On a Plausible Solution to the Hubble Tension via the Hypothesis of Cosmologically Varying Fundamental Natural Constants

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We herein present what we propose could be a plausible solution to the current, interesting and topical problem in cosmology — the *Hubble Tension*. This problem of the Hubble tension seems to have thrown all of cosmology into a crisis. By employing the seemingly temarious hypothesis of varying *Fundamental Natural Constants* (FNCs), namely Planck's constant, $\hbar$, we demonstrate that for the case where the cosmological Interstellar Medium (ISM) is a perfect *vacuo* with a refractive index of unity, the supernovae derived $\mathcal{H}_0$-value can be brought down from its current lofty height of: $\mathcal{H}_0^{\rm SNe} = 73.30 \pm 1.03\,{\rm km\,s^{-1}\,Mpc^{-1}}$, down to a more humble and modest value of: $68.70 \pm 0.30\,{\rm km\,s^{-1}\,Mpc^{-1}}$, and within the margins of error, this new value is in agreement with the Tip of the Red Giant Branch (TRGB) derived $\mathcal{H}_0$-value, namely: $\mathcal{H}_0^{\rm TRGB} = 69.80 \pm 2.20\,{\rm km\,s^{-1}\,Mpc^{-1}}$, and this is much closer to the CMB-derived $\mathcal{H}_0$-value: $\mathcal{H}_0^{\rm CMB} = 67.40 \pm 0.50\,{\rm km\,s^{-1}\,Mpc^{-1}}$. At a 2.2σ-level of statistical significance in discrepancy, this new $\mathcal{H}_0$-value reduces the tension by 88%, and this surely is a most welcome development. On the other hand, if the ISM is assumed to be homogeneous and isotropic with a slightly varying, if not near constant refractive index, $n_r^{\rm ISM}$, for most photon wavelengths, then, a refractive index value of: $n_r^{\rm ISM} = 1.010 \pm 0.006$, does bring the new SNe-derived $\mathcal{H}_0$-value into complete and total concordance with the CMB-derived $\mathcal{H}_0$-value, thus resolving the tension altogether. The final concordance $\mathcal{H}_0$-value that matches or resolves both measurements after a final correction of the ISM's refractive index is found to be: $\mathcal{H}_0 = 68.00 \pm 0.90\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

Cosmology is peculiar among the sciences for it is both the oldest and the youngest. From the dawn of civilization man has speculated about the nature of the starry heavens and the origin of the world, but only in the present century has physical cosmology split away from general philosophy to become an independent discipline.

Gerald James Whitrow (1912–2000)*

1 Introduction

Without an *iota* of doubt, the Hubble constant, denoted by the symbol $\mathcal{H}_0$, is an all important constant in all of modern cosmology and astrophysics [1–4]. It, amongst others, measures the expansion rate of the Universe and is pivotal in the measurement of the age of the Universe [1–4]. Since the theoretical discovery [5] of the expansion of the Universe by the Belgian Catholic priest, theoretical physicist, mathematician, astronomer, and then professor of physics at the Catholic University of Louvain, Georges Henri Joseph Édouard Lemaître (1894–1966), and the subsequent observational confirmation [6] of this hypothetical expansion by the great American astronomer, Edwin Powell Hubble (1889–1953), a great many efforts have been made to measure this constant with the highest and optimum possible precision available at the time. The importance of this parameter in cosmology cannot be overstated. Hence, accurate knowledge of this constant is not only a *sine qua non*, but very important as all of cosmology and the cosmological models thereof, depend on it.

Rather worrisomely, initial measurements of this constant in the past century were marred by serious scattering with the resultant values thereof ranging from: $\sim 40\,{\rm km\,s^{-1}\,Mpc^{-1}}$ to $\sim 100\,{\rm km\,s^{-1}\,Mpc^{-1}}$ [4]. However, recent 21st century advances in science and technology have made it all possible to obtain very accurate measurements of this constant using at least three different methods — which methods measure the Hubble constant on two different evolutionary epochs of the Universe, namely the early-and-late Universe. Values of $\mathcal{H}_0$ from the early Universe are typically referred to as global measurements of $\mathcal{H}_0$, while those from the late Universe are commonly referred to as local values of $\mathcal{H}_0$. Global $\mathcal{H}_0$ values measure the Hubble constant in the early Universe (distant past) while the local $\mathcal{H}_0$ values measure this constant in our local neighbourhood which is the present epoch in the Universe.

According to the widely accepted Standard ΛCDM cosmology model that is used to describe the Universe, $\mathcal{H}_0$ must be the same for any evolutionary epochs of the Universe — be it in early or late Universe, it does not matter, the value of $\mathcal{H}_0$ aught to be the same. To the chagrin and against the desideratum of the cosmologically searching mind, the local and global values of $\mathcal{H}_0$ seem to not be in agreement — each yielding at a 4.9σ-level of statistical significance [10], two

*In "Theories of the Universe" (1958)

Table 1: Critical Measurements of the Hubble Constant

Group	Cosmic Epoch	Measurement Type	$\mathcal{H}_0$ ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)	Reference
Supernova Cosmology (SCG)	Late Universe	Far Local	73.30 ± 1.04	[7]
Carnegie-Chicago Hubble Project (CCHP)	Late Universe	Near Local	69.80 ± 2.20	[8]
Planck Collaboration (PC)	Early Universe	Global	67.40 ± 0.50	[9]

different values that are not only $\sim 10\%$ apart, but also outside of the provinces of their error margins. This interesting and topical problem or discrepancy in the local and global measurements of the Hubble constant has come to be known as the *Hubble tension* and has thrown cosmology into a serious crisis.

From a fundamental theoretical stand point, before suspecting that there possibly may be errors in the measurements and/or systematics thereof, one needs to first trust that — those that have made these measurements have done so meticulously with due and requisite diligence, and with the best precision at hand. Of course, one cannot rule out errors in the measurements and/or systematics — we are human after all, we err. Be that as it may, as our point of departure, we shall assume that these measurements are flawless. With that having been said, we must say that there are three main popular and common techniques used to measure $\mathcal{H}_0$:

1. Supernovae Type Ia (SNe Ia) method;
2. Tip-of-the-Red-Giant-Branch (TRGB) method;
3. Cosmic Microwave Background (CMB) radiation method.

We shall discuss in detail these techniques in §4. Our interest in taking a deeper look into these methods is to unravel their dependence on FNCs because it is in these FNCs that we believe the source of our error in the determination of the Hubble constant may lay.

For further clarity, as already aforementioned, we shall elaborate that the methods to measure the Hubble constant fall into two classes: a) *Local* measurements, and, b) *Global* measurements, i.e.:

1. Local $\mathcal{H}_0$ measurements: measure $\mathcal{H}_0$ in the present (local) evolutionary epoch of the Universe. The present epoch is the late Universe, hence, these type of measurements are also referred to as late Universe measurements.
2. Global $\mathcal{H}_0$ measurements: are all-sky measurements of $\mathcal{H}_0$, measuring the Hubble constant across the entire sky, hence, they being referred to as global $\mathcal{H}_0$ measurements. These measurements typically measure, $\mathcal{H}_0$, in the very early Universe hence they also being referred to as early Universe measurements.

The TRGB and SNe Ia measurements are classified as local $\mathcal{H}_0$ measurements as they measure $\mathcal{H}_0$ in the present (and not past) evolutionary epoch of the Universe. The TRGB method measures $\mathcal{H}_0$-values in galaxy systems much closer to us (yielding: $\mathcal{H}_0^{\mathrm{TRGB}} = 69.80 \pm 2.20\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [8]), while the SNe Ia measurement $\mathcal{H}_0$-values in galaxy systems relatively far in the local Universe (yielding: $\mathcal{H}_0^{\mathrm{SNe}} = 73.30 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [7]). We shall say that the TRGB method measures $\mathcal{H}_0$-values in the near-local Universe, while, the SNe Ia methods measures $\mathcal{H}_0$-values in the far-local Universe. The near and far-local $\mathcal{H}_0$-values do not agree ($69.80 \pm 2.20\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [8] and $73.30 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [7], respectively), thus, giving raise to yet another tension within an already existing tension.

On the other hand, the CMB $\mathcal{H}_0$ measurements are classified as a global $\mathcal{H}_0$ measurements as these measurements are all-sky measurements of $\mathcal{H}_0$ measuring the Hubble constant across the entire sky, hence, they being referred to as a global $\mathcal{H}_0$ measurements. The CMB method is a state-of-the-art precision method of global $\mathcal{H}_0$ measurements by Aghanim *et al.* [9] and this has yielded: $\mathcal{H}_0^{\mathrm{CMB}} = 67.40 \pm 0.50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. A summary of these key measurements is presented in self-explanatory Table 1.

