Accelerations of the Closed Time-Like Gödel Curves

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In a paper published in 1949 in honor of his close friend Albert Einstein on the occasion of his 70th birthday, Kurt Gödel described a homogeneous and rotating universe by discovering the existence of closed timelike curves (CTCs). In a series of papers, we replaced the constant a of the Gödel metric with a simple conformal factor, which easily induces a pressure term that leads directly to the ideal fluid field equation. Gödel introduced this special term a, relating it to the cosmological constant, to make his solution satisfying Einstein's field equations. This theory is now endowed with physical sense, and the dynamics no longer apply to space, but to a fluid. Eventually, the Gödel CTCs are considered to be flow lines of a charged fluid, which preserve the properties of the model. The resulting acceleration of these flow lines can then be adequately controlled.

Notations

Space-time indices: $\mu, \nu = 0, 1, 2, 3$;
Space-time signature: -2;
Einstein's constant: $\varkappa$;
The velocity of light: $c = 1$.

1 The Gödel Universe

1.1 General

In his original paper [1], K. Gödel derived an exact solution to Einstein's field equation that describes a homogeneous and non-isotropic universe where matter takes the form of a shear-free fluid. This metric exhibits a rotational symmetry that allows for the existence of closed timelike curves (CTCs).

Gödel's model is usually regarded as a mathematical curiosity and is rejected because it requires a cosmological constant related to a constant Ricci scalar finely tuned to the mass density of the Universe.

In several publications, we have been able to relax our requirement that the Gödel metric be a description of our real Universe, which is still observed to be expanding.

1.2 Gödel's metric

The classical Gödel line element is given by:

$$ds^2 = a^2\left(dt^2 + \frac{1}{2}\,e^{2x}dy^2 - 2\,e^x\,dt\,dy - dx^2 - dz^2\right), \qquad (1.1)$$

where $a > 0$ is a constant.

The normalized unit vector $\boldsymbol{u}$ of matter has components:

$$u^\mu = (a^{-1},\, 0,\, 0,\, 0)\,, \quad u_\mu = (a,\, 0,\, ae^x,\, 0)\,, \qquad (1.2)$$

thus the Ricci tensor takes the value

$$R_{\mu\nu} = u_\mu u_\nu a^{-2} \qquad (1.3)$$

and the Ricci scalar is

$$R = u^\mu u_\mu = a^{-2}. \qquad (1.4)$$

Since R is a constant, then the field equations

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R = \varkappa\rho u_\mu u_\nu + \Lambda g_{\mu\nu} \qquad (1.5)$$

are satisfied (for a given value of the density ρ), if we put:

$$a^{-2} = \varkappa\,, \qquad (1.6)$$

$$\Lambda = -\frac{1}{2}\,R = \frac{1}{2a^2} = -\frac{1}{2}\,\varkappa\rho\,. \qquad (1.7)$$

The sign of the cosmological constant Λ here is opposite to that in Einstein's field equations. Bearing in mind that a is a constant, finetuning the density of the universe with the cosmological constant and the Ricci scalar appears as a dubious hypothesis. One then clearly sees that such cosmological constraints are physically irrelevant.

2 The Gödel model as a homogeneous perfect fluid

2.1 Reformulation of the Gödel metric

In our publications [2, 3], we assumed that a is slightly space-time variable and we set:

$$a^2 = e^{2U} \qquad (2.1)$$

(the positive scalar $U(x)$ will be explicited below). Thus, the Gödel metric takes the form:

$$ds^2 = e^{2U}\left(dt^2 + \frac{1}{2}\,e^{2x}dy^2 - 2\,e^x dt\,dy - dx^2 - dz^2\right). \qquad (2.2)$$

With the Euler variational derivation detailed in [4–6], this conformal metric leads to the Einstein field equations for a perfect fluid [7]:

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R = \varkappa\left[(\rho + P)\,u_\mu u_\nu - Pg_{\mu\nu}\right]. \qquad (2.3)$$

