

## LETTERS TO PROGRESS IN PHYSICS

## Machine-Checked Formalization of Earlier Arguments on $\mathbb{P}$ versus $\mathbb{NP}$ Using Isabelle/HOL

Craig Alan Feinstein

2712 Willow Glen Drive, Baltimore, Maryland 21209, USA.

E-mail: cafeinst@msn.com

This letter revisits an earlier argument concerning  $\mathbb{P}$  versus  $\mathbb{NP}$  based on the SUBSET-SUM problem and examines its formalization in Isabelle/HOL. The formal development clarifies the argument's logical structure by separating its deductive combinatorial core from the broader universality principle required to extend it to all exact deterministic algorithms.

### Introduction

In earlier articles published in *Progress in Physics* I presented information-theoretic arguments concerning the Collatz  $3n+1$  conjecture and the  $\mathbb{P}$  versus  $\mathbb{NP}$  problem [1–3].

Almost twenty years after my first article, I undertook a formalization of portions of those arguments in Isabelle/HOL [8]. The Collatz development formalized without conceptual difficulty: definitions were made precise and the reasoning was expressed in machine-checked deductive form [4].

The SUBSET-SUM lower-bound argument proved more subtle [5]. Its combinatorial component formalizes directly, but extending that component to a universal lower bound over all exact deterministic algorithms does not.

The purpose of this letter is to describe what emerged from the formalization — namely, a separation between the deductive combinatorial core of the argument and the broader general principle required to extend it to algorithmic universality.

### The SUBSET-SUM argument

The SUBSET-SUM decision problem asks: given integers  $s_1, \dots, s_n$  and a target  $t$ , does there exist a subset  $I \subseteq \{1, \dots, n\}$  such that

$$\sum_{i \in I} s_i = t?$$

There are  $2^n$  possible subsets.

Fix  $k$  and partition the index set into  $\{1, \dots, k\}$  and  $\{k+1, \dots, n\}$ . This induces the families

$$L_k = \left\{ \sum_{i \in I^+} s_i : I^+ \subseteq \{1, \dots, k\} \right\},$$

$$R_k = \left\{ t - \sum_{i \in I^-} s_i : I^- \subseteq \{k+1, \dots, n\} \right\}.$$

The verification condition is equivalent to

$$L_k \cap R_k \neq \emptyset.$$

If the subset-sums on each side are distinct, then

$$|L_k| = 2^k, \quad |R_k| = 2^{n-k}.$$

The quantity  $2^k + 2^{n-k}$  is minimized when  $k = \lfloor n/2 \rfloor$ , yielding  $\Theta(2^{n/2})$ .

All of these identities formalize directly in Isabelle/HOL. The combinatorial structure is explicit and fully verified.

### The missing step

What does not formalize is the further claim that this combinatorial structure alone entails exponential worst-case behavior for all exact deterministic algorithms.

In [3], the exponential conclusion was motivated by the observation that the verification equation does not appear to admit a fundamentally simpler equivalent formulation. All known algebraic rearrangements and equivalent reformulations preserve the exponential number of candidate values.

The transition from this observation to a universal lower bound, however, is not a deductive consequence of the combinatorial identities themselves. It is an additional general principle.

The proof assistant makes this distinction explicit. It verifies the combinatorial theorems, while requiring universal claims to be stated as explicit assumptions or independently proved results. In particular, a universal conclusion cannot be obtained solely from the absence of a known simplification.

This does not alter the substance of the argument; it clarifies its logical structure: the split analysis is deductive, whereas the universal lower-bound claim rests on an additional principle.

It is natural to conjecture that the verification equation, in its standard form, admits no genuine simplification. All familiar algebraic rearrangements preserve the exponential proliferation of candidate values revealed by the split construction. In this sense, the equation appears structurally rigid.

One might hope to prove such rigidity formally. A theorem of this kind would show that, within a fixed formal framework, no nontrivial simplification can be derived. Yet

even such a result would not completely settle the issue. It would only show that no simplification is provable there, not that alternative formulations of SUBSET-SUM cannot exist.

The essential question is therefore not merely whether this verification equation can be simplified, but whether every formulation of SUBSET-SUM must exhibit the same combinatorial structure.

### The LR-read principle

Within the formal development, the additional step can be isolated as an explicit assumption.

**LR-read principle (informal statement).** Any exact deterministic algorithm for SUBSET-SUM must, in the worst case, distinguish values arising from both sides of some partition of the index set into subsets of sizes  $k$  and  $n - k$ .

Equivalently, no exact deterministic algorithm can decide

$$L_k \cap R_k \neq \emptyset,$$

while avoiding distinction among exponentially many induced values.

Under this principle, the combinatorial structure yields an exponential lower bound. Without it, the counting identities alone do not imply a universal lower bound.

The original argument implicitly relied on this principle. The formal development makes it explicit.

### Deductive core and universality

The formal development separates the argument into two logically distinct components.

**Deductive core.** The split construction, cardinality identities, and minimization. These are exact theorems and fully machine-verified.

**Universality step.** The assertion that every exact deterministic algorithm must respect the distinction captured by LR-read.

The proof assistant does not alter the substance of the argument; it makes explicit which parts are formally derived and which depend on an additional principle.

### Conclusion

This development does not establish  $\mathbb{P} \neq \mathbb{NP}$  as an unconditional deductive theorem. Rather, it shows that under the additional modelling principle introduced here, the combinatorial structure of SUBSET-SUM yields an exponential lower bound.

The central question becomes:

*Does there exist an exact deterministic algorithm for SUBSET-SUM whose structure fundamentally avoids the exponential distinction revealed by the split analysis?*

The formalization makes explicit the precise point at which the argument moves beyond deduction. Accordingly, the issue reduces to whether the verification equation admits an equivalent formulation that avoids the exponential structure revealed by the split analysis.

It is intuitively plausible that no such equivalent formulation exists, since all known reformulations preserve the same combinatorial structure. If such a formulation exists, the lower-bound argument would not apply. If no such formulation exists, then the argument would extend to all exact deterministic algorithms, and the exponential distinction would follow.

More broadly, this clarification applies not only to the present SUBSET-SUM argument but also to the author's earlier work on complexity-theoretic questions. In each case, the deductive core is accompanied by additional assumptions that are intuitively plausible but not themselves derived as formal theorems [1, 2, 6, 7].

Submitted on February 21, 2026

### References

1. Feinstein C.A. An elegant argument that  $\mathbb{P}$  is not  $\mathbb{NP}$ . *Progress in Physics*, 2011, v. 7, issue 2, 30–31.
2. Feinstein C.A. Complexity science for simpletons. *Progress in Physics*, 2006, v. 2, issue 3, 35–42.
3. Feinstein C.A. Dialogue concerning the two chief world views. *Progress in Physics*, 2016 v. 12, issue 3, 280–283.
4. Feinstein C.A. Isabelle formalization of Collatz information barriers. *Zenodo*, January 25, 2026. <https://zenodo.org/records/18364577>
5. Feinstein C.A. Information-flow lower bounds for SUBSET-SUM. *Zenodo*, February 19, 2026. <https://zenodo.org/records/18688260>
6. Feinstein C.A. A mathematical definition of “simplify”. *Progress in Physics*, 2019 v. 15, issue 2, 75–77.
7. Feinstein C.A. The computational complexity of the traveling salesman problem. *Global Journal of Computer Science and Technology*, December 2011, v. 11, issue 23, 1–2.
8. Isabelle Proof Assistant. <https://isabelle.in.tum.de/index.html>