, if we initially postulate the origin of the cosmological redshift effect, say, as a result of the Doppler effect or something else, then no matter what mathematical operations we perform next, we get the same effect that we postulated at the beginning, but only expressed in a mathematically more extended and elegant form.

Therefore, if we like to find the truly origin of the cosmological redshift effect, we should newer postulate its origin. In addition, in order to be honest, if we like to deduce the cosmological redshift law as a space-time effect, i.e., as an effect in the framework of a cosmological model provided by General Relativity, we should follow only with the equations of General Relativity, and never use the equations and laws of classical physics (such as the Doppler effect formula). In other words, the cosmological redshift law should be obtained from the equations of General Relativity, and without any preliminary assumptions about its origin. This is the solely right way how to do things in theoretical physics.

In this letter, we list the newest solutions that are most fit for explaining the observed cosmological redshift, including its non-linearity. These solutions have been obtained since 2009 using the same original derivation method that has never been used for this purpose before — solving the scalar geodesic equation (energy equation) for a photon travelling from a source to an observer in the space-time of General Relativity. These solutions were obtained using only the equations of General Relativity, and without any prior assumptions about the nature of the cosmological redshift.

The solutions are different only because of the geometric structure of space, which is different for different space metrics (cosmological models). In other words, the mathematical derivation merely follows the geometric structure of the space in which it is performed. Thus, the resulting redshift law merely shows how the frequency of a travelling photon changes according to the geometric structure of the particular space (cosmological model) in which the photon travels.

The mentioned new method used to derive the cosmological redshift law dates back to our research studies of the 1990s, which we summarized in 2001 in our two monographs [33, 34]. The first monograph focuses on the geodesic (free) motion of mass-bearing and massless (light-like) particles in the space-time of General Relativity, and the second monograph examines their non-geodesic (non-free) motion.

As always in our studies we used the mathematical apparatus of chronometric invariants, which are physically observable quantities in the space-time of General Relativity. Such quantities are obtained as the projections of four-dimensional (general covariant) quantities onto the three-dimensional spatial section and the time line associated with a particular observer and his laboratory. Such quantities depend on the geometric and physical properties of the real physical space of the observer, as well as the laboratory standards to which he compares his measurement results. Therefore, if we have all quantities and equations of General Relativity expressed in the chronometrically invariant form, then we do not need to think about which of the obtained solutions are physically observable (that was a common problem in General Relativity in the past), since all the obtained solutions are, by definition, measurable on practice. The mathematical apparatus of chronometric invariants is also known as the Zelmanov chronometric invariants in honor of Abraham Zelmanov, who developed it in 1944; see our detailed survey of chronometric invariants [35] and references therein.

As for the mentioned new method used to derive the cosmological redshift law, it is simple.

The four-dimensional equations of motion of a particle in space-time have two physically observable projections. The projection onto the time line of the observer is a scalar equation showing how the particle's energy changes in time, depending on the properties of the observer's space. In other words, this is the *energy equation* of the particle. The projection onto the spatial section associated with the observer (his three-dimensional space) is the three-dimensional vector equation of motion of the particle, which also depends on the properties of the observer's space.

Integrating the scalar equation of motion (energy equation) of mass-bearing particles, Dmitri in 2009–2011 derived that the observable masses of cosmic bodies depend on their distance from the observer. He had called this the *cosmological mass-defect* [36], which is a new effect predicted according to General Relativity. The cosmological mass-defect depends on the specific metric of space, i.e., on the geometric structure of the specific space (particular cosmological model). Dmitri had calculated the cosmological mass-defect in the space of the most commonly used space metrics (cosmological models), such as Schwarzschild's mass-point metric, Reissner-Nordström's metric of the space of an electrically charged mass-point, Gödel's metric of the rotating space with self-closed time-like geodesics, Schwarzschild's metric inside a sphere filled with an incompressible liquid, de Sitter's metric inside a sphere filled with the physical vacuum, Einstein's metric inside a sphere filled with an ideal liquid and the physical vacuum, and also Friedmann's metric of a deforming (expanding or compressing) space.