Now, the real 4-unit vector $\boldsymbol{u}$ of the Gödel fluid displays the following components:

$$u^\mu = (1,\, 0,\, 0,\, 0)\,, \qquad u_\mu = (1,\, 0,\, e^x,\, 0)\,. \qquad (2.4)$$

2.2 Differential geodesic system

The 4-unit vector u^μ is normalized on $(\mathrm{M}, \boldsymbol{g})$:

$$g_{\mu\nu} u^\mu u^\nu = g^{\mu\nu} u_\mu u_\nu = 1 .$$

By differentiating we get:

$$u^\nu \nabla_\mu u_\nu = 0 . \tag{2.5}$$

Let us define the vector L_ν by the relation

$$\nabla_\mu P \delta^\mu_\nu = \mathbf{r} L_\nu \tag{2.6}$$

having set $\mathbf{r} = \rho + P$.

The conservation law for $T_{\mu\nu} = \mathbf{r} u_\mu u_\nu - P g_{\mu\nu}$ is expressed by $\nabla_\mu T^\mu_\nu = 0$, i.e.:

$$\left.\begin{aligned} &\nabla_\mu (\mathbf{r} u^\mu u_\nu) = \mathbf{r} L_\nu \\ &\nabla_\mu (\mathbf{r} u^\mu) u_\nu + \mathbf{r} u^\mu \nabla_\mu u_\nu = \mathbf{r} L_\nu \end{aligned}\right\} . \tag{2.7}$$

Multiplying through this relation with u^ν and taking into account (2.5), we obtain, by substituting in (2.7) and after dividing by $\mathbf{r}$:

$$u^\mu \nabla_\mu u_\nu = (g_{\mu\nu} - u_\mu u_\nu) L^\mu \tag{2.8}$$

with the projection tensor $h_{\mu\nu} = (g_{\mu\nu} - u_\mu u_\nu)$

$$u_\nu = h_{\mu\nu} L^\mu . \tag{2.9}$$

With setting $L_\nu = \partial_\nu U$, the equation (2.9) takes the form ${}^*u_\nu = h_{\mu\nu} \partial^\mu U$ and (2.6) reads

$$(\rho + P) L_\nu = \nabla_\mu P \delta^\mu_\nu ,$$

$$L_\nu = \frac{\partial_\nu P}{\rho + P} .$$

As a result we find:

$$U(x^\mu) = \int_{P_1}^{P_2} \frac{dP}{\rho + P} ,$$

where the pressures P_1 and P_2 are related to the points x_1 and x_2, respectively.

The flow lines of a perfect fluid with a density ρ and a pressure P with the equation of state $\rho = f(P)$ obey the differential system:

$$u^\mu \nabla_\mu u_\nu = h_{\mu\nu} \partial^\mu U = {}^*u_\nu . \tag{2.10}$$

The 4-vector ${}^*u_\nu$ must be regarded as the 4-acceleration of the flow lines given by the pressure gradient orthogonal to those lines [8, p.70].

Controlling this acceleration is almost impossible: varying the pressure P through the equation of state appears as physically unrealistic. There is however a way to solve this problem: the fluid encoding the CTCs should be characterized by a charged current density acted upon by a variable electromagnetic field. Next we will show that the resulting 4-acceleration of this fluid only depends on the charge and the 4-potential of the field.

3 Controlling the CTCs

3.1 Charged fluid

At first, we consider a simple charged fluid in the connected domain where exists a field vector A_δ represented by the Maxwell tensor

$$F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma . \tag{3.1}$$

To this 4-potential-vector is associated the linear form:

$$dA = A_\lambda dx^\lambda . \tag{3.2}$$

The energy-momentum tensor reads:

$$T^{\mu\nu} = \rho u^\mu u^\nu + t^{\mu\nu} , \tag{3.3}$$

where

$$t^{\mu\nu} = -\frac{1}{4\pi} \left(\frac{1}{4} g^{\mu\nu} F_{\gamma\delta} F^{\gamma\delta} + F^{\mu\beta} F^{\ \nu}_{\beta} \right) \tag{3.4}$$

is the energy-momentum of the electromagnetic field.

