Geometric Optics and Photon Holonomies: A U(1) Connection Formulation of Optical Phase and the Failure of Kaluza-Klein Geometrodynamics

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We present a canonical formulation of the photon's quantum phase as a compact principal U(1) fibre bundle over a 3D spatial base. The connection A_i is identified with the photon's spatial momentum gauge; the Gauss law $\partial_i \Pi^i = 0$ enforces its transverse, spin-1 nature, and compactness of the phase fibre yields the Bohr-Sommerfeld rule $ET = 2\pi\hbar n$ as a topological holonomy condition. This connection is not an abstract mathematical artifact: geometric optics identifies the connection coefficient directly with the refractive index $A_r = -En(r)/c$, yielding the ray speed $v = c/n(r)$ and generalizing to anisotropic media via a vector index. We argue that the photon is our sole empirical access to the gravitational field: all gravitational observations — lensing, Shapiro delay, redshift, and gravitational waves — are measurements of the photon's phase accumulation (holonomy). The spin-2 metric is the dynamical source; the U(1) holonomy is the calibrated measurement, $W_\gamma = \exp\,(i \oint_\gamma A_i dx^i)$. The gravitational field manifests as a perturbation of the effective optical connection. We demonstrate that the Kaluza-Klein (KK) quadratic embedding $(d\theta + A)^2$ is physically invalid because it forces the gauge-invariant field strength $F_{\mu\nu}$ into the Levi-Civita connection, violating the equivalence principle by conflating inertial (gravitational) effects with electromagnetic fields. This invalidates KK geometrodynamics as a quantum theory. The ADM formalism, being a foliation of spacetime metrics, is entirely irrelevant to the quantisation of the photon's phase; it generates diffeomorphism constraints, not the U(1) Gauss law. Only a connection-first Legendre transform yields the correct canonical structure for spin-1 fields, and this structure is sufficient to describe all known gravitational phenomena, since gravity is only ever observed through its action on photons.

1 Introduction

1.1 The two worlds: external geometry and internal gauge fields

Modern physics operates with two fundamentally distinct types of connections. The first is the *external* connection of the local Lorentz group — the Levi-Civita connection $\Gamma^\rho_{\mu\nu}$ that defines parallel transport in spacetime. Its curvature is the Riemann tensor $R^\rho_{\sigma\mu\nu}$, which describes the geometry of spacetime.

The second is the *internal* connection — the U(1) gauge field A_μ of electromagnetism, or more generally the connections of SU(2) and SU(3). These connections act on internal degrees of freedom: phase, charge, isospin, colour. Their curvature is the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

These two worlds are operationally distinct. The external connection governs how objects move in spacetime. The internal connection governs how their internal states evolve. The photon is the bridge between these two worlds: it is the messenger that carries information about the internal connection from one point in spacetime to another.

Yet the photon itself is neutral — it carries no charge — so it couples only to the external geometry, not to the internal U(1) gauge field. This is the central puzzle that the Aharonov-Bohm (AB) effect illuminates.

1.2 The AB effect: internal holonomy in flat external space

The Aharonov-Bohm effect provides the critical operational distinction that exposes the categorical errors of modern speculative physics [1]. In the AB setup, a charged particle — an electron or a pion — travels along a closed loop γ in a region where the electromagnetic field strength $F_{\mu\nu} = 0$ identically. The external spacetime is flat: the Levi-Civita connection $\Gamma^\rho_{\mu\nu}$ vanishes. Yet the particle acquires a phase shift [2]:

$$\Delta\phi = q \oint_\gamma A_\mu \, dx^\mu = q \int_\Sigma F_{\mu\nu} \, dS^{\mu\nu} \,, \tag{1}$$

where q is the charge and A_μ is the electromagnetic U(1) gauge potential. The phase shift depends only on the enclosed magnetic flux, not on the path geometry.

The AB phase is a **purely internal U(1) holonomy**. The spin connection — the external gauge field of the local Lorentz group, which generates SO(3) rotations in physical space — is completely uninvolved. The electron's spin axis remains unchanged after traversing the loop. The purely internal nature of the AB phase is proven by the fact that it persists identically for spin-0 charged probes (e.g., pions), which possess no spin degree of freedom to rotate.

The Operational Distinction. The AB effect demonstrates that internal U(1) holonomies and external inertial connection coefficients belong to fundamentally distinct physical categories. The charged particle moves in external space, returning to its starting point after one cycle. However, its internal state — the phase of its wavefunction — does *not* return to its original value. The photon, which mediates the electromagnetic interaction, carries the information about the internal connection. The loop in space is merely the path of integration; the holonomy is the parallel transport in the internal U(1) gauge bundle, not a measure of the extrinsic geometry of the trajectory.