Accordingly, by integrating the scalar equation of motion (energy equation) of a massless light-like particle, such as a photon, we obtain its physically observable frequency as a function of the travelled distance. This is the way to derive the cosmological redshift law in the space of any specific metric (particular cosmological model), without any prior assumptions about the nature of the cosmological redshift. This is how Dmitri in 2011 derived the cosmological redshift law in the space of each of the aforementioned cosmological models [37] (see also his 2012 short paper [38]), by analogy with the cosmological mass-defect.

The above work [37] has its own background and continuation. A year earlier, in 2010, Larissa considered a Sitter sphere in the state of gravitational collapse (its radius coincides with its gravitational radius). She showed that a de Sitter collapsar (*de Sitter bubble*) is fit to the observed Universe [39]. Integrating the scalar equation of motion (energy equation) of photons, based on her *de Sitter bubble model*, showed a parabolic redshift law [37, §6], which remains valid outside the state of gravitational collapse. Then in 2013, in our monograph on astrophysics [40, §6.4–6.5] (§5.1 in the 1st edition), we proved that the parabolic redshift law takes place a de Sitter space, in which $\lambda > 0$, the physical vacuum has a positive density, and the observable curvature radius of space is positive (otherwise it is a blueshift). Our redshift studies in a de Sitter universe were summarized in a short paper in 2018 [41] and then in an extended paper in 2023 [42].

The same method as above for deriving the cosmological redshift law was used in the 2009 papers [43–45], in which Dmitri had derived an exponential cosmological redshift due to the global non-holonomity of space.

The term *holonomity* dates back to Schouten's theory of non-holonomic manifolds and was first used in General Relativity in 1944 by Zelmanov [35]. If the time lines that "pierce" a three-dimensional spatial section are everywhere orthogonal to it, then the space (space-time) is *holonomic*. Otherwise it is *non-holonomic*. Zelmanov had proved that $g_{0i} \neq 0$ in non-holonomic spaces, which manifests itself in the form of a rotation of the spatial section (three-dimensional space) with a speed depending on g_{0i}, and this rotation cannot be removed by coordinate transformations. See [35] for detail.

Dmitri had showed in the third paper [45] that although each particular space (space-time) has its own specific metric and does not necessarily have a three-dimensional rotation, its light-like sub-space (home of photons) always rotates with the speed of light (varying depending on the gravitational potential). The light-speed rotation of the light-like space cannot be removed by coordinate transformations and is due to the sign-alternating structure of any space-time metric (which distinguishes the time axis from the spatial coordinate axes). In other words, the light-like space (in which photons travel) is always strictly non-holonomic. This rotation creates a centrifugal force that affects only particles in the light-like space (such as photons). By assuming the mentioned rotation when integrating the scalar equation of motion (energy equation) of photons, Dmitri had derived the exponential redshift law. This law should take place even in a flat unperturbed space (space-time), while each particular space metric creates only an addition to the exponent.

As for the origin of the cosmological redshift and the cosmological mass-defect, it can be understood from the scalar equation of motion (energy equation), which for photons and mass-bearing particles has the form, respectively,

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\, F_i\, c^i + \frac{\omega}{c^2}\, D_{ik}\, c^i c^k = 0\,,$$

$$\frac{dm}{d\tau} - \frac{m}{c^2}\, F_i\, \mathrm{v}^i + \frac{m}{c^2}\, D_{ik}\, \mathrm{v}^i \mathrm{v}^k = 0\,,$$

in which m is the relativistic mass of a mass-bearing particle, travelling with the velocity $\mathrm{v}^i = \frac{dx^i}{d\tau}$, and ω is the frequency of a photon (photons travel with the velocity of light $c^i = \frac{dx^i}{d\tau}$, for which $c_i\, c^i = c^2 = const$).

