A Simple Method for Finding the Properties of the Physical Vacuum Using Einstein's Field Equations

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Here we introduce a simple original method for finding the physical properties of the physicsal vacuum (known also as the λ-field). This method is based on Einstein's field equations for a space, the metric of which is satisfied in emptiness, i.e., when the left-hand side of the field equations is satisfied itself (and, hence, the right-hand side of the equations is zero). The obtained result coincides with the properties derived in the late 1990s in the framework of the theory of the physical vacuum based on its energy-momentum tensor (Borissova L., Rabounski D. Fields, Vacuum and the Mirror Universe. Chapter 5). Namely, — this is a homogeneous, non-viscous and non-emitting medium, which is in the state of inflation (i.e., this medium expands) and is stressed exactly in accordance with the observable geometric structure of the space itself.

1 The physical vacuum

The physical vacuum, known also as the λ-field is known due to Einstein's space metric and de Sitter's space metric. This is the medium introduced by Albert Einstein as a part of his cosmological model, based on the first "cosmological" space metric he introduced for this purpose. This metric describes a finite spherical static and homogeneous universe, the space of which is free of a gravitational field and does not rotate or deform, but is filled with the physical vacuum (λ-field) and an ideal liquid. Later Willem de Sitter modified this metric in the sense that the space contains a gravitational field, created by a distribution of the physical vacuum (de Sitter's space metric). But the question of which properties of the physical vacuum are physical observables remained open for decades.

This question was answered only in the late 1990s thanks to the theory of the physical vacuum [1, Chapter 5], created using the mathematical apparatus of physically observable quantities in General Relativity (known as the chronometrically invariant formalism). Assuming that the term $\lambda g_{\alpha\beta}$ on the right-hand side of Einstein's field equations is the energy-momentum tensor of the physical vacuum, and then calculating its physically observable components, we had obtained that the physical vacuum (i.e., the λ-field) is a homogeneous, non-viscous and non-emitting medium, which is in the state of inflation (at a positive density of the medium, the pressure inside it is negative — the medium expands).

2 Instrumentarium

In fact, the question of which properties of the physical vacuum are physical observable dates back to the general problem of how to mathematically define physically observable quantities in the space-time of General Relativity. It was only in 1944 that Abraham Zelmanov created the *mathematical apparatus of chronometric invariants*, in the framework of which four-dimensional general covariant quantities are projected onto the three-dimensional spatial section belonging to a real observer (his observable three-dimensional space) and onto his own time line using special projection operators that follow the geometric structure of the space-time. Since the resulting time and spatial projections are invariant throughout the observer's space and also depend on its geometric and physical properties, these projections (called *chronometric invariants*) are physically observables in the space-time of General Relativity. See Zelmanov's original publications [2–4] or, even better, the comprehensive compendium of this mathematical formalism, published in 2023 [5].

In this short note we do not need detailed derivations using the chronometrically invariant formalism. We only need Einstein's field equations in their general covariant form

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\varkappa\, T_{\alpha\beta} + \lambda g_{\alpha\beta}\,, \tag{1}$$

and also their chr.inv.-projections, deduced by Zelmanov in 1941–1944 and called the *chr.inv.-Einstein equations*

$$\left.\begin{aligned}
&\frac{{}^*\partial D}{\partial t} + D_{jl} D^{jl} + A_{jl} A^{lj} + {}^*\nabla_j F^j - \frac{1}{c^2} F_j F^j = \\
&\qquad\qquad = -\frac{\varkappa}{2}\left(\varrho c^2 + U\right) + \lambda c^2 \\
&{}^*\nabla_j \left(h^{ij} D - D^{ij} - A^{ij}\right) + \frac{2}{c^2} F_j A^{ij} = \varkappa J^i \\
&\frac{{}^*\partial D_{ik}}{\partial t} - \left(D_{ij} + A_{ij}\right)\left(D_k^j + A_{k\cdot}^{\cdot j}\right) + D D_{ik} + 3 A_{ij} A_{k\cdot}^{\cdot j} - \\
&\qquad - \frac{1}{c^2} F_i F_k + \frac{1}{2}\left({}^*\nabla_i F_k + {}^*\nabla_k F_i\right) - c^2 C_{ik} = \\
&\qquad = \frac{\varkappa}{2}\left(\varrho c^2 h_{ik} + 2 U_{ik} - U h_{ik}\right) + \lambda c^2 h_{ik}
\end{aligned}\right\}. \tag{2}$$