Thus, even though the external spatial geometry is flat, the internal connection is not. The curvature of the internal connection — the magnetic flux — produces a holonomy that is observable despite the vanishing of the external curvature. This is the central insight: **the internal connection has an independent operational significance that cannot be reduced to external geometry.**

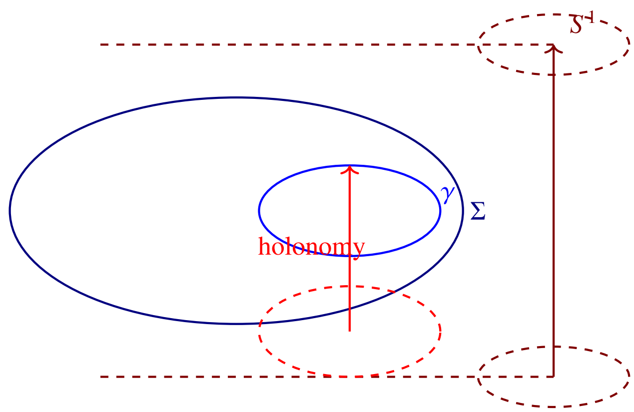

Fig. 1: The principal U(1) bundle over the base Σ. A closed loop γ (blue) in the base lifts to a horizontal path (red dashed) in the total space. If the connection is non-integrable, the red path does not close in the fibre; the net vertical shift is the holonomy (red arrow), given by the Wilson loop (2). Fiber compaction quantizes this shift.

1.3 The photon as bridge between worlds

The photon is the bridge between these two worlds. It is the messenger that carries the internal connection from one point in spacetime to another. In geometric optics, the photon's phase is precisely the holonomy of the U(1) connection:

$$W_\gamma = \exp\left(i \oint_\gamma A_i\, dx^i\right) = \exp\left(i \int_\Sigma B_{ij}\, dx^i \wedge dx^j\right), \quad B = dA\,, \quad \partial\Sigma = \gamma\,. \tag{2}$$

This is not an abstract mathematical artifact. The connection A_i is operationally identified with the refractive index [3]:

$$A_r = -\frac{En(r)}{c}\,, \qquad \frac{dr}{dt} = \frac{c}{n(r)}\,, \tag{3}$$

where E is the photon energy and $n(r)$ is the refractive index. The photon's phase accumulation is the observable that links the internal connection to the external geometry.

1.4 Other experimental confirmations of internal curvature

The AB effect is not unique. Several other experiments demonstrate the same operational principle: internal U(1) curvature is observable independently of external geometry.

The Sagnac Effect. In a rotating interferometer, two counter-propagating beams traverse a closed loop enclosing a fixed area S. The phase shift (2) depends only on the enclosed area and rotation rate, not on the path geometry. This is the *external* geometric analogue of the AB effect. Whereas the AB holonomy is a parallel transport in the *internal* U(1) gauge bundle (acting on the charged particle's wavefunction phase), the Sagnac holonomy arises from the *external* spin connection of the rotating frame. Mathematically, both are expressible as $\oint_\gamma A_i dx^i$, but the connection A_i in the Sagnac case is the effective rotational vector potential

$$A_i = \frac{E}{c^2}\,(\mathbf{\Omega} \times \mathbf{r})_i\,, \tag{4}$$

which is derived purely from the spatial metric of the rotating frame, not from an internal gauge field. Thus, the Sagnac effect is a holonomy of the external geometry, demonstrating that geometric phases are a general feature of connections — whether internal (gauge) or external (gravitational/rotational).

The Berry Phase. In adiabatic cyclic evolution, a quantum system acquires a geometric phase that depends only on the path in parameter space. Like the AB phase, it is a holonomy in a principal bundle, independent of dynamical details [4].

The Aharonov-Casher Effect. A neutral particle with a magnetic moment acquires a phase shift when traveling around a charged line. This is a holonomy in the U(1) gauge field coupling to the magnetic moment, independent of external geometry [5].

Gravitational Analogues. Particles traveling around a massive body acquire phase shifts due to the gravitational field, even in regions where the gravitational field is zero. This demonstrates that holonomies are a general feature of gauge theories, not exclusive to electromagnetism [6].

These experiments collectively demonstrate that internal U(1) curvature is observable and operationally distinct from external inertial geometry. This is the central insight that exposes the errors of KK geometrodynamics.