If the space is static (the tensor of the space deformation rate is $D_{ik} = 0$), then $d\tau$ is reduced in the equations, which then are integrated with respect to the radial coordinate $x^1 = r$. As a result, we obtain the mass-bearing particle's mass m and the photon's frequency ω as a function of the distance r from the observer (for whom $r = r_0 = 0$).

If the gravitational inertial force is $F_i = 0$ (there is no gravitational field and rotation of space), but the space is deforming (expanding or compressing), then when multiplying the equations by the metric tensor h_{ik}, the multiplier $h_{ik}\, c^i c^k = c^2$ is reduced and the equations are integrated with respect to the travel time τ. In this case, we obtain the mass-bearing particle's mass m and the photon's frequency ω as a function of the time t travelled from the source (where $t = t_0 = 0$) to the observer (which is the reverse path of integration, changing the sign of the integration result).

Therefore, the origin of the cosmological redshift and the cosmological mass-defect is clearly seen from the equations. If the gravitational inertial force, consisting of a term given by the gravitational potential and a term given by the centrifugal force, is a force of repulsion ($F_1 > 0$) or the space is expanding ($D_{11} > 0$), then the repulsive force decelerates photons travelling to the observer, thereby producing a loss of the photon energy $E = \hbar\omega$ (*photon redshift*). In the case of mass-bearing particles such as cosmic bodies, their masses (and energies $E = mc^2$) registered by the observer are less than their actual masses (and energies) at their distant locations.

Otherwise, if the gravitational inertial force is a force of attraction ($F_1 < 0$) or the space is compressing ($D_{11} < 0$), then the force accelerates photons travelling to the observer, thereby producing a gain of their energy (*photon blueshift*), and the masses of distant cosmic bodies registered by the observer are greater than their actual masses at their distant locations.

This means that both the cosmological mass-defect and the cosmological redshift arise from the specific geometric structure of each particular space.

Below we list three different solutions for the cosmological redshift law, which can be considered to fit to the observed Universe. The first two were derived in 2011 [37], while the third solution — in 2009 [43–45], all using the above method of integrating the scalar equation of motion (energy equation) for photons.

Cosmological redshift in an expanding Friedmann universe. In such a universe, the frequency ω of a photon registered by an observer away from the emitted photon is

$$\omega = \omega_0 \, e^{-\frac{\dot{R}}{R}\,t},$$

where R is the curvature radius of space (the Universe's radius in this case), and $\dot{R}$ is the rate of its expansion. This exponential law transforms into the linear

$$\omega \simeq \omega_0 \left(1 - \frac{\dot{R}}{R}\,t\right)$$

at short duration of the photon's travel (and, respectively, at small distances from the photon's emitter to the observer).

We see from the above formulae that the photon's frequency ω registered by the observer is lower that its frequency ω_0 at the initial moment of time $t = t_0 = 0$, when it was emitted by a source in the far cosmos. The farther the photon's emitter is located from the observer, the lower the photon's frequency ω registered by him: the photon's frequency is redshifted upon arrival at the observer.

The above formulae for the photon's frequency result in the *exponential redshift law*

$$z = \frac{\omega_0 - \omega}{\omega} = e^{\frac{\dot{R}}{R}\,t} - 1\,, \qquad z > 0\,,$$

which transforms into the *linear redshift law* at short duration (and small distances) of the photon's travel

$$z \simeq \frac{\dot{R}}{R}\,t\,.$$

As was shown in [37], the above formulae for the photon's frequency and redshift are the same in both a constant-speed expanding Friedmann universe ($\dot{R} = const$) and a constant-deformation Friedmann universe (where $\frac{\dot{R}}{R} = const$).

So, the cosmological redshift in an expanding Friedmann universe increases with distance to cosmic bodies according to the *exponential redshift law*, which transforms into the *linear redshift law* at short duration (and small distances) of photons' travel.