Einstein's equations show how the geometry of the observer's space (the left-hand side of the equations) is linked to the distributed matter that fills it (the right-hand side). Therefore, the left-hand side of the equations (2) is a combination only of the observable properties of the space: the chr.inv.-vector of the acting gravitational inertial force F_i, the chr.inv.-tensor A_{ik} of the angular velocity of the space (if it rotates), the chr.inv.-tensor D_{ik} of the deformation rates of the space (if it deforms), and the chr.inv.-curvature tensor C_{ik}. The sign $^*\partial$ denotes chr.inv.-differentiation, $^*\nabla_i$ is the sign of chr.inv.-derivative (it is similar to general covariant derivative, but is based on chr.inv.-quantities), and $^*\nabla_i - \frac{1}{c^2} F_i = {}^*\widetilde{\nabla}_i$ is the sign of physically observable chr.inv.-physical derivative, which takes into account the fact that the flow of time is different on the opposite walls of an elementary volume; see the compendium of chronometric invariants [5, p.18–19] for detail.

The right-hand side of the chr.inv.-Einstein equations is a combination only of the observable properties of the distributed matter that fills the space (a medium and the physical vacuum). First, these are the chr.inv.-projections of the energy-momentum tensor $T_{\alpha\beta}$ of the medium

$$\varrho = \frac{T_{00}}{g_{00}}, \qquad J^i = \frac{c\,T_0^i}{\sqrt{g_{00}}}, \qquad U^{ik} = c^2 T^{ik}, \tag{3}$$

which are the chr.inv.-mass density ϱ of the medium, its chr. inv.-momentum density J^i, and its chr.inv.-stress tensor U^{ik}. Second, there is also the scalar λ, which determines the density of the physical vacuum. It is multiplied by the chr.inv.-metric tensor h_{ik}, which in turn is determined by g_{ik} with correction for g_{0i} and g_{00}, and has all properties of the fundamental metric tensor $g_{\alpha\beta}$ throughout the observer's spatial section (his observable space). This means that the stresses of the physical vacuum exactly correspond to the observable geometric structure of the space itself.

3 Problem statement

Let us now consider a spherical space that does not rotate or deform ($A_{ik} = 0$, $D_{ik} = 0$), is not filled with any distributed matter ($T_{\alpha\beta} = 0$, $\lambda = 0$), but contains a gravitational field created in emptiness by a massive spherical island of substance (approximated by a point mass). This is a space of Schwarzschild's mass-point metric

$$ds^2 = \left(1 - \frac{r_g}{r}\right)c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{4}$$

where r is the radial distance from the mass M, $r_g = \frac{2GM}{c^2}$ is the gravitational radius of the mass, G is the constant of gravitation, and $x^1 = r$, $x^2 = \varphi$, $x^3 = \theta$ are spherical coordinates.

As is known, Schwarzschild's mass-point metric is satisfied itself. This means that as soon as we calculate F_i based on g_{00} taken from the mass-point metric (4), and then substitute it (and also $A_{ik} = 0$ and $D_{ik} = 0$) into the left-hand side of the chr.inv.-Einstein equations (2), we obtain that the left-hand side of these three equations becomes zero. This also means the right-hand side of the equations for the mass-point metric is also equal to zero.

On the other hand, we can consider a case, where a space of the mass-point metric is filled with the physical vacuum together with a special kind of medium, which compensates the effect created by the physical vacuum. Thus, by equating the right-hand side of the chr.inv.-Einstein equations (2) to zero, we can easily deduce the physically observable properties of the physical vacuum. This is what will be done next.

4 Solution and conclusions

So, let us equate the right-hand side of the chr.inv.-Einstein equations (2) to zero. We obtain

$$\left.\begin{aligned} \varrho c^2 + U &= \frac{2\lambda c^2}{\varkappa} \\ J^i &= 0 \\ U_{ik} &= -\varrho c^2 h_{ik} \end{aligned}\right\}. \tag{5}$$

To understand the equations (5), remind that the chr.inv.-stress tensor of any medium is $U_{ik} = p_0 h_{ik} - \alpha_{ik} = p\,h_{ik} - \beta_{ik}$. Here p_0 is the equilibrium pressure (coming from the equation of state, which is a relation between the pressure and density), p is the true pressure, α_{ik} is the viscous stress-tensor of the 2nd kind, $\alpha = \alpha_i^i$ is its trace, and $\beta_{ik} = \alpha_{ik} - \frac{1}{3}\alpha h_{ik}$ is its anisotropic part (the viscous stress-tensor of the 1st kind, manifested in anisotropic deformations); see [4] and [1, §5.3] for detail. Since any space of the mass-point metric and, hence the medium that fills it, is isotropic, $\beta_{ik} = 0$. Thus, and since the chr.inv.-metric tensor trace is $h_{ik}h^{ik} = 3$, we have

$$U_{ik} = p\,h_{ik}, \qquad U = h^{ik}U_{ik} = 3p\,. \tag{6}$$

Now, taking into account these conclusions, we transform the equations (5), obtained by equating the right-hand side of the chr.inv.-Einstein equations to zero, into the form

$$\left.\begin{aligned} \varrho &= -\frac{\lambda}{\varkappa} \\ J^i &= 0 \\ p &= -\varrho c^2 \end{aligned}\right\}, \tag{7}$$

revealing the physically observable properties of the medium that compensates the presence of the physical vacuum.