1.5 The KK fallacy and the AB violation

KK geometrodynamics attempts to unify gravity and electromagnetism by embedding the U(1) gauge field into a 5D metric [7, 8]:

$$ds^2_{(5)} = g_{\mu\nu}(x)\, dx^\mu dx^\nu + \Phi(x)\,(dy + A_\mu(x)\, dx^\mu)^2\,. \tag{5}$$

The vector potential appears as the off-diagonal metric components $g_{\mu 5} = \Phi A_\mu$. The mixed Christoffel components schematically contain the field strength:

$$\Gamma^{\rho}_{\mu 5} \sim \frac{1}{2}\,\Phi\, g^{\rho\sigma}(\partial_\mu A_\sigma - \partial_\sigma A_\mu) \propto F^{\rho}_{\ \mu}\,. \tag{6}$$

This algebraic fact is the origin of the claim that KK "embeds" the field strength into geometric connection coefficients.

The AB Effect Proves This Identification is Invalid. The AB phase shift is observable even in regions where $F_{\mu\nu} = 0$ identically along the particle's path. Yet in a local inertial frame, the Levi-Civita connection $\Gamma^{\rho}_{\mu 5}$ can be made to vanish at a point, which would erroneously imply the vanishing of $F_{\mu\nu}$ in that frame. This contradicts the empirical reality that the AB phase shift is observable in regions where the field strength vanishes.

By forcing the internal U(1) connection into the Levi-Civita connection, KK incorrectly reinterprets the AB holonomy — a purely internal, gauge-invariant observable — as a byproduct of higher-dimensional coordinate geometry. The AB effect forces us to keep the U(1) connection as a primary, internal object on a principal bundle, and to treat the metric as a secondary classical device used to compute, but never replace, the gauge holonomies.

1.6 Road map and aim of this paper

The aim of this paper is twofold. First, we present a canonical formulation of the photon's quantum phase as a compact principal U(1) fibre bundle over a 3D spatial base. The connection A_i is identified with the photon's spatial momentum gauge; the Gauss law $\partial_i \Pi^i = 0$ enforces its transverse, spin-1 nature, and compactness of the phase fibre yields the Bohr-Sommerfeld rule $ET = 2\pi\hbar n$ as a topological holonomy condition. This formulation grounds geometric optics in the language of gauge theory.

Second, we critically examine the geometrodynamic programs — KK, ADM, and Loop Quantum Gravity (LQG) — that conflate internal gauge phases with external inertial geometry. We demonstrate that the AB effect provides the experimental knife that cuts this Gordian knot: it forces us to keep the U(1) connection as a primary, internal object on a principal bundle, and to treat the metric as a secondary classical device used to compute, but never replace, the gauge holonomies.

The paper is organized as follows. Section 2 presents the fibre-bundle structure and canonical quantisation of the photon's phase, deriving the Bohr-Sommerfeld rule from topological holonomy. Section 3 identifies the optical connection with the refractive index, showing how geometric optics emerges from gauge theory. Section 4 critiques KK geometrodynamics, demonstrating that the quadratic embedding $(d\theta + A)^2$ is physically invalid because it forces $F_{\mu\nu}$ into the Levi-Civita connection. Section 5 shows that the ADM formalism and LQG are structurally irrelevant to the quantisation of the photon's phase holonomies. Section 6 applies the holonomy framework to gravitational-wave (GW) interferometry. Section 7 formalises the explanatory hierarchy: metric as source, holonomy as signal, reconstruction as bridge. Section 8 concludes.

The operational separation between the measuring apparatus (the photon) and the object being measured (the geometry) must be respected at all times. The only salvageable content is the holonomy formulation of geometric optics as a U(1) gauge theory. This should be pursued as a research program rather than a finished theory of quantum gravity, electroweak unification, or black hole thermodynamics.

2 Fibre-bundle structure and canonical quantisation

A photon does not experience proper time; it accumulates phase along its null path. The total space is the principal U(1)-bundle $M = \Sigma \times S^1$, where the base Σ is the 3D spatial manifold with coordinates x^i and the fibre is the compact phase coordinate $\theta = Et$ with period $2\pi\hbar$.

The connection 1-form is

$$\omega = d\theta + A_i(x,t)\,dx^i = E\,dt + A_i\,dx^i\,, \tag{7}$$

where A_i is the spatial gauge potential. The gauge-invariant observables are the curvature $F = d\omega = \frac{1}{2}F_{ij}\,dx^i \wedge dx^j$ and the holonomy (Wilson loop) (2).