Since Lemaître [5] and Hubble [6]'s initial estimates, there has been numerous measurements of the Hubble constant. For our purposes here, the above three measurements (presented in [7–9], which are summarised in a clear and succinct manner in Table 1) shall constitute our focal point in all the $\mathcal{H}_0$ measurements as these three important measurements sufficiently capture the morass substance contained in our current musings and at the same time — they drive our point home regarding this important tropical issue of the Hubble tension.

Astronomers, astrophysicists and cosmologists are hard at work to figure out why the discrepancy in the values of $\mathcal{H}_0$ from the two different methods as a number have wondered if this discrepancy is heralding some hitherto yet unknown physics [3, 11, 12] or there might be some serious, albeit subtle, error in our methods and analysis? We herein present a suggestion to this problem and this suggestion is to the effect that varying Fundamental Natural Constants (FNCs) may be the cause of this tension. As will be demonstrated, a simple hypothesis regarding the nature of the said variation on the FNCs seem to deliver a bold solution to this problem.

In closing this introductory section, we shall give a synopsis of the reminder of this article, i.e.: the reminder of this article is arranged as follows: for no other than smoothness, completeness and self-containment purpose, we present in the next §2, a pedestrian derivation of the distance modulus formula used in astronomy, astrophysics and cosmology. Thereafter in §3, we discuss distances in cosmology with emphasis on how the luminosity and Light travel distances are used in the distance modulus formula in order to derive the Hubble constant and having done this, in §4, we discuss the three common and popular methods to measure the Hubble constant. In §5, we present what we believe is the source of the problem in our endeavour to compute the Hubble constant leading to the current tension in the measurement of this constant using the two major methods. In §6, we justify the idea of variable fundamental natural constants. It is this idea that our proposed solution to the Hubble tension is to be found, hence there is need to justify the idea. In §7, we present our proposed solution and application of this solution to real data in §8. Lastly, in §10 and §11, we present a general discussion and the conclusion drawn thereof.

2 Distance modulus

In this section, we are going to go through some necessary trivialities and this is for no other purpose other than for self-containment and smooth flow of the paper thereof. As is well known, in astronomy, astrophysics and cosmology, the distance modulus, denoted by the symbol μ, is a way of expressing distances to stellar objects. It is a measure of the difference between the apparent (m) and absolute magnitude (M), of an astronomical object, i.e.: $\mu = m - M$. For a star (or any stellar body of radius, R, mean temperature, T, and, with surface emissivity, ϵ) whose luminosity: $L = 4\pi R^2 \epsilon \sigma_0 T^4$ (where: σ_0 is the Stefan-Boltzmann constant), with a total flux of: $F(d_L)$, and with this flux reaching at the arbitrary distance, d_L, away from the star — the flux received at the said arbitrary distance d_L, obeys the following inverse square law:

$$F(d_L) = \frac{L}{4\pi d_L^2}\,. \tag{1}$$

The absolute magnitude is by definition defined as follows:

$$M = -2.5 \log_{10} F(d_L)\,, \tag{2}$$

while the apparent magnitude is by definition defined:

$$m = -2.5 \log_{10} F(10\,\text{pc})\,, \tag{3}$$

where: $F(10\text{pc}) = L/4\pi(10\,\text{pc})^2$ is the flux of the given stellar object at a distance: $d_L = 10\,\text{pc}$, away. Hence:

$$\begin{aligned} \mu_L &= m - M\,, \\ &= -2.5 \log_{10}\left(\frac{F(d_L)}{F(10\,\text{pc})}\right), \\ &= -2.5 \log_{10}\left(\frac{10\,\text{pc}}{d_L}\right)^2. \end{aligned} \tag{4}$$

This further simplifies to:

$$\mu_L = 5 \log_{10}\left(\frac{d_L}{10\,\text{pc}}\right), \tag{5}$$

In cosmology, one often works with distances in mega-parsec (Mpc), so, it is convenient to write (5) with, d_L, in Mpc and not in in units of 10pc. Written in the units of Mpc, (5) becomes:

$$\mu_L = 5 \log_{10}\left(\frac{d_L}{\text{Mpc}}\right) + 25\,. \tag{6}$$

Now, (6) applies in the case where the flux does not experience attenuation as a result of interstellar material along its path — i.e., in the case where there is no extinction of the flux.

In the case were there is extinction, the flux undergoes attenuation. Let, τ, be the optical depth of the Interstellar Medium (ISM) along the intervening spaces along the path of the photons reaching our telescopes and let, F_0, be the flux at the surface of the star (or stellar body). Then, the flux at distance d_L away is such that:

$$F(d_L) = F_0\left(\frac{4\pi R^2}{4\pi d^2}\right) e^{-\tau}\,. \tag{7}$$

For the absolute magnitude, we need the flux, $F(10\text{pc})$, at a distance of 10 parsecs as this is to be evaluated without extinction, i.e.:

$$F(10\text{pc}) = F_0\left(\frac{4\pi R^2}{4\pi(10\text{pc})^2}\right). \tag{8}$$

Therefore, from (7) and (8), it follows that:

$$\frac{F(d_L)}{F(10\text{pc})} = \frac{(10\text{pc})^2}{d_L^2} e^{-\tau}\,, \tag{9}$$

hence:

$$\mu_{L^\tau} = 5 \log_{10}\left(\frac{d_{L^\tau}}{10\,\text{pc}}\right) + A_\tau\,, \tag{10}$$

where:

$$A_\tau = -2.5 \log_{10}(e^{-\tau}) = 5 \log_{10}(e^{0.5\tau})\,, \tag{11}$$

is the extinction correction term to the distance modulus, and: μ_{L^τ}, is the extinction-corrected distance modulus. With, d_L, expressed in Mpc, the above can be written as follows:

$$\mu_{L^\tau} = 5 \log_{10}\left(\frac{d_{L^\tau}}{\text{Mpc}}\right) + 25\,, \tag{12}$$

where:

$$d_{L^\tau} = e^{0.5\tau} d_L\,, \tag{13}$$

is the extinction-corrected luminosity distance. Eq. (12) is what is used in cosmology in the study of supernovae to estimate the distance to the Cepheid variables that are resident in the Host galaxy of supernovae.

In closing this section, allow us to say that we are very much aware that we have presented an elementary and textbook derivation of the distance modulus formula. We want to rest assure our reader that this has been done for a very good reason and the reason is that there is an esoteric subtlety associated with this derivation that we want to *exegetically* unmask (point out) and "correct", all this in the hope that this may be one of the problems from which the discrepancy in the measurement of the Hubble constant might lie. Therefore, we kindly ask our reader for their due indulgence as we unpack this esoteric subtlety.

3 Distances in cosmology

If we get our distances wrong in astronomy, astrophysics and cosmology, so is our interpretation of the results — they will be wrong as well. So, the importance of the measures that we use to obtain these distances cannot be overstated. Different distance measures are used in astronomy, astrophysics and physical cosmology. These distance measures give a natural notion of the distance between two objects or events in the Universe. They are often used to tie some observable quantity to another quantity that is not directly observable, but is more convenient for calculations such as the comoving coordinates of quasars, galaxy, *etc*. The observable quantities in question are quantities such as the luminosity of a distant star (or quasar), the redshift of a distant galaxy, or the angular size of the acoustic peaks in the CMB power spectrum. For low redshift objects, these distance measures reduce to the common notion of Euclidean distance. Of particular interest in our present expedition are the luminosity and Light travel distances.

3.1 Light travel distance

Herein denoted by the symbol d_{LT}, the *Light Travel Distance*, is a cosmological concept that refers to the distance Light travels from one point (A) to the other (B), in particular, the distance Light could travel say from one galaxy to our own telescope at the time of observation. The Light travel distance can be important for understanding phenomenon such as the age of the Universe, its expansion rate and the spatial size of the observable Universe for example. Wholly within the framework of Einstein [13–15]'s General Theory of Relativity (GTR), the Light travel distance is calculated with respect to proper time $d\tau$, i.e.:

$$d_{LT} = \int_{\tau_e}^{\tau_r} c\, d\tau = \int_{\tau_e}^{\tau_r} \left(\frac{c_0}{n_r}\right) d\tau\,, \qquad (14)$$

where in this case: n_r, is the refractive index of the Interstellar Medium (ISM). In most considerations in the definition and calculation of the Light travel distance, the refractive index does not appear in the formulae, the meaning of which is that, the ISM is, in the said cases, being assumed to be a perfect *vacuo* with a refractive index of unity. In the present expedition, we shall be meticulous and exercise equanimity by not assuming a perfect *vacuo* for the ISM. This is going to help us in our effort to explain the remaining discrepancy in the resulting Hubble constant after the correction of the FNCs has been made.