Here we should make a short remark about Lemaître's linear redshift law. With all our respect to Georges Lemaître, he did not solve any equations. His 1927 paper focused on how to find the expansion rate of the Universe from Friedmann's metric. Then he substituted this rate into the Doppler redshift formula taken from classical physics. In fact, he merely renamed the emitter's velocity in Doppler's formula as the expansion rate of the Universe (and justified this renaming by showing how the expansion rate is found from Friedmann's metric). But by doing this, Lemaître could not obtain anything other than the linear redshift law, because it initially follows from Doppler's formula at the velocity of the emitter, much lower than the velocity of light.

In contrast to what Lemaître did, the exponential redshift law formula that above is a mathematical solution obtained directly by solving the scalar equation of motion (energy equation) for photons travelling in an expanding Friedmann universe. It was derived without any prior assumptions about the form of the redshift law. This is the solely right way how to do things in theoretical physics.

The said does not affect the memory about Abbé Lemaître as an outstanding scientist and good Catholic, an exemplar of human decency and honesty, and does not diminish his fundamental contribution to relativistic cosmology.

Cosmological redshift in a static de Sitter universe. In a de Sitter universe, the frequency ω_0 of a photon registered by an observer (for whom $r = r_0 = 0$) upon its arrival is also lower than its frequency ω at the location of its distant source, from which it was emitted. This dependence is expressed with the parabolic (square) law

$$\omega = \frac{\omega_0}{\sqrt{1 - \frac{\lambda r^2}{3}}}\,,$$

which at small distances r between the photon's source and the observer transforms into the simplified law

$$\omega \simeq \omega_0 \left(1 + \frac{\lambda r^2}{6}\right).$$

The farther the emitter is located from the observer, the lower the photon's frequency ω_0 registered by him. Thus, the photon's frequency is redshifted upon arrival at the observer in a de Sitter universe.

These formulae for the photon's frequency result in the *parabolic* (*square*) *redshift law*

$$z = \frac{\omega - \omega_0}{\omega_0} = \frac{1}{\sqrt{1 - \frac{\lambda r^2}{3}}} - 1\,, \qquad z > 0\,,$$

which at small distances r takes the simplified form

$$z \simeq \frac{\lambda r^2}{6}\,.$$

At the ultimately large distance in space (event horizon, where $r = a$), which is determined in a de Sitter universe by the condition $\frac{\lambda r^2}{3} = \frac{\lambda a^2}{3} = 1$, the photon's frequency and redshift are maximum: $\omega_{max} = \infty$ and $z_{max} = \infty$.

So, the cosmological redshift in a static de Sitter universe increases with distance to cosmic bodies according to the *parabolic* (*square*) *redshift law*.

This redshift law depends on the sign of the λ-term and, accordingly, the sign of the density of the physical vacuum (which is the filler of de Sitter space) and the sign of the physically observable curvature of space (since these quantities are connected with λ). It was proved in [40, §6.4–6.5] (§5.1 in the 2013 edition) and then summarized in [41, 42] that the cosmological redshift ($z > 0$) takes place in a de Sitter universe, where $\lambda > 0$, the physical vacuum has a positive density (like substance, and not a negative density like field), the curvature radius of space is positive (the geometry of space is spherical, and not hyperbolic), and the non-Newtonian gravitational forces that act in any de Sitter space and increase with distance from the observer are repulsive forces. These repulsive forces cause photons to lose energy as they travel to the observer, thereby producing a redshift in the frequency of the photons. Otherwise (if $\lambda < 0$), there is not a cosmological redshift, but a blueshift ($z < 0$) and the curvature radius of space takes an imaginary numerical value (the geometry of space is hyperbolic).

Cosmological redshift due to the global non-holonomity of the light-like space. The term *non-holonomity* dates back to Schouten's theory of non-holonomic manifolds and was first used in General Relativity in 1944 by Zelmanov. If the time lines that "pierce" a three-dimensional spatial section are everywhere orthogonal to it, then the space (space-time) is *holonomic*. Otherwise it is *non-holonomic*. Zelmanov had proved that $g_{0i} = 0$ in holonomic spaces and $g_{0i} \neq 0$ in non-holonomic spaces. The latter manifests itself as a rotation of the spatial section (three-dimensional space) with a speed depending on g_{0i}, which cannot be removed by coordinate transformations. For detail, see our survey [35] and references therein.