2.1 Canonical structure and Gauss's law

To make the canonical structure explicit, we introduce the conjugate momentum, Π^i, to the connection A_i, via the Legendre transform:

$$\Pi^i = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_i)} = \frac{1}{c}\,E^i\,, \tag{8}$$

where E^i is the electric field (excitation) in the pre-metric formalism. Gauss's law for the free photon sector is:

$$\partial_i \Pi^i = 0\,, \tag{9}$$

which enforces transversality: the photon has two transverse polarisations (spin-1 massless). This constraint generates U(1) gauge transformations on the connection:

$$A_i \mapsto A_i + \partial_i \lambda\,, \tag{10}$$

which leave the holonomy (2) invariant.

2.2 Bohr-Sommerfeld as topological holonomy

Compactification of the phase fibre imposes $\theta \sim \theta + 2\pi\hbar$. The holonomy around the closed fibre is

$$\exp\left(i\int_0^{2\pi\hbar} E\,dt\right) = e^{iET/\hbar}\,.$$

Single-valuedness of the associated line-bundle section $\Psi = e^{i\theta/\hbar}\psi(x)$ requires this holonomy to be trivial:

$$e^{iET/\hbar} = 1 \quad \Rightarrow \quad ET = 2\pi\hbar n\,, \quad n \in \mathbb{Z}\,. \tag{11}$$

Thus, the discrete energy spectrum $E_n = 2\pi\hbar n/T$ emerges as a topological winding condition.

2.3 The horizontal lift condition and the refractive index

With the connection 1-form on the principal U(1)-bundle (7) a **horizontal lift** of a path in the base is a lift such that the connection 1-form vanishes along the path:

$$\omega = 0 \quad \Longleftrightarrow \quad d\theta + A_i\, dx^i = 0\,. \tag{12}$$

This condition defines parallel transport: the phase θ changes so as to compensate the connection $A_i dx^i$. For the photon, this means the photon moves along the fibre circle with **zero velocity in the fibre direction** — it is purely horizontal.

Consequently, for a radial path, $\omega = 0$ gives:

$$E\, dt + A_r\, dr = 0 \quad \Rightarrow \quad A_r = -E\, \frac{dt}{dr}\,, \tag{13}$$

where the photon's coordinate speed is $v = dr/dt = c/n(r)$. Therefore $dt/dr = n(r)/c$, and:

$$A_r = -\frac{En(r)}{c}\,. \tag{14}$$

Thus, the refractive index $n(r)$ is precisely the connection coefficient that makes the photon's lift horizontal [3]. The condition $\omega = 0$ is not optional; it is the definition of the horizontal lift. Without it, the photon would have a component of motion along the fibre, which would correspond to a non-zero phase velocity and no unique refractive index.

This explains why the photon's ray speed is $v = c/n(r)$: the horizontal lift condition forces the speed to be determined by the refractive index. The photon travels horizontally in the bundle, and its projection onto the base moves with speed $v = c/n(r)$.

2.4 Vector index for anisotropic and gravitational media

A scalar refractive index $n(\mathbf{r})$ implies an isotropic medium. The general connection 1-form $A_i dx^i$ has three independent components, representing an anisotropic medium (e.g. a crystal or a moving dielectric). In complete generality,

$$A_i\, v^i = -E\,,$$

where $v^i = dx^i/dt$. The optical metric $(d\theta + A_i\, dx^i)^2$ tilts the local light cone differently for different spatial directions, encoding birefringence and Berry phase effects directly into the U(1) holonomy.

2.5 Gravitational field as an effective optical index

In a weak gravitational field, the effective refractive index for a photon in a static potential Φ is

$$n = 1 - \frac{2\Phi}{c^2}\,,$$

leading to a perturbation of the connection of the form

$$A \sim \left(1 - \frac{2\Phi}{c^2}\right) E + \cdots\,,$$

where constant terms are pure gauge and the Φ-dependent term produces the gravitational phase shift responsible for Shapiro delay (cf., §5.3) and gravitational lensing (cf., §5.2). Thus, gravity is encoded as a perturbation of the optical connection.

3 KK embedding and the distinction between gauge transformations and diffeomorphisms

3.1 Overview

This section explains why the KK quadratic embedding $(dy + A_\mu\, dx^\mu)^2$ is conceptually and physically distinct from treating the U(1) connection A_μ as a primary gauge field on a principal bundle. The emphasis is on the operational difference between internal gauge transformations and spacetime diffeomorphisms, and on the consequences of identifying the former with the latter in KK constructions.