From the homogeneous and isotropic metric tensor of Friedmann (1924) [16], Lemaître (1933) [17], Robertson (1935, 1933a,b,c) [18–20] and Walker (1937) [21] (hereafter, FLRW-metric), which is what is used in the ΛCDM cosmology model — by setting the proper time in this metric to equal zero for the propagation of Light in an FLRW-Universe — one can show from it that, the Light travel distance, d_{LT}, defined in (14), is such that:

$$d_{LT} = \frac{d_{\rm H}}{n_r} \int_0^z \frac{dz}{(1+z_\lambda)\, n_r \sqrt{\Omega}} = d_{\rm H} f(z_\lambda)\,, \qquad (15)$$

where off cause:

$$f(z_\lambda) = \int_0^{z_\lambda} \frac{dz_\lambda}{(1+z_\lambda)\sqrt{\Omega}}\,, \qquad (16)$$

and: $d_{\rm H} = c_0/\mathcal{H}_0$, is what is called the Hubble distance with:

$$\Omega = \frac{1}{\mathcal{H}_0^2}\frac{\dot{a}^2}{a^2} = \Omega_{\rm m} + \Omega_\Lambda + \Omega_k\,. \qquad (17)$$

The Ω's appearing in (17) are the usual Ω-parameters used in cosmology, with Ω, being the total Ω-parameter; while, $\Omega_{\rm m}$, is the Ω-matter parameter; Ω_Λ, is the Ω-vacuum parameter for the Λ-cosmological field; and, Ω_k is the Ω-curvature parameter.

Now as is the usual case, using the Light travel distance, d_{LT}, one can calculate from it the corresponding distance modulus, μ_{LT}, of the given supernovae, it is given by:

$$\mu_{LT} = 5\log_{10}\left(\frac{d_{LT}}{\rm Mpc}\right) + 25\,. \qquad (18)$$

Inserting (15) into (18), we obtain:

$$\mu_{LT} = 5\log_{10}\left[f(z_\lambda)\right] + K\,, \qquad (19)$$

where:

$$K = 25 + 5\log_{10}\left(\frac{c_0}{\rm Mpc}\right) - 5\log_{10}\left(n_r\mathcal{H}_0\right)\,. \qquad (20)$$

It is from the value of, K, as given in (20), that one is able to calculate the Hubble constant.

3.2 Luminosity distance

We have already met the concept of luminosity distance in our derivation of the distance modulus in §2, which distance we have denoted by the symbol, $d_{\rm L}$. There are two concepts relating to luminosity distance that we shall call the observationally derived luminosity distance and the redshift derived luminosity distance. The former is what we have met. We shall discuss these two concepts below:

1. *Observationally Derived Luminosity Distance*: The observationally derived luminosity, is the luminosity distance that is defined as the distance at which an object would need to be located in order for its observed (apparent) luminosity to match its intrinsic (absolute) luminosity. This is the distance, d_{L}, as defined in (1). That is to say, the luminosity distance, d_{L}, is related to the observed (apparent) flux (F) from the given object and its intrinsic (absolute) luminosity (L) through (1). At the instance of (8) leading to (13), the luminosity distance has been corrected for extinction and the extinction-corrected luminosity distance has been denoted by the symbol, $d_{L^{\tau}}$. We will argue in §5 that our understanding of the luminosity may need to be updated if FNCs are variable over cosmic epochs. It is this dearth and paucity of knowledge in our understanding of the luminosity distance that may very well be the cause of the Hubble tension.
2. *Redshift Derived Luminosity Distance*: The redshift derived luminosity distance, $d_L(z_\lambda)$, depends on cosmology under probe and is given by:

$$\frac{d_L(z_\lambda)}{d_{\mathrm{H}}} = \frac{1+z_\lambda}{a_0}\int_0^{z_\lambda}\frac{dz_\lambda}{\sqrt{\Omega}}\,, \tag{21}$$

where: $d_{\mathrm{H}} = c_0/\mathcal{H}_0$, is the Hubble distance and Ω is the total Ω-parameter already defined in (17). The cosmology is defined by the total Ω-parameter.

What happens in the supernovae determinations of the Hubble constant is that two distance moduli are constructed and equated and the resulting equation, the Hubble constant is determined. That is to say, from the observationally derived luminosity distance, $d_{L^{\tau}}$, the distance modulus, $\mu_{L^{\tau}}$, is constructed as given in (12). From (21), one constructs the corresponding the redshift derived distance modulus:

$$\mu_L(z_\lambda) = 5\log_{10}\left(\frac{d_L(z_\lambda)}{\mathrm{Mpc}}\right) + 25\,. \tag{22}$$

Now, from the equation: $\mu_{L^{\tau}} = \mu_L(z_\lambda)$, the Hubble constant is determined.

4 Measuring the Hubble constant

The Hubble constant, can be determined through several different methods, each with its own advantages, disadvantages, and limitations. Here are some of the primary methods:

1. The *Distance Ladder Method* makes use of standard candles such as Cepheid variable stars and type Ia supernovae and from these standard candle distance measures and the the corresponding redshift, one can infer the Hubble constant.
2. The *Cosmic Microwave Background* observations from missions like the Planck satellite provide a measurement of the Hubble constant based on the early Universe's conditions.
3. The *Tip-of-the-Red-Giant Method* makes use of stars at the tip of the red giant branch on a IV-color-color diagram. These stars have known fixed intrinsic brightness, hence, they are standard candles. Using this fact other with their redshift, one can infer the Hubble constant.
4. The *Baryon Acoustic Oscillations (BAO) Method* uses the distribution of galaxies to infer distances and hence the expansion rate of the Universe.
5. The *Gravitational Lensing Method* uses the bending of light from distant objects by massive foreground objects as this can be analyzed to estimate the Hubble constant.
6. The *Time Delay Measurements Method* in systems with multiple images of the same astronomical event (like a supernova), the time delays in these systems can be used to calculate the Hubble constant.
7. The *Tying to Local Measurements Method* links the Hubble constant to local measurements in the Solar System, such as the motion of nearby galaxies.
8. The *Galaxy Cluster Dynamics Method* utilizes the motion of galaxies within clusters providing insights into the expansion rate.

In the next two subsections [i.e., §4.1 and §4.2], we shall give an *exegetic exposition* of the first two methods, namely the *Distance Ladder Method* and the *CMB-Method*. The exegesis that we institute is meant to pinpoint the plausible sources of error that may need to be corrected so as to bring about concordance in the $\mathcal{H}_0$-values derived from these two state-of-the-art methods.

4.1 SNe Ia distance ladder method

In the SNe Ia method, three things are necessary:

1. A type Ia supernovae and its redshift, z_λ.
2. A host galaxy for the given supernova.
3. A Cepheid variable star or Cepheid variable stars in the supernovae host galaxy.

Cepheids are stars that vary periodically in brightness in a predictable way, and their brightness can be used to determine their distance from Earth. The Cepheid distances are then used to calibrate type Ia supernova luminosities, whose luminosities are then applied to SN Ia out into the far-field to measure $\mathcal{H}_0$ [22]. With the distance to the supernova known, the distance modulus, $\mu_{L^{\tau}}$, corrected for extinction is known. From the calibrated supernova luminosity, the redshift of the supernova is known. With the redshift of the supernova now known, the theoretically derived redshift dependent luminosity distance, $d_L(z_\lambda)$, is then calculated and the value of, $\mathcal{H}_0$, is deduced from the equation: $\mu_{L^{\tau}} = \mu_{L^{z_\lambda}}$.

4.2 CMB method

BAO experiments essentially measure two quantities, one parallel to the line-of-sight:

$$\beta_{\|} = \mathcal{H}(z)r_s(z_\star)\,, \tag{23}$$

and the other perpendicular to the line-of-sight:

$$\beta_{\perp} = \frac{r_s(z_\star)}{D_A(z)} = \theta_s(z_\star)\,, \tag{24}$$

where $\mathcal{H}(z)$ is the Hubble parameter, $r_s(z_\star)$ is the comoving sound horizon at recombination (i.e., the standard ruler) and $D_A(z)$ is the comoving angular distance to the observation redshift, z. The latter is computed as:

$$D_A(z) = d_{\rm H} \int_0^z \frac{dz}{\sqrt{\Omega}}\,. \tag{25}$$

The standard ruler $r_s(z_\star)$ is well constrained by CMB experiments. For the shape of $\mathcal{H}(z)$, one needs to assume some model (such as ΛCDM). Thus, by fitting the theoretical predictions for $\beta_{\|}$ and $\beta_{\perp}$ to the BAO data, we get indirect constraints on the expansion history of the Universe, $\mathcal{H}(z)$, and thus on the Hubble constant $H_0 = H(z = 0)$. In a similar way to other probes of the early Universe (as the CMB), this method gives a value of H_0 that is in tension with the direct measurement in the local Universe (using the cosmic distance ladder). Note that even if BAO observations are made in the late Universe (by looking at the large-scale distribution of galaxies), it is considered as an early probe because it provides a constraint on $r_s(z_\star)$, that gives information about the primordial plasma.