It was proved [45] that the light-like sub-space of any space-time metric rotates with the speed of light, thereby creating a repulsive centrifugal force. This repulsive force only acts on particles in the light-like space (i.e., photons) in the direction away from the observer (coordinate origin), thereby causing photons to lose energy and frequency as they travel to him

$$\omega = \omega_0 \, e^{-\Omega t}, \qquad \Omega \equiv H_0 \,,$$

resulting in the *exponential redshift law*

$$z = \frac{\omega_0 - \omega}{\omega} = e^{\Omega t} - 1 \,, \qquad z > 0 \,,$$

where ω_0 is the photon's frequency at the initial moment of time $t = t_0 = 0$, when it was emitted by a distant source in the cosmos, ω is its frequency upon arrival at the observer, and Ω is the angular rotational velocity of the light-like space due to its global non-holonomity (light-speed rotation), which is equal to the Lemaître-Hubble constant $H_0 = 2.3 \pm 0.3 \times 10^{-18}$ sec^{-1} (as measured in the framework of the Hubble Space Telescope Key Project for 2001 [46]).

We see that the repulsive centrifugal force, which is always present in the light-like space (home of photons) due to its light-speed rotation, causes a redshift (loss of energy) in the frequency of a photon arrived from a distant source at the observer. The farther the photon's emitter (and longer is its travel time t), the lower the photon's frequency ω registered by the observer upon its arrival.

At short duration (and small distances) of the photon's travel we have the linear approximation for the photon's frequency

$$\omega \simeq \omega_0 \, (1 - H_0 \, t)$$

and the *linear redshift law*

$$z \simeq H_0 \, t \,.$$

So, the cosmological redshift due to the light-speed rotation of the light-like space (its global non-holonomity) increases with distance to cosmic bodies according to the *exponential redshift law*, which at short duration (and small distances) of photons' travel transforms into the *linear redshift law*.

Since the light-like space rotates with the speed of light due to only the sign-alternating structure of any space-time metric (which distinguishes the time axis from the spatial coordinate axes), and this rotation cannot be removed by coordinate transformations, the above exponential redshift law and its linear approximation at small distances should take place even in a flat unperturbed space. Any particular space metric should create only an addition to the above exponential redshift law, straightening this exponential curve or making it more curved.

Thus, the following three versions have been proposed according to General Relativity to explain the observed non-linear cosmological redshift law.

1. If the redshift in the spectra of nearby galaxies increases linearly with distance to them, then it turns into exponential for distant galaxies, and triangulation of galaxies reveals non-zero curvature of space, then our Universe is an expanding Friedmann world. In this case, photons lose energy as they travel to the observer due to the fact that they are decelerated by the expansion of space.

2. If the linear redshift law turns into parabolic for distant galaxies, then our Universe is a static de Sitter world with $\lambda > 0$, in which the physical vacuum has a positive density, the observable curvature is positive, and the non-Newtonian gravitational forces acting there are repulsive forces increasing with distance from the observer (which cause photons to lose energy as they travel to the observer).

3. If for distant galaxies the linear redshift law turns into exponential, but triangulation of galaxies does not reveal even the slightest curvature of space, then our Universe has a flat space, where the redshift in the spectra of distant objects is due only to the light-speed rotation of the light-like sub-space (home of photons) in any metric space-time, which creates a repulsive centrifugal force causing photons to lose energy as they travel to the observer. But in this case we should assume that the device with which the observer measures the redshift is connected with a light-like reference frame, which creates a problem for an ordinary observer, since he and his laboratory reference frame are related to ordinary substance.

Which of the above three options best explains the cosmological redshift in our Universe will be decided in accordance with astronomical observations.

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