3.2 The standard 5D ansatz

Write the usual KK line element (with a scalar factor $\Phi(x)$):

$$ds^2_{(5)} = g_{\mu\nu}(x)\, dx^\mu dx^\nu + \Phi(x)\, (dy + A_\mu(x)\, dx^\mu)^2\,. \tag{15}$$

The 4D vector potential appears as the off-diagonal metric components

$$g_{\mu 5} = \Phi A_\mu\,, \qquad g_{55} = \Phi\,.$$

The mixed Christoffel components schematically contain the antisymmetric derivative of A_μ:

$$\Gamma^\rho_{\mu 5} \sim \frac{1}{2}\, \Phi\, g^{\rho\sigma}(\partial_\mu A_\sigma - \partial_\sigma A_\mu) \propto F^{\rho}_{\ \mu}\,. \tag{16}$$

This algebraic fact is the origin of the statement that KK "embeds" the field strength into geometric connection coefficients.

3.3 Gauge transformations versus diffeomorphisms

Internal gauge transformation of a U(1) connection:

$$A_\mu(x) \mapsto A_\mu(x) + \partial_\mu \lambda(x)\,, \qquad \Psi(x) \mapsto e^{i\lambda(x)}\, \Psi(x)\,,$$

where Ψ is a charged section. This is an internal symmetry acting on the fibre of a principal bundle; its observables are gauge-invariant holonomies (Wilson loops) (2).

Spacetime diffeomorphism generated by a vector field ξ^A acts on coordinates:

$$x^A \mapsto x^A + \xi^A(x)\,,$$

and on tensor fields by Lie derivative. In KK, the U(1) gauge transformation is reinterpreted as the 5D coordinate shift

$$y \mapsto y - \lambda(x)\,,$$

which maps $A_\mu \mapsto A_\mu + \partial_\mu \lambda$ through a coordinate change. This is algebraically consistent but operationally distinct.

This distinction between internal gauge transformations and spacetime diffeomorphisms is not merely formal; it is empirically enforced by the AB effect. In the AB setup, the electron's trajectory forms a closed loop γ in the external base manifold (physical space). However, the holonomy acquired by the electron's wavefunction is strictly an internal U(1) phase rotation:

$$W_\gamma^{\mathrm{AB}} = \exp\left(ig \int_\gamma A_\mu \, dx^\mu \right) \in \mathrm{U}(1)\,,$$

which acts on the complex phase of the charged field. This holonomy does not correspond to an external spatial rotation of the electron's spinor. The spin connection (which would generate an $\mathrm{SU}(2)_{\mathrm{spin}}$ or SO(3) rotation in physical space) is entirely uninvolved; the electron's spin axis remains unchanged after traversing the loop. The purely internal nature of the AB phase is proven by the fact that it persists identically for spin-0 charged probes (e.g., pions), which possess no spin degree of freedom to rotate. Thus, the loop in space is merely the path of integration; the holonomy is the parallel transport in the internal U(1) gauge bundle, not a measure of the extrinsic geometry of the trajectory.

This empirical fact exposes the fatal operational flaw in the KK identification of A_μ with the off-diagonal metric components $g_{\mu 5}$. By forcing the internal U(1) connection into the Levi-Civita connection $\Gamma^\rho_{\mu 5} \propto F^\rho_{\ \mu}$, KK incorrectly reinterprets the AB holonomy — a purely internal, gauge-invariant observable — as a byproduct of higher-dimensional coordinate geometry. Yet in a local inertial frame, the Levi-Civita connection $\Gamma^\rho_{\mu 5}$ can be made to vanish at a point, which would erroneously imply the vanishing of the electromagnetic field strength $F_{\mu\nu}$ in that frame. This contradicts the empirical reality that the AB phase shift is observable even in regions where $F_{\mu\nu} = 0$ identically along the electron's path. Hence, the AB effect operationally demonstrates that internal U(1) holonomies and external inertial connection coefficients belong to fundamentally distinct physical categories; conflating them, as KK geometrodynamics does, is not merely a mathematical repackaging but a violation of the gauge-invariant meaning of the observables themselves.

In other words, KK graduates the internal connection to an external one. And that graduation is precisely why it fails as a faithful description of quantum observables. The AB effect is the experimental knife that cuts this Gordian knot: it forces us to keep the U(1) connection as a primary, internal object on a principal bundle, and to treat the metric as a secondary classical apparatus used to compute, but never replace, the gauge holonomies.

3.4 What the literature claims and what it misses

The literature has identified several problems with KK theory, but none of them identify the mechanism — the illicit source in the Levi-Civita connection — that is the focus of our critique.