To determine, $\mathcal{H}_0$, from the CMB data one calculates a *Monte Carlo Markov Chain* (MCMC) which involves evaluation of the likelihood of parameter values and their associated spectra at tens to hundreds of thousands of points in the parameter space, and then one uses this chain to infer the posterior density of, $\mathcal{H}_0$, or any other cosmological parameter of interest [12, 23]. Apart from laying down the method leading to the calculation of, $\mathcal{H}_0$, what we want at the end of this section is a generic formula of how one proceeds to calculate $\mathcal{H}_0$.

The Hubble constant is inferred from CMB temperature anisotropies measurements. That is, measurements of temperature anisotropies in the CMB have revealed a series of (damped) acoustic peaks [12, 23]. These acoustic peaks constitute the esoteric fingerprint of the early Universe's BAO during the era of the pre-recombination plasma — i.e.: sound waves propagating in the baryon-photon plasma prior to photon decoupling, set up by the interplay between gravity and radiation pressure [24–28]. The first acoustic peak is set up by an oscillation mode which had exactly the time to compress once before freezing as photons decoupled shortly after recombination and this peak is precisely determined at: $\theta_s = 1^\circ$.

The first acoustic peak of the CMB carries the indelible imprint of the comoving sound horizon at last scattering $r_s(z_\star)$, given by the following:

$$r_s(z_\star) = \int_0^{z_\star} \frac{c_s(z_\lambda)dz_\lambda}{\mathcal{H}(z_\lambda)} = \frac{c_0}{\mathcal{H}_0} \int_0^{z_\star} \frac{c_s(z_\lambda)dz_\lambda}{c_0\sqrt{\Omega}}\,, \tag{26}$$

where: $z_\star \sim 1100$, denotes the redshift of last scattering, $\mathcal{H}(z_\lambda)$ denotes the expansion rate, and $c_s(z_\lambda)$ is the sound speed of the photon-baryon fluid. For most of the expansion history prior to last scattering, $c_s(z_\lambda)/c_0 \simeq 1/\sqrt{3}$, before dropping rapidly when matter starts to dominate.

On the other hand, the spatial temperature fluctuations at last scattering are projected to us as anisotropies on the CMB sky. As a consequence, the first acoustic peak actually carries information on the angular scale θ_s (usually referred to as the angular scale of the first peak), given by:

$$\theta_s(z_\star) = \frac{r_s(z_\star)}{D_A(z_\star)}\,, \tag{27}$$

where: $D_A(z_\star)$, is the angular diameter distance to the surface of last scattering, given by:

$$\begin{aligned} D_A(z_\star) &= \frac{c_0}{1+z_\star} \int_0^{z_\star} \frac{dz_\lambda}{\mathcal{H}(z_\lambda)} \\ &= \frac{c_0}{\mathcal{H}_0\,(1+z_\star)} \int_0^{z_\star} \frac{dz_\lambda}{\sqrt{\Omega}}\,, \end{aligned} \tag{28}$$

From this, one can determine the CMB-derived Hubble constant, $\mathcal{H}_0^{\rm CMB}$, from the following:

$$\mathcal{H}_0^{\rm CMB} = \frac{\theta_s(z_\star)}{r_s(z_\star)} \left(\frac{c_0}{1+z_\star} \int_0^{z_\star} \frac{dz_\lambda}{\sqrt{\Omega}} \right). \tag{29}$$

According (*e.g.*) to Vagnozzi (2020) [12], measurements of anisotropies in the temperature of the CMB, and in particular the position of the first acoustic peak (which appears at a multipole $\ell \simeq \pi/\theta_s$), accurately fix θ_s, therefore, any modification to the standard cosmological model aimed at solving the Hubble tension should not modify θ_s in the process.

In (29), we see that the CMB-derived redshift is not affected by the variation of FNCs. Apart from, $\mathcal{H}_0$, the only other FNC in the CMB $\mathcal{H}_0$ determination is the speed of Light and in-accordance with the very strong reservations laid down by [29] and [30], we are not going to vary this. The sound speed, c_0, in the pre-recombination plasma medium is the only quantity that can depend on FNC *via* the radiation density term, that is to say, the sound speed is such that: $c_s = c_0/\sqrt{3(1+\varrho_b/\varrho_\gamma)}$, where: ϱ_b and ϱ_γ, are the densities of baryonic matter and radiation in this plasma, respectively. Because during the plasma era, radiation dominated the Universe, hence, it is generally assumed that: $\varrho_b/\varrho_\gamma \lll 1$, so that the sound speed in this cosmic plasma medium is approximately equal to $c_0/\sqrt{3}$. Hence, the CMB measurements of, $\mathcal{H}_0$, are not affected by the variation of FNCs.

5 Problem

So what is the problem? We are of the strong view that the problem with the discrepancy leading to the Hubble tension may arise from an underestimate of the distance modulus (μ_L) from its determination using the luminosity distance and this underestimate may be a result of the variation of the FNCs:

most probably Planck's constant, $\hbar^*$. We will show in §7, that if indeed FNCs are to vary with cosmological time, then, this variation will introduce a form of "dark extinction" that is not accounted for in the typical calibrations leading to the Hubble constant and this is so for the case of the cosmic distance ladder method. The reason for this omission is that at present, the idea of a variable FNCs is not taken with the seriousness it so deserves despite observations [31–39] of the FSC strongly pointing to this possibility.

The two distance moduli, μ_{L^τ} and μ_{LT}, are determined and then compared (i.e.: $\mu_{L^\tau} = \mu_{LT}$), with μ_{L^τ} being determined from the brightness of the Cepheids resident in the supernovae galaxy, while, μ_{LT}, is determined from the supernova's redshift and in addition to the redshift, it relays on the chosen parameters of the Friedmann model. It is in this comparison: $\mu_{L^\tau} = \mu_{LT}$, that the Hubble constant, $\mathcal{H}_0$, is determined. One thing that one can immediately deduce without fail from this comparison is that $d_{L^\tau} \neq d_{LT}$. That is to say, from (12) and (18), we have that:

$$\mu_{L^\tau} = 5\log_{10}\left(\frac{d_{L^\tau}}{\text{Mpc}}\right) + 25\,, \tag{30a}$$

$$\mu_{LT} = 5\log_{10}\left(\frac{d_{LT}}{\text{Mpc}}\right) + 25\,, \tag{30b}$$

and from (30), it is not difficult to deduce that the said comparison of μ_{L^τ} and μ_{LT} ($\mu_{L^\tau} = \mu_{LT}$) implies that:

$$d_{L^\tau} = d_{LT}\,. \tag{31}$$

So, the luminosity and Light travel distances are generally not equal and are only equal in the case of the ISM having a vanishing optical depth. Now, before we deliver our suggested solution, we shall first motivate for our working model on the variation of FNCs.

6 Variable fundamental natural constants

If we blindly were to go by their verbatim name, then *Fundamental Natural Constants* (FNCs) aught to be what is purported or suggested by their very name "Fundamental", "Natural" and "Constant".

1. Fundamental — meaning intrinsic, inherent and foundational in all reality where they are involved;
2. Natural — meaning that these FNCs must arise naturally in our theories and are not imposed by our finite and limited *intellect*, *whim*, *will* or *desideratum*;
3. Constant — meaning they are sacrosanct and unchanging throughout the entire evolution of the Universe.

Pristinely and succinctly stated, the term *Fundamental Natural Constant* expresses a somewhat "divine" notion of the sacrosanctity of these seemingly immutable and divinely imposed physical quantities.

How far true is this assumption of sacrosanctity, immutability and constancy of these FNCs? For all we know, physics is an experimental human endeavour where answers to the questions that we pause regarding the inner and outer workings of Nature are to be sought by way of physical enquiry *via* ponderable measurements. That is to say, only measurements can decisively and conclusively answer this deep and very interesting question about the possible variation the FNCs. Fortunately, this question of the possible variation of FNCs is now a question capable of being answered from both experimental and observational science — thanks to the capabilities of modern state-of-the-art precision technology that has made this a reality.

The path to the road of inquiry into the possible variation of the FNCs began sometime in 1935 and 1937 with the great British theoretical physicists Edward Arthur Milne (1896-1950) and Paul Adrian Maurice Dirac (1902-1984). That is to say, Milne [40, 41] and Dirac [42] were perhaps the first (in the recorded scientific literature) to question this *status quo* by suggesting that this long held assumption that Newton's supposed universal constant of gravitation, G, was a sacrosanct and sacred constant of Nature that has remained constant since the Universe came into being.

To that end, if current observations [31–39] indicating the cosmological variation of the *Fine Structure Constant* (FSC) stand up to the most ruthless scientific scrutiny, then Milne [40, 41] and Dirac [42] may have been right after all, albeit not on the possible variation of Newton's constant G, but the cosmological variation of the FSC which involves four FNCs, namely: the electronic charge, $e = 1.602176634 \times 10^{-19}$ C; the permittivity of free space, $\varepsilon_0 = 8.8541878128(13) \times 10^{12}$ F m^{-1}; Planck's constant, $h = 6.62607015 \times 10^{-34}$ J s; and, the speed of Light in *vacuo*, $c_0 = 299792458 \times 10^8$ m s^{-1} (2022, CODATA Values).