From the conservation condition of the tensor $T^{\mu\nu}$

$$\nabla_\mu T^\mu_\nu = 0 \tag{3.5}$$

it follows that

$$\nabla_\mu t^\mu_\nu = - F_{\mu\nu} j^\mu , \tag{3.6}$$

where the 4-current density $j^\mu = \mu u^\mu$ carrying the charge μ is defined by the second group of Maxwell's equations:

$$\nabla_\mu F^{\mu\nu} = -4\pi j^\nu . \tag{3.7}$$

Equation (3.5) yields:

$$\nabla_\mu (\rho u^\mu u_\nu) = \mu F_{\mu\nu} u^\mu ,$$

$$\rho u^\mu \nabla_\mu u_\nu + u_\nu \nabla_\mu (\rho u^\mu) = \mu F_{\mu\nu} u^\mu .$$

The 4 current density is conserved:

$$\nabla_\mu (\mu u^\mu) = 0 .$$

Then, using the relation $u_\mu u_\nu = 0$ and due to the antisymmetry of $F_{\mu\nu}$, we obtain:

$$\nabla_\mu (\rho u^\mu) = 0 ,$$

therefore

$$u^\mu \nabla_\mu u_\nu = \frac{\mu}{\rho} F_{\mu\nu} u^\mu .$$

By setting $\mathrm{k} = \mu / \rho$, the equation

$$u^\mu \nabla_\mu u_\nu = \mathrm{k} (F_{\mu\nu} u^\mu) = {}^*u_\nu \tag{3.8}$$

represents the equation of geodesics for a charged homogeneous fluid (i.e., its acceleration).

The flow lines of this current form the geodesics of the Finsler metric [9], which extremizes the integral:

$$s = \int_{x_1}^{x_2} (ds + \mathrm{k} d\boldsymbol{A}) . \tag{3.9}$$

Relations (2.5) and (2.6) can be written in the form:

$$\nabla_\mu u^\mu + \frac{u^\mu \partial_\mu \mu}{\mu} = 0\,, \qquad \nabla_\mu u^\mu + \frac{u^\mu \partial_\mu \rho}{\rho} = 0\,,$$

then, subtracting, we obtain:

$$u^\mu \partial_\mu \left(\ln \frac{\mu}{\rho} \right) = 0\,. \tag{3.10}$$

It should be noted that throughout along these trajectories, the ratio $\mathrm{k} = \mu/\rho$ remains constant.

3.2 Charged perfect fluid

Let us now turn to the perfect fluid scheme. In this case, the energy-momentum tensor reads:

$$T^{\mu\nu} = (\rho + P)\, u^\mu u^\nu - P g^{\mu\nu} + t^{\mu\nu}. \tag{3.11}$$

Introduce the scalar:

$$\rho' = \frac{\rho + P}{e^U}\,. \tag{3.12}$$

Observing that

$$\frac{d\rho'}{\rho} = \frac{d(\rho + P)}{\rho + P} - \frac{dP}{\rho + P} = \frac{d\rho}{\rho + P}\,,$$

we derive an equation similar to (3.10):

$$u^\mu \partial_\mu \ln \left(\frac{\mu}{\rho'} \right) = 0\,.$$

This shows that the ratio $\mathrm{k}' = \mu/\rho'$ should also remain constant along the Finsler trajectory:

$$ds' = \left(e^{2U} g_{ab}\, dx^a dx^b \right)^{1/2} + \mathrm{k}'\, d\boldsymbol{A}\,,$$

$$s' = \int_{x_1}^{x_2} \left(e^U ds + \mathrm{k}'\, d\boldsymbol{A} \right).$$

Let us apply this system to the Gödel interval:

$$ds_{\mathrm{G}} = \\ = \left[e^{2U} \left(dt^2 + \frac{1}{2}\, e^{2x} dy^2 - 2e^x dt\, dy - dx^2 - dz^2 \right) \right]^{1/2}. \tag{3.13}$$