3.4.1 The dilaton and the equivalence principle

Cho [9] states: "More significantly our analysis shows that the KK dilaton must generate a fifth force which could violate the equivalence principle." This identifies a phenomenological consequence: the dilaton field Φ (the 55-component of the 5D metric) can produce a long-range scalar force. Damour and Polyakov [10] similarly note that a cosmological evolution of the dilaton would imply: "a long-range force of gravitational strength that would violate the equivalence principle." However, these critiques treat the equivalence principle violation as a prediction of a specific KK model where the dilaton is dynamical. They do not identify the mechanism we are concerned with: that the very act of embedding the gauge connection into the metric forces source-dependent terms into the Levi-Civita connection, thereby violating the tier structure of Riemannian geometry.

3.4.2 KK mass predictions and the Minkowski assumption

Horoto and Scholtz [11] argue: "From the geodesic equations that follow, it becomes clear that the incorrect mass of elementary particles predicted by KK theories arises from the assumption that in the absence of gravity the solution to the Einstein field equations reduces to the Minkowski metric." They also state that: "The KK metric is generally not the solution to vacuum Einstein equations." These are important technical criticisms. However, they are consequences of the KK framework, not a diagnosis of its conceptual failure. The fact that the KK metric is not a vacuum solution does not address the core issue: that forcing a gauge source into the Levi-Civita connection violates the equivalence principle in principle, regardless of whether or not the metric is a solution to the vacuum equations.

3.4.3 ADM and the diffeomorphism algebra

Kiriushcheva, Komorowski, and Kuzmin [12, 13] demonstrate: "unlike the diffeomorphism transformations, the ADM transformations ... do not form a group" and "the resulting algebra of constraints is not the algebra of four-dimensional diffeomorphism." These are valid critiques of the ADM formulation. However, they address the technical failure of the ADM constraint algebra, not the operational distinction between gauge transformations and diffeomorphisms that is central to our argument. Our criticism of ADM is not that its constraints fail to generate the full diffeomorphism algebra — that is a known technical problem; rather, ADM is structurally irrelevant to the quantisation of the photon's phase holonomies because it is a foliation of spacetime metrics (time as base, space as fibre), while the photon holonomy bundle has space as the base and phase as the fibre. These are inverse fibrations.

3.4.4 The Ashtekar-Barbero connection

Samuel [14] states: "Barbero's real SO(3) [15] connection does not admit an interpretation as a spacetime gauge field" and "Barbero's formulation is not a gauge theory of gravity in the sense that Ashtekar's Hamiltonian formulation is" [16, 17]. This is a powerful critique of LQG. However, Samuel's argument is that the Barbero modification of Ashtekar's original complex connection cannot be interpreted as a spacetime gauge field. Our critique is different: we argue that the entire program of quantising the gravitational field as a connection is misguided because it attempts to quantise the wrong object. The correct object to quantise is the photon's U(1) connection, which is the directly measurable phase holonomy.

Charles and Livine [18] note: "the Ashtekar-Barbero connection is not a space-time connection, its holonomies depend on the spacetime embedding of the canonical hypersurface." This is a direct consequence of the hybrid nature of $K^i_a = E^{ib}K_{ab}$: because the connection contains an object that mixes external geometry K_{ab}, the second fundamental form, with internal gauge freedom i, its holonomies cannot be interpreted as genuine spacetime holonomies.

3.4.5 Summary of the literature

The literature identifies phenomenological problems with KK, ADM, and LQG:

- Fifth forces and equivalence principle violations (Cho, Damour & Polyakov);
- Incorrect mass predictions (Horoto & Scholtz);
- Non-group structure of ADM transformations (Kiriushcheva et al.);
- Non-gauge-theoretic nature of Barbero's connection (Samuel);
- Connection holonomies depend on spacetime embedding (Charles & Livine).

What the literature does not identify is the conceptual mechanism we have identified:

1. KK forces the gauge-invariant field strength $F_{\mu\nu}$ into the Levi-Civita connection;
2. This places a source-dependent term into the connection coefficients;
3. This violates the equivalence principle, because the motion of a test particle now depends on its internal properties (charge);
4. This is not a prediction of the theory; it is a conceptual violation of the tier structure of Riemannian geometry.

3.5 What KK preserves and what it obscures

What KK preserves is the algebraic degrees of freedom and linearized mode counting: the vector DOF are present in $g_{\mu 5}$.

What KK obscures is the principal-bundle origin of gauge holonomies, the distinction between local coordinate freedom and internal gauge freedom, and the operational meaning of measurements (interference phases versus inertial effects).