The dimensionless FSC, denoted by the symbol α_0, is such that:

$$\alpha_0 = \frac{e^2}{4\pi\varepsilon_0\hbar c_0} = \frac{1}{137.035999074(44)}\,, \tag{32}$$

hence:

$$\frac{\Delta\alpha}{\alpha_0} = 2\left(\frac{\Delta e}{e}\right) - \frac{\Delta\varepsilon_0}{\varepsilon_0} - \frac{\Delta\hbar}{\hbar} - \frac{\Delta c}{c_0}\,, \tag{33}$$

that is to say, a cosmological variation in α_0, directly points to a variation in any one, or any possible combination, of the four FNCs: $e, \varepsilon_0, \hbar$, and, c_0.

At present, there exists no properly constituted and fairly accepted theory that explains why any of the supposed FNCs must vary. Most theories that do make the endeavour to explain the possibility of the variation of the FSC are speculative theories based on exotic and exogenous ideas [43–48] and some of these theories are yet to make contact with experience such as string and string-related theories.

*Typically, $\hbar$ is referred to as the reduced or normalized Planck constant. Fully cognisant of this fact, we shall however refer to this constant $\hbar$, simply as Planck's constant.

Following Dirac [42] on the variation of the Newtonian gravitational constant that it must vary in proportional to the age of the Universe, which also translates to a variation with respect to the cosmological scale factor $\mathfrak{a} = \mathfrak{a}(t)$, we shall assume that the expansion of the Universe is what is responsible for the variation of FNCs. That is to say, if for example, K, is some arbitrary FNC, then, its variation will scale in proportion to the scale factor, $\mathfrak{a}$, that is to say: $K \propto \mathfrak{a}^{\beta_{\text{K}}}$, and as a mathematical equation, this can be written as follows:

$$K = K_{\text{H}}\mathfrak{a}^{\beta_{\text{K}}} = K_{\text{H}}\,(1 + z_{\lambda})^{-\beta_{\text{K}}}\,, \tag{34}$$

where: K_{H}, is the value of this constant at the beginning of time where: $t = \tau_{\text{P}}$, and β_{K}, is the proportionality index for this constant and $\mathfrak{a}_0$, is the scale factor of the Universe while, $\mathfrak{a}$, is the scale factor of the Universe at the time of emission of the photon whose redshift we measure with our telescopes today. We hypothesize that the Universe began when the cosmic clock was reading one Planck second $\tau_{\text{P}} = \sqrt{G\hbar/c_0^5}$. From this, it follows that:

1. If: $\beta_{\text{K}} > 0$, then, the FNC in question increases with time, i.e., its value gets larger as the Universe gets older.
2. If: $\beta_{\text{K}} < 0$, then, the FNC in question decreases with time, i.e., its value gets smaller as the Universe gets older.
3. If: $\beta_{\text{K}} = 0$, then, the FNC in question is indeed a true constant of Nature.

In the present exploration of ideas, we shall assume that one of, or all of, or any possible combination of the four FNCs $(e, \varepsilon_0, \hbar, c_0)$ making up the FSC will vary with cosmological time, i.e.:

$$e = e_{\text{H}}\mathfrak{a}^{\beta_e} = e_{\text{H}}\,(1 + z_{\lambda})^{-\beta_e}\,, \tag{35a}$$

$$\varepsilon_0 = \varepsilon_{0\text{H}}\mathfrak{a}^{\beta_{\varepsilon_0}} = \varepsilon_{0\text{H}}\,(1 + z_{\lambda})^{-\beta_{\varepsilon_0}}\,, \tag{35b}$$

$$\hbar = \hbar_{\text{H}}\mathfrak{a}^{\beta_{\hbar}} = \hbar_{\text{H}}\,(1 + z_{\lambda})^{-\beta_{\hbar}}\,, \tag{35c}$$

$$c_0 = c_{0\text{H}}\mathfrak{a}^{\beta_{c_0}} = c_{0\text{H}}\,(1 + z_{\lambda})^{-\beta_{c_0}}\,, \tag{35d}$$

where: $e_{\text{H}}, \varepsilon_{0\text{H}}, \hbar_{\text{H}}$ and $c_{0\text{H}}$, are the values of the fundamental electronic charge, the permittivity of free space, Planck's constant and the speed of Light in *vacuo* at the beginning of time and: $\beta_e, \beta_{\varepsilon_0}, \beta_{\hbar}$, and, β_{c_0}, are the corresponding indices of the variation of these FNCs, respectively.

We want to be clear to our reader in that we are not proposing that all the four FNCs $e, \varepsilon_0, \hbar$, and, c_0, do vary with cosmic time. What we are saying is that the variation of the FSC allows us to entertain the possibility of the variation of at least one of these four constants. If we were asked our inclination regarding which of the four do we really think are varying, we would say, it is probably Planck's constant. We have our reasons, for we have pondered on this matter in our on-going ideas that we are still working on and are yet to be published; from the said ideas, we strongly holdfast that the speed of Light and as well the electronic charge must be true FNCs, thus leaving $\hbar$ and ε_0 as variables.

For our purpose here, it really does not matter as to which FNC is varying, as long just one of them is variable, this would lead to the Stefan-Boltzmann-Planck constant, σ_0, being a variable as it does depend on the Planck constant and the speed of Light in *vacuo*. That is to say, we know that:

$$\sigma_0 = \frac{2\pi^5 k_{\text{B}}^4}{15\hbar^3 c_0^2} = 5.670374419 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}\,, \tag{36}$$

where: $k_{\text{B}} = 1.380649 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$ (2022, CODATA Value) is Boltzmann's constant. From (36), it follows that if say, $\hbar$, or, c_0, did vary with cosmological time, then, σ_0, will vary cosmologically as well, i.e.:

$$\sigma_0 = \sigma_{0\text{H}}\mathfrak{a}^{\beta_{\sigma}}\,, \tag{37}$$

where as before: $\sigma_{0\text{H}}$, is the Stefan-Boltzmann-Planck constant at the beginning of time and, $\beta_{\sigma} = 4\beta_{k_{\text{B}}} - 3\beta_{\hbar} - 2\beta_{c_0}$, is the corresponding index of the cosmological variation of σ_0. For our purposes here, following the strong advice of Ellis & Uzan [29, 30], we shall assume that: $\beta_{c_0} = 0$, and also following our own intuition, we shall assume: $\beta_{k_{\text{B}}} = 0$; hence, we shall have: $\beta_{\sigma} = -3\beta_{\hbar}$ and this implies that the luminosity of a star, L, will vary with the scale factor as follows:

$$L \propto \mathfrak{a}^{-3\beta_{\hbar}}\,. \tag{38}$$

Equipped with this seemingly strange and exotic hypothetical idea of the cosmological variation of FNCs, we are going to suggest in the next section a plausible solution to the Hubble tension problem.

7 Proposed solution

From the thesis just laid down in the previous section, it is pristine clear that if FNCs are variable, then there aught to be a discrepancy in the values of early and late measurements of $\mathcal{H}_0$, and the reason is simple because these epochs have different values of these FNCs that drive the physics thereof. For example, late-type values are those from the local neighbourhood where the FNCs $(k_{\text{B}}, \hbar, c_0)$ in those galaxies are just about the same as in our own galaxy, whereas in the early-type $\mathcal{H}_0$-measurements, the FNCs are significantly different from our own, hence we are comparing two significantly different cosmological epochs. Thus, from the foregoing, it is clear that late-type $\mathcal{H}_0$-measurements aught to be the true and correct values of $\mathcal{H}_0$, whereas those from the early-type measurements are going to contain a hitherto intrinsic and inherent additional signal (term) which is not accounted for in contemporary measurements, hence the tension.

Now, in order to see how this variation of FNCs comes in, from (38), we now have the FNC variation term, $\mathfrak{a}^{-3\beta_{\hbar}}$, in the flux emitted by the source at distance, d, i.e.:

$$F(d_L) = F_0\left(\frac{4\pi R^2}{4\pi d_L^2}\right)\mathfrak{a}^{-3\beta_{\hbar}}e^{-\tau}\,, \tag{39}$$

where in (39), we see that in comparison to (7), we have in addition to the traditional extinction term, $e^{-\tau}$, there now is supplemented a new extinction term $\mathfrak{a}^{-3\beta_\hbar}$. Our claim is that it is this term $\mathfrak{a}^{-3\beta_\hbar}$ that is not accounted for in contemporary cosmology models that do not embrace the variation of the FNCs.