The flow lines of the charged fluid encoding the Gödel CTCs are described by:

$$s_{\mathrm{G}} = \int_{x_1}^{x_2} \left[e^U \left(dt^2 + \frac{1}{2}\, e^{2x} dy^2 - 2e^x dt\, dy - dx^2 - dz^2 \right)^{1/2} + \\ + \mathrm{k}'\, d\boldsymbol{A} \right]. \tag{3.14}$$

The 4-acceleration vector of the charged fluid encoding the CTCs is now:

$$u^\mu \nabla_\mu u_\nu = \mathrm{k}' (F_{\mu\nu} u^\mu) = {}^*u_\nu\,. \tag{3.15}$$

For a given value of the charge μ, this simple formula can be modified through a variable electromagnetic field.

Conclusions

When Gödel wrote down his metric, he was forced to introduce a distinctive constant factor a to re-write the field equations with a cosmological constant together with additional restrictions. Our theory is free from all these restrictions and, moreover, it gives a physical meaning to the term a.

The Gödel space-time is no longer a cosmological model, but a bounded region in which the dynamics of a physical fluid takes place, preserving all the basic properties associated with closed timelike curves. These CTCs are not geodesics, as shown in [10], so they are a subject to accelerations that were obtained using our conformal formalism.

It is obvious that the properties of Gödel CTCs are preserved for a charged fluid, and the modified Gödel metric can be locally reproduced. Moreover, the acceleration of this fluid seems to be physically feasible by means of an alternating electromagnetic field.

As mentioned earlier [11], these results shed new light on the possibilities of time travel, confirming earlier work started in [12–14].

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References

1. Gödel K. An example of a new type of cosmological solutions of Einstein's field equations of gravitation. *Review of Modern Physics*, 1949, v. 21, no. 3, 447–450.
2. Marquet P. The exact Gödel metric. *Progress in Physics*, 2021, v. 17, no. 2, 133–138.
3. Marquet P. The time machine. *Progress in Physics*, 2025, v. 21, no. 1, 16–28.
4. Eisenhart L.P. Space-time continua of perfect fluids in general relativity. *Trans. Amer. Math. Soc.*, 1924, v. 26, 205–220.
5. Synge J.L. Relativistic hydrodynamics. *Proc. London Math. Soc.*, 1937, v. 43, 376–416.
6. Lichnerowicz A. Les Théories Relativistes de la Gravitation et de l'Electromagnétisme. Masson et Cie, Paris, 1955.
7. Landau L., Lifshitz E. The Classical Theory of Fields. Addison-Wesley, Reading, 1962.
8. Hawking S.W., Ellis G.F.R. The Large Scale Structure of Space-Time. Cambridge University Press, 1973.
9. Marquet P. Geodesics and Finslerian equations in the EGR theory. *The Abraham Zelmanov Journal*, 2010, v. 3, 90–100.
10. Chandrasekhar S., Wright J.P. The geodesics in Gödels's universe. *Proc. of the National Academy of Sciences*, 1961, v. 47(3), 341–347.
11. Marquet P. Gödel time travel with warp drive propulsion. *Progress in Physics*, 2022, v. 18, no. 1, 82–88.
12. Lanczos C. Über eine Stationäre Kosmologie im Sinne der Einsteinischer Gravitationstheorie. *Zeitschrift für Physik*, 1924, Bd. 21, 73–110.
13. Van Stockum W.J. The gravitationnal field of a distribution of particles rotating around an axis of symmetry. *Proc. Roy. Soc. Edinburgh*, 1937, v. 57, 135–154.
14. Tipler F.J. Rotating cylinders and the possibility of global causality. *Phys. Rev. D*, 1974, v. 9, no. 8, 2203–2206.