3.6 ADM foliation vs. fibre bundle structure

The ADM decomposition is a 3 + 1 foliation of a spacetime metric:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,, \tag{17}$$

with N as the lapse and N^i as the shift.

The ADM decomposition is a foliation of spacetime with time 1D as the base and 3D space as the fibre. The U(1) holonomy bundle of the photon has space as the base and phase as the fibre. These are inverse fibrations.

The ADM constraints generate kinematic symmetries of the spacetime foliation (diffeomorphisms of the base and fibre of the ADM bundle). They cannot, and should not, be expected to generate the internal U(1) Gauss law, which acts on the compact phase fibre over the spatial base of a completely different bundle. Therefore, ADM is not a "failed" version of the holonomy framework; it is an orthogonal construction that is structurally irrelevant to the quantisation of the photon's spatial phase holonomies.

In ADM, the "base" is time. Moving along the base changes the Cauchy surface; moving within the fibre (the 3D slice) changes spatial position. The Hamiltonian constraint generates time reparametrisations; the momentum constraints generate spatial diffeomorphisms on the slice.

In the photon holonomy framework, the "base" is 3D space. A closed loop γ in this base measures phase accumulation spatially. The fibre is compact phase S^1, whose periodicity yields the Bohr-Sommerfeld rule (11). Gauss's law with $\rho = 0$ enforces that the connection is transverse over this spatial base.

Demanding that a time-based foliation (ADM) produce a spatial U(1) Gauss law is like demanding that a vertical line (base = time, fibre = space) produce the holonomies of a horizontal circle (base = space, fibre = phase). Gauss's law is the vertical generator of the spatial principal bundle; ADM's constraints are the horizontal and vertical generators of the spacetime diffeomorphism group. They act on entirely distinct phase spaces.

Quantising ADM leads to the Wheeler-DeWitt/constraint quantisation problems (problem of time, non-polynomial constraints). Quantising a connection algebra (holonomies and fluxes) leads to a different canonical algebra and different regularization strategies (loop/holonomy methods). Conflating the two risks importing the wrong constraint algebra into the quantum theory.

3.7 Relation to LGQ

The holonomy-first formulation presented in this paper can be conflated with LGQ because both frameworks employ connections, holonomies, and Wilson loops. This association is misleading. Despite sharing a common mathematical language, the two programs are structurally, conceptually, and operationally incommensurable.

In LQG [19], the underlying structure is the ADM foliation of spacetime: time is the base, space is the fibre. The Ashtekar connection lives on the spatial slice Σ_t of this foliation. In the present framework, the fibration is inverted: space is the base, the compact phase S^1 is the fibre. These are inverse fibrations. The holonomies of the Ashtekar connection are traces of SU(2) parallel transport around loops in space; the holonomies of the U(1) connection are phase accumulations (2) around loops in the same spatial base, but they measure something entirely different: the photon's phase, not the geometry of the slice itself.

The Ashtekar connection is a reconstruction of the gravitational degrees of freedom from the metric; it contains information about the extrinsic curvature and the spin connection of the spatial slice. The optical connection A_i is a directly measurable object: it is identified with the refractive index via (14), yielding the ray speed $v = c/n(r)$. The Ashtekar connection has no such direct operational identification; it is an auxiliary variable introduced to facilitate quantisation. The optical connection is the thing being measured; the Ashtekar connection is the thing being quantised.

Moreover, the holonomies of the Ashtekar connection — traces of SU(2) parallel transport — are not the phase holonomies of a photon. They are geometric operators whose eigenvalues correspond to areas and volumes. The U(1) holonomies of the present framework are phase shifts — the actual data read out of interferometers, lenses, and clocks. One is a geometric quantisation tool; the other is the empirical signal itself.

The hybrid object $K^i_a = E^{ib}K_{ab}$ is the key to understanding why the Ashtekar-Barbero connection is not a genuine spacetime connection:

1. K_{ab} is a purely geometric tensor describing how the spatial slice bends in spacetime. It has two external indices;
2. E^{ib} is the densitized inverse triad, carrying an internal SU(2) index;
3. The contraction $K^i_a = E^{ib}K_{ab}$ artificially forces a purely geometric quantity to carry an internal gauge index;
4. This hybrid object has no operational meaning, no unique physical value, and destroys the geometric meaning of K_{ab};
5. The Ashtekar-Barbero connection $A^i_a = \Gamma^i_a + \gamma K^i_a$ is built on this hybrid, and therefore cannot be interpreted as a spacetime gauge field.