Now, just as before for the absolute magnitude, we need the flux, $F(10\text{pc})$, at a distance of 10 parsecs as this is to be evaluated without any form extinction — either the optical (τ) term or the FNC-variation term ($\beta_\hbar$), i.e.:

$$F(10\text{pc}) = F_0\left(\frac{4\pi R^2}{4\pi(10\text{pc})^2}\right). \tag{40}$$

From (39) and (40), it follows that:

$$\frac{F(d_L)}{F(10\text{pc})} = \frac{(10\text{pc})^2}{d_L^2}\mathfrak{a}^{-3\beta_\hbar}e^{-\tau}, \tag{41}$$

hence, the *variation* of FNCs corrected-distance modulus, μ'_L, is given by:

$$\mu'_L = \overbrace{5\log_{10}\left(\frac{d_L}{\text{Mpc}}\right) + 25 + A_\tau}^{\mu_{L^\tau}} + \underbrace{\overbrace{5\log_{10}\left(\mathfrak{a}^{1.5\beta_\hbar}\right)}^{\mu_\text{D}}}_{\textbf{Dark-Term}}. \tag{42}$$

That is to say, (42) reads: $\mu'_L = \mu_{L^\tau} + \mu_\text{D}$, where: μ_D, is a new emergent dark-term that arises from the variation of the Planck constant (if the Planck constant is not variable, then it must either be, k_B, and, c_0). Since: $\mu'_L = \mu_{LT}$, it follows that:

$$\begin{aligned} &\overbrace{5\log_{10}\left(\frac{d_L}{\text{Mpc}}\right) + 25 + A_\tau}^{\mu_{L^\tau}} + \underbrace{\overbrace{5\log_{10}\left(\mathfrak{a}^{1.5\beta_\hbar}\right)}^{\mu_\text{D}}}_{\textbf{Dark-Term}} = \\ &\qquad = 5\log_{10}\left(\frac{d_{LT}}{\text{Mpc}}\right) + 25. \end{aligned} \tag{43}$$

Taking the dark-term to the right hand-side of (43), we will have:

$$\begin{aligned} &5\log_{10}\left(\frac{d_L}{\text{Mpc}}\right) + 25 + A_\tau = \\ &\qquad = 5\log_{10}\left(\frac{d_{LT}}{\text{Mpc}}\right) + 25 - 5\log_{10}\left(\mathfrak{a}^{1.5\beta_\hbar}\right). \end{aligned} \tag{44}$$

We can re-write (44), as follows:

$$\begin{aligned} &\overbrace{\underbrace{\mu_{L^\tau} = 5\log_{10}\left(\frac{d_{L^\tau}}{\text{Mpc}}\right)}_{\textbf{Observationally Derived}} + 25}^{\textbf{Flux-Dependent}} = \\ &\qquad = \overbrace{\underbrace{5\log_{10}\left(\frac{d_{LT}^{\Delta\sigma_0}}{\text{Mpc}}\right) + 25}_{\textbf{Theoretically Derived}}}^{\textbf{Redshift-Dependent}} = \mu_{LT}^{\delta\hbar}, \end{aligned} \tag{45}$$

where:

$$d_{LT}^{\Delta\sigma_0} = \mathfrak{a}^{-1.5\beta_\hbar}d_{LT}, \tag{46}$$

is what we shall call the *FNC variation-corrected Light travel distance*, where in the present case, the FNC for which the Light travel distance has been corrected for, is the Planck constant because it is the particular FNC that we have chosen is variable, while the other two (k_B, c_0) have been held constant.

Now, given that in the ΛCDM cosmology model, the redshift, z_λ, and the scale factor, a, are related as follows: $1+z_\lambda = a_0/a$, i.e.:

$$a = \frac{a_0}{1+z_\lambda}, \tag{47}$$

where: a_0, is the present day scale factor of the Universe while, a, is the Universe's scale factor at the time of emission of the photon that we receive here on Earth. The present scale factor of the Universe is set: $a_0 = 1$. From this, it follows that if we are to insert this into (42), we will obtain:

$$d_{LT}^{\Delta\sigma_0} = (1+z_\lambda)^{1.5\beta_\hbar}\, d_{LT}. \tag{48}$$

Now, since: $d_{L^\tau} = d_{LT}^{\Delta\sigma_0}$, it follows that:

$$d_{L^\tau} = (1+z_\lambda)^{1.5\beta_\hbar}\, d_{LT} = d_\text{H}\,(1+z_\lambda)^{1.5\beta_\hbar}\, f(z_\lambda), \tag{49}$$

hence:

$$\mu_{L^\tau} = 5\log_{10}\left[(1+z_\lambda)^{1.5\beta_\hbar}\, f(z_\lambda)\right] + K, \tag{50}$$

where, K, is no longer as has been defined in (20), but is now defined as follows:

$$K = 25 + 5\log_{10}\left(\frac{c_0}{\text{Mpc}}\right) - 5\log_{10}(n_r\mathcal{H}_0). \tag{51}$$

This completes our theoretical exegesis on the plausible origins of the Hubble tension. What is now left is for us to calibrate this result (50) against real data. In order to to do this, there is need to first figure out what, $f(z_\lambda)$, is. This function, $f(z_\lambda)$, is dependent on the cosmology model that one adopts. In our present case, we shall adopt a cosmology for which the total Ω-parameter is identically equal to unity, i.e.: $\Omega \equiv 1$. That is to say, Ω, does not happen to be equal to unity in the present epoch of the Universe's evolution, but is eternally so for all times — i.e., from antiquity to eternity. If as declared: $\Omega \equiv 1$, it follows from (16), that:

$$f(z_\lambda) = \ln(1+z_\lambda), \tag{52}$$

hence:

$$\mu_{L^\tau} = 5\log_{10}\left[(1+z_\lambda)^{1.5\beta_\hbar}\ln(1+z_\lambda)\right] + K. \tag{53}$$

Thus, (53) is what we are going to test against observational evidence and we must hasten to say that (53) has not been *priori* designed to fit the observational data that it will excellently fit. It actually came as nothing short of a *non-posteriori* surprise that this model [(53)] agrees very well with empirical evidence.

8 Application of theory

We are now ready to apply our ideas onto some real and tangible data and for this, we are going to use the Supernova Cosmology Project (SCP) Union2.1 dataset spanning the redshift range: $0.015 \leq z_\lambda \leq 1.414$, [49]. This dataset is a compilation of 580 SNe type Ia drawn from 19 datasets [50–67]. We must say that this dataset may very well be the most comprehensive and most accurate SNe data available to date. Further, according to Suzuki *et al.* [49], all SNe were fitted using a single light-curve fitter (SALT2-1) and uniformly analyzed in blind-mode, i.e., without due consideration of a particular cosmology model. With 580 data points in the sufficiently large redshift range: $0.015 \leq z_\lambda \leq 1.414$, we certainly do have a statistically significant dataset to make a meaningful conclusion on the present model (53) of the plausible time variability of FNCs.

What we really want in this section is to test the proposed model presented in (53). We want to find the value of $\beta_\hbar$, and, K; and from the value of K, we can deduce $\mathcal{H}_0$. To that end, in Fig. 1, we have plotted the distance modulus, μ_{L^τ}, *vs* the redshift, z_λ, of the 580 SNe from the Union2.1 dataset and with this dataset, we perform a non-linear curve fitting on the data and from this non-linear curve fitting exercise, we obtain:

$$\beta_\hbar = +0.77 \pm 0.02\,, \tag{54a}$$

$$K = 43.20 \pm 0.01\,\text{mag}\,. \tag{54b}$$

From the value of K, obtained (43.20 ± 0.01 mag.), we find for the Hubble constant, the value:

$$\mathcal{H}_0 = \frac{68.70 \pm 0.30\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}}{n_r} = \frac{\mathcal{H}_0^{\text{SNe}}}{n_r}\,. \tag{55}$$

If the ISM is a perfect *vacuo* (which it obviously is not), then:

$$\mathcal{H}_0 = \mathcal{H}_0^{\text{SNe}} = 68.70 \pm 0.30\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}\,. \tag{56}$$

This value given (56) is the corrected *vacuo* SNe $\mathcal{H}_0$-value where the correction made is that hypothesised variation in the Planck constant and the tension in this value when compared with the CMB-value is significant at a 2.2σ-level (97%) of statistical significance.

Of this value, within the provinces of its own error margins, one can safely say that this rather unexpected result is in very good agreement (0.5σ-level of statistical significance in discrepancy) with that of Freedman *et al.* [8]'s TRGB-midpoint value: $\mathcal{H}_0 = 69.80 \pm 2.20\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. Further, this value is in agreement with the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) data for the CMB data — where: $\mathcal{H}_0 = 69.30 \pm 1.60\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ [68, 69], and, the *Planck 2013* data — where: $\mathcal{H}_0 = 69.80 \pm 2.20\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ [70]. In the $\mathcal{H}_0$ values of Anderson *et al.* [68], Mehta *et al.* [69] & Ade *et al.* [70], the BAO data has been admitted together with the CMB data, thus allowing Ω_k to be a free parameter [68, 70, 71], and this is unlike in Aghanim *et al.* [9]'s case were the curvature parameter has been tightly constrained to: $\Omega_k \sim 0$. Furthermore, applying the *WMAP & CMB* constraints to both BAO and SNe data together with the CMB, Blake *et al.* [72] obtained: $\mathcal{H}_0 = 68.70 \pm 1.90\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, and Anderson *et al.* [68] obtained: $\mathcal{H}_0 = 69.60 \pm 1.70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. Within the margins of error — all these results are in very good agreement with our result: $\mathcal{H}_0 = 68.70 \pm 0.30\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. While this is the case — that our derived value is an improvement in matching the two discontent $\mathcal{H}_0$-values, if at all possible, there is need to get a most perfect agreement between these two values and this can be done by considering the fact that the ISM is not a perfect *vacuo*, the meaning of which is that we need not assume a refractive index of unity for the ISM.

9 Concordance $\mathcal{H}_0$–value

As stated above, the two discontent $\mathcal{H}_0$-values ($\mathcal{H}_0^{\text{SNe}}$ and $\mathcal{H}_0^{\text{CMB}}$) can be brought into concordance by considering the fact that the ISM is not a perfect *vacuo*. That is to say, in the derivation of $\mathcal{H}_0^{\text{SNe}}$, leading to (55), the refractive index was taken into account but later in (56), it (refractive index) was then set to equal unity. We shall drop this assumption that the refractive index is unity. On the same pedestal, we must realize that this same assumption that the refractive index of ISM is unity is employed in the CMB-derivation of $\mathcal{H}_0^{\text{CMB}}$ in (29).

In order for us to take the refractive index into account in (29), what we need to do is to replace c_0 with c_0/n_r. So doing, we obtain:

$$\mathcal{H}_0^{\text{CMB}} = \frac{\theta_s(z_\star)}{r_s(z_\star)}\left(\frac{c_0/n_r}{1+z_\star}\int_0^{z_\star}\frac{dz_\lambda}{\sqrt{\Omega}}\right) = \frac{\mathcal{H}_0}{n_r}\,. \tag{57}$$

From (57), we obtain: $\mathcal{H}_0 = n_r\mathcal{H}_0^{\text{CMB}}$, and proceeding to substitute this into (55), we obtain:

$$\begin{aligned} n_r &= \sqrt{\frac{\mathcal{H}_0^{\text{SNe}}}{\mathcal{H}_0^{\text{CMB}}}}\,,\\ &= \sqrt{\frac{68.70 \pm 0.30\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}}{67.40 \pm 0.50\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}}}\,,\\ \therefore\; n_r^{\text{ISM}} &= 1.010 \pm 0.006\,. \end{aligned} \tag{58}$$

In all probity, this value ($n_r^{\text{ISM}} = 1.010 \pm 0.006$) is not at all in bad agreement with the measured refractive index ($n_r^{\text{ISM}} =$ 1.0001 to 1.0003 [73–75]) of the ISM. With this ISM refractive index value (1.010 ± 0.006), the concordance $\mathcal{H}_0$-value is:

$$\mathcal{H}_0 = 68.00 \pm 0.90\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}\,. \tag{59}$$

Within the margins of error, this concordance $\mathcal{H}_0$-value is in good agreement with Freedman *et al.* [8]'s TRGB $\mathcal{H}_0$-value. This good agreement can very well be understood from the fact that the TRGB stars, from which these measurement are inferred, are nearby stars and as a direct result

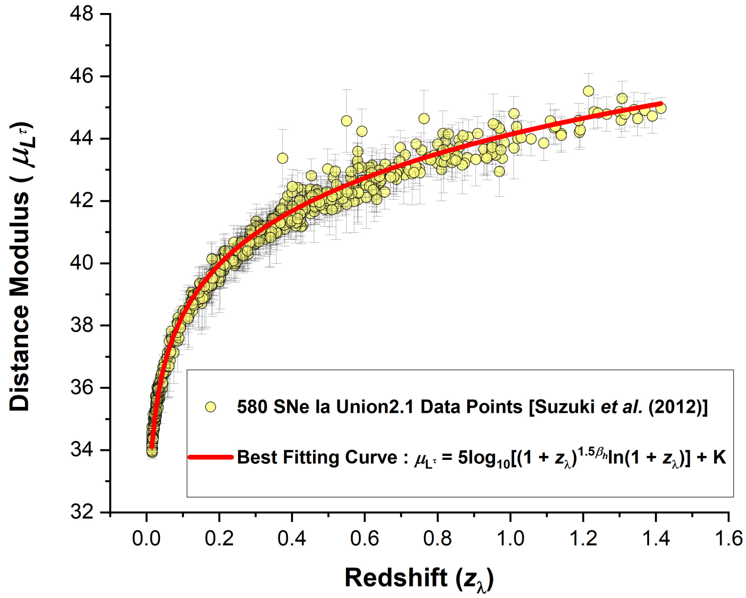

Fig. 1: Graph of Distance Modulus (μ_L) *vs* Redshift (z_λ) from the Union2.1 data of [49]. The Best Fit Graph (RED) is described by the non-linear curve: $\mu_{L^\tau} = 5\log_{10}\left[(1+z_\lambda)^{1.5\beta_\hbar}\ln(1+z_\lambda)\right] + K$, and, from it we obtain the following best parameter fittings: $\beta_\hbar = 0.77 \pm 0.02$, and, $K = 43.20 \pm 0.01$ mag. The R^2-value or Coefficient of Determination (COD) of the fit to data is: 99.49%. Assuming an ISM refractive index of unity — i.e.: $n_r^{\rm ISM} \equiv 1$, the obtaining K-value leads to: $\mathcal{H}_0 = 68.70 \pm 0.30\,{\rm km\,s^{-1}\,Mpc^{-1}}$. In order to bring the CMB and SNe Ia measurements into unity and harmony, an ISM refractive index of: $n_r^{\rm ISM} = 1.010 \pm 0.006$, is needed and this leads to a concordance $\mathcal{H}_0$-value of: $\mathcal{H}_0 = 68.00 \pm 0.90\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

of this fact, the value of Planck's constant for these systems is pretty much the same as the value of Planck's constant here on Earth, hence, the correction of the variation of Planck's constant needed on these measurements may very well be negligible. Be that as it may, there is need to subject TRGB $\mathcal{H}_0$-measurements to the present idea of a variable Planck constant.

10 General discussion

We have herein suggested that cosmologically varying FNCs may very well present a viable and perdurable solution to the current crisis in cosmology, namely, the Hubble tension. That is to say, from the same SNe Ia data that usually produces values of the Hubble constant in the range ∼ 70–to–76 km s^{-1} Mpc^{-1}, we have downgraded this old value to the new concordance $\mathcal{H}_0$-value: 68.00 ± 0.10 km s^{-1} Mpc^{-1}, and in the same exercise, the Planck collaboration value of: 67.40 ±0.50 km s^{-1} Mpc^{-1} has been upgraded to this concordance $\mathcal{H}_0$-value. This has required two major ideas to be evoked, namely, the:

1. Assumption of a cosmologically varying Planck constant, $\hbar$.
2. Adoption of a non-unity value for the refractive index of the ISM.

The assumption of a cosmologically varying Planck constant reduces the SNe Ia derived value of the Hubble constant from the: 70–to–76 km s^{-1} Mpc^{-1}, territory, to exactly: 68.70 ± 0.30 km s^{-1} Mpc^{-1}. As pointed out in the penultimate of §4.2, this assumption of a cosmologically variable Planck constant does not apply to the derivation of the CMB-derived Hubble constant because none of the physical parameters that enter in the formulae leading to the $\mathcal{H}_0^{\rm CMB}$ depend on $\hbar$. Effectively, what this means is that the tension in $\mathcal{H}_0^{\rm SNe}$ and $\mathcal{H}_0^{\rm CMB}$ is reduced and not resolved. The initial tension* (gap in the two values) is: 7.44 km s^{-1} Mpc^{-1}, and this is reduced to: 2.10 km s^{-1} Mpc^{-1}, and this is an 88% reduction. In order to "resolve" the tension completely, the fact that the ISM is not a perfect *vacuo* is taken into account and this fact affects both measurements — i.e., the CMB and SNe Ia measurement and is seen that a refractive index: $n_r = 1.010 \pm 0.006$, resolves the tension completely, leading to the concordance value: $\mathcal{H}_0 = 68.00 \pm 0.90$ km s^{-1} Mpc^{-1}.