Based on the above equations, we easily obtain the physically observable properties of the physical vacuum itself

$$\left.\begin{aligned} \breve{\varrho} &= \frac{\lambda}{\varkappa} \\ \breve{J}^i &= 0 \\ \breve{p} &= -\breve{\varrho} c^2 \end{aligned}\right\}, \tag{8}$$

which are denoted here with the sign "breve" and differ from the properties of the "vacuum-compensating" medium (7) only in the sign of the physically observable density.

The first of the obtained equations (8) is the definition of the physically observable density $\breve{\varrho}$ of the physical vacuum. The second equation says that the chr.inv.-momentum density of the physical vacuum is zero: $\breve{J}^i = 0$. This means that the physical vacuum does not create momentum (the *medium is non-emitting*). The third is the *equation of state* of the physical vacuum, according to which, if the density of the physical vacuum is positive $\breve{\varrho} > 0$, then the observable pressure inside it is negative $\breve{p} < 0$: we observe that the medium expands. This state of distributed matter is called *inflation*.

A few more words about the found physically observable properties of the physical vacuum. The chr.inv.-stress tensor trace $\breve{U} = h^{ik}\breve{U}_{ik}$ shows how much the physical vacuum is compressed or stretched in all directions at once. It is responsible for changing the volume of the medium without changing its shape. Therefore, the relation $\breve{U}_{ik} = \frac{1}{3}\,\breve{U}h_{ik}$ (coming from the third chr.inv.-Einstein equation) means that the field of stresses of the physical vacuum is isotropic, and this medium is stressed exactly in accordance with the observable geometric structure of the space itself (which is expressed with the physically observable chr.inv.-metric tensor h_{ik}).

In summary, all physically observable properties of the physical vacuum deduced here can be grouped as follows

$$\left.\begin{aligned} &\breve{\varrho} = \frac{\lambda}{\varkappa} \\ &\breve{\alpha}_{ik} = 0\,, \quad \breve{\beta}_{ik} = 0 \\ &\breve{J}^i = 0 \\ &\breve{p} = -\breve{\varrho}c^2 \\ &\breve{U} = 3\breve{p} = -3\breve{\varrho}c^2 = -\frac{3\lambda c^2}{\varkappa} \\ &\breve{U}_{ik} = -\breve{\varrho}c^2 h_{ik} = \frac{1}{3}\,\breve{U}h_{ik} \end{aligned}\right\}. \tag{9}$$

They are easy to verify. Substituting them into the right-hand side of the chr.inv.-Einstein equations (2) in place of the "vacuum-compensating" medium, we obtain $2\lambda c^2$ in the first equation and $2\lambda c^2 h_{ik}$ in the third equation, as it should be if the space is "doubly filled" with the physical vacuum. While the second equation $\breve{J}^i = 0$ remains the same.

In conclusion, we can say the following about the physical vacuum (λ-field) based on its properties (9):

The physical vacuum is a homogeneous ($\breve{\varrho} = const$), non-viscous ($\breve{\alpha}_{ik} = 0$, $\breve{\beta}_{ik} = 0$) and non-emitting ($\breve{J}^i = 0$) medium, which is in the state of inflation $\breve{p} = -\breve{\varrho}c^2$ (this means that the medium expands) and is stressed exactly in accordance with the physically observable geometric structure of the space itself ($\breve{U}_{ik} = \frac{1}{3}\,\breve{U}h_{ik}$).

The same result was obtained in the late 1990s and published in 2001 in the 1st edition of the book [1, §5.2–5.3], and then explained in detail in the previous article [6], wherein the λ-term and the cosmological redshift were calculated in spaces of de Sitter's metric (constant curvature spaces). But those derivations followed another way — by assuming that the term $\lambda g_{\alpha\beta}$ on the right-hand side of Einstein's field equations is the energy-momentum tensor of the physical vacuum (which is arbitrary), and then calculating its physically observable components according to de Sitter's space metric (i.e., in a de Sitter space).

However, the simple method presented in this short note does not need any assumptions, but only pure mathematics.

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