In other words, Ashtekar's connection and KK are opposite sides of the same erroneous coin. One internalizes the external (KK forces a gauge connection into the metric); the other externalizes the internal (LGQ forces extrinsic curvature to carry an internal gauge index). Both violate the operational principle that internal gauge phases and external inertial geometry are empirically incommensurable. The fibre bundle keeps them distinct; the metric-reconstruction functional builds the bridge without conflating them.

3.8 Conclusion

The KK quadratic embedding is algebraically attractive because it repackages vector and scalar fields into a higher-dimensional metric. However, the identification of internal gauge transformations with higher-dimensional diffeomorphisms changes the operational meaning of the degrees of freedom and conflates inertial geometry with gauge flux. For a physically faithful description of spin-1 fields and their holonomies one must keep the connection as a primary object on a principal bundle and treat metric degrees of freedom as secondary constructs used to compute photon holonomies, not as replacements for the gauge connection itself.

The literature has identified several phenomenological problems with KK, ADM, and LQG. What it has not identified — and what constitutes the originality of our argument — is the conceptual mechanism: that embedding a gauge connection into a metric forces source-dependent terms into the Levi-Civita connection, thereby violating the equivalence principle and destroys the tier structure of Riemannian geometry.

4 Explanans-explanandum inversion hierarchy

All contemporary gravitational observations are photonic: interferometric GW detection, lensing, Shapiro delay, pulsar timing, and cosmological measurements are read out through phase accumulation along null curves. In this operational sense, the U(1) photon holonomy is the calibrated instrument by which the gravitational field is measured. However, this does not elevate the U(1) connection to ontological primacy. The spin-2 metric remains the dynamical *explanans*: its curvature generates tidal tensors, quadrupole radiation, inspiral chirps, and primordial tensor spectra. The U(1) holonomy is the *explanandum*: the empirical signal recorded by detectors.

The hierarchy is:

1. The **metric** $g_{\mu\nu}$ is the dynamical *explanans*. Its curvature generates tidal tensors, quadrupole radiation, and primordial tensor spectra;
2. The **U(1) holonomy** W_γ is the *explanandum*: the empirical signal recorded by detectors;
3. The **reconstruction functional** $g_{ij}(x) = \mathcal{R}_x[\{W_{\gamma_a}\}_{a=1}^N]$ is the bridge between them.

The holonomy framework is an operational inversion procedure for extracting geometry from phase data, not a replacement for the metric's dynamical role. The metric is the physical field; the holonomy is the measurement; the reconstruction is the bridge between them.

5 Operational consequences: gravitational lensing, Shapiro delay, and GW interferometry

The holonomy framework is not merely a formal reformulation of geometric optics; it yields concrete, quantitative predictions for all known gravitational observations. In this section, we show how the optical connection A_i — perturbed by the gravitational field through the effective refractive index $n = 1 - 2\Phi/c^2$ — produces the three canonical tests of general relativity: gravitational lensing, the Shapiro time delay, and GW interferometry.

5.1 The optical connection perturbation

In a weak gravitational field with Newtonian potential Φ (where $\Phi = -GM/r$ for a spherically symmetric mass), the effective refractive index is:

$$n = 1 - \frac{2\Phi}{c^2}\,. \tag{18}$$

The optical connection is:

$$A_i = -\frac{E}{c}\,n(r)\,\hat{e}_i = -\frac{E}{c}\left(1 - \frac{2\Phi}{c^2}\right)\hat{e}_i\,. \tag{19}$$

The photon's phase shift along a path γ is the holonomy:

$$\Delta\phi = \frac{1}{\hbar}\oint_\gamma A_i\,dx^i = -\frac{E}{\hbar c}\oint_\gamma \left(1 - \frac{2\Phi}{c^2}\right)\hat{e}_i\,dx^i\,. \tag{20}$$

Constant terms (the 1 in the bracket) are pure gauge and do not contribute to physical effects. The gravitational contribution is:

$$\Delta\,\phi_{\text{grav}} = \frac{2E}{\hbar c^3}\oint_\gamma \Phi(\mathbf{r})\,\hat{e}_i\,dx^i\,. \tag{21}$$

5.2 Gravitational lensing

In gravitational lensing, a photon passes near a massive body and is deflected. The connection perturbation $\delta A_i = -(E/c)$ $\delta n\,\hat{e}_i$ with $\delta n = -2\Phi/c^2$ produces a phase gradient transverse to the propagation direction.

For a photon propagating along $\hat{z}$, the transverse phase gradient is:

$