We candidly must say that our choice in the Planck constant, $\hbar$, as the likely culprit is informed by what we believe to

*By tension here we mean the difference within the margins of error between the two values: $\mathcal{H}_0^{\rm SNe} = 68.00 \pm 0.10$ km s^{-1} Mpc^{-1}, and, $\mathcal{H}_0^{\rm CMB} = 67.40 \pm 0.50$ km s^{-1} Mpc^{-1}. That is to say, the difference in: MIN($\mathcal{H}_0^{\rm SNe}$) = 71.97 km s^{-1} Mpc^{-1}, and, MAX($\mathcal{H}_0^{\rm CMB}$) = 67.90 km s^{-1} Mpc^{-1}. Clearly, this difference is equal to: 4.07 km s^{-1} Mpc^{-1}. Following the same line of thought and reasoning, the tension in the new variable-$\hbar$ corrected: $\mathcal{H}_0^{\rm SNe} = 68.00 \pm 0.10$ km s^{-1} Mpc^{-1}, and the old CMB-derived $\mathcal{H}_0$-value: $\mathcal{H}_0^{\rm CMB} = 67.40 \pm 0.50$ km s^{-1} Mpc^{-1}, is: 0.50 km s^{-1} Mpc^{-1}. Clearly, the percentage reduction in tension is: $(1 - 0.50\,{\rm km\,s^{-1}\,Mpc^{-1}}/4.07\,{\rm km\,s^{-1}\,Mpc^{-1}}) \times 100\% = 88\%$.

be our strong intuition rather than scientific objectivity. Because of a general lack or consensus on the variation of the FSC; without fail, we must say that ideas of variable FNCs are by their nature largely considered to be speculative and may very well be outside of the realm of the general support of contemporary scientific understanding because despite claims of a variable FSC [31–39], at present, there is no direct "incriminating" and invigorating evidence to suggest that the Planck constant could change over cosmic times [76–78].

The most widely considered FNCs to vary over cosmic times is the speed of Light [79–87] but we have our deep-seated reasons for holding back on taking this position*. We are not going to take a position simply because everyone is taking that position because we are aware that no amount of research on the candle would have led mankind to discover the Light bulb. To discover the Light bulb, it was needed to consider ideas alien to our common experience. We have here chosen to take the "road less travelled if not the road not travelled" and vary Planck's constant.

Because the Planck constant sets the scale for the quantum nature of particles and their interactions with its value determining the granularity of atomic energy levels and the scale at which quantum effects become significant — in a Universe with an increasing Planck constant such as the one that we are suggesting, over cosmic times, the behaviour of the Universe would tend to be more classical rather than quantum mechanical. From a quantum probability calculus view point, this means the initial state of the Universe must have been less probabilistic (i.e., highly unpredictable) and has been evolving into a more probabilistic state (i.e., more predictable). This evolutionary sequence of the Universe resonates well with the *Second Law of Thermodynamics* (SLT) as this implies that the Universe must have started in a state of lowest entropy and has been, and is, evolving into a state of highest entropy.

Further, if the Planck constant $\hbar$, were to vary as suggested here, it could help solve one of the outstanding problems in the Universe's expansion to do with the conservation of the photon's energy and the expansion of the spacetime. The problem is a simple one and is as follows. We know that the energy, E_γ, of a photon is related to the photon's wavelength, λ, as follows: $E_\gamma = 2\pi\hbar c_0/\lambda$. As a result of cosmic expansion, the wavelength of the photon increases. If $\hbar$, and, c_0, are to remain constant as the spacetime expands, it follows that the energy of the photon will diminish without any foreseeable compensation — i.e.: $\Delta E_\gamma \neq 0$, and this obviously violates the *Law of Conservation of Energy*.

*Completely in agreement with Ellis [29] and Ellis & Uzan [30], we are of the view that the speed of Light cannot be varied in a "part or portion of physics" but must be done wholesomely in a consistent manner at a most fundamental level. At the very least, this requires a complete and total rewrite of physics. Varying the speed of Light is unimaginable at the very least. We have held fast in the present exploration the idea of a sacrosanct and invariant speed of Light.

Where does the diminished energy go to? This is something that has bothered the desideratum of the foremost theoretical physicist since this issue was first noticed and to this day, it has not been resolved. In the advent of a time-variable Planck constant and an invariant Light speed c_0, one can postulate that the photon energy is conserved ($\Delta E_\gamma = 0$) and the compensation in the increase in its wavelength comes in the wake of an equal compensation in the increase of the Planck constant — i.e.:

$$z_\lambda = \frac{\Delta\lambda}{\lambda} = \frac{\Delta\hbar}{\hbar}\,. \tag{60}$$

What (60) means is that the redshift, z_λ, that we measure must be a measure in the change of the Planck constant.

Regarding the evidence of a varying Planck constant, there have been few direct references in the literature on the subject of a variable Planck constant [88–90]. [88, 89] approaches the subject from a laboratory view point while [90] does this on a purely speculative theoretical standpoint. Searches for a variable Planck constant have been under the guise of a variable FSC [31, 39] which amongst others also implies a variable electronic charge, the speed of Light and/or the permittivity of free space.

Hutchin [89] reports that a gradual and systematic drop has been observed in the decay rates of 8 radionuclides $\left[{}^{226}\text{Ra},\ {}^{154}\text{Eu},\ {}^{238}\text{Pu},\ {}^{3}\text{H},\ {}^{54}\text{Mn},\ {}^{60}\text{Co},\ {}^{90}\text{Sr},\ {}^{36}\text{Cl}\right]$ over a 20 year span by six organizations on three continents (German, American and Russian labs), including beta decay (weak interaction) and alpha decay (strong interaction) and in the search for a common cause, Hutchin [89] hypothesizes that small variations in Planck's constant might account for the observed synchronized variations in these strong and weak decays.

Hutchin [88] further suggests that this proposed variation of $\hbar$, may very well be a good candidate for the cause of the Casimir radiation and further proposes that if this Casimir radiation were emitted by stars *via* a changing $\hbar$, then:

> ... this could provide an alternative explanation for the Hubble constant, where the distant galaxies are redder simply because $\hbar$ is smaller back in time, making local time move more slowly. In contrast to the expanding model of the Universe, we could now consider whether our Universe might simply be static, where gravity is everywhere balanced on a large scale. Such a conclusion would end the search for dark energy since such a Universe is essentially static while the usual red shift would still be observed.

Unlike Hutchin [88], we do not believe that a variable $\hbar$ necessarily rules out "the expansion of the Universe and points to a *Static Universe*."

As apparent fissures in the standard model have been emerging, there are also indications that there may be cracks that need attention in the local distance scale as well. For example, the tip of the red giant branch (TRGB) method and the Cepheid distance scale result in differing values of $\mathcal{H}_0$ =

69.60 ± 1.90 km s^{-1} Mpc^{-1} [8,91] for the TRGB and 73.30 ± 1.04, km s^{-1} Mpc^{-1} [7], for the Cepheids. This divergence raises the question of whether the purported tension is being driven by yet-to-be-revealed systematic errors in the local Cepheid data, rather than in the cosmological models.

11 Conclusion

Assuming what has been presented herein is acceptable, we hereby present the following as the logical conclusion that can be drawn thereof:

1. We have shown that the Hubble tension can in principle be alleviated if we assume a cosmologically varying Planck constant and as well as a dispersive non-zero refractive index ISM.
2. Further — we have shown that the current supernovae derived $\mathcal{H}_0$-value can be brought down from its current lofty value: $\mathcal{H}_0^{\text{SNe}} = 73.30 \pm 1.03$ km s^{-1} Mpc^{-1}, down to: 68.70 ± 0.30 km s^{-1} Mpc^{-1}, and this new value is not in dire disagreement with the CMB-derived $\mathcal{H}_0$-value: $\mathcal{H}_0^{\text{CMB}} = 67.40 \pm 0.50$ km s^{-1} Mpc^{-1}. That is to say, at a 2.2σ-level of statistical significance in discrepancy, this new $\mathcal{H}_0$-value reduces the tension by 88%.
3. Furthermore — in order to "resolve" the tension completely, the fact that the ISM is not a perfect *vacuo* is taken into account and this fact affects both measurements — i.e., the CMB and SNe Ia measurement and it is seen that a refractive index: $n_r = 1.010 \pm 0.006$, resolves the tension completely, leading to the concordance value: $\mathcal{H}_0 = 68.00 \pm 0.90$ km s^{-1} Mpc^{-1}.
4. Additionally — apart from providing a viable solution to the Hubble tension problem, a time variable Planck constant has the potential to solve the problem of the conservation of the photon's energy in an expanding Universe if it is to be assumed that the photon's redshift, $\Delta\lambda/\lambda$, is compensated by a change in Planck's constant, $\Delta\hbar/\hbar$. Ultimately, the photon's redshift under this model emerges as a measure in the change in Planck's constant.
5. Lastly — as demonstrated herein, the idea of varying FNCs aught to be taken much more seriously than currently done as this has the potential to solve the darkenergy and darkmatter problem because if FNCs are really variable, this variation may bring in "dark" effects that might explain away darkenergy and darkmatter.

Dedication

This reading is dedicated to my friend *Anna Neff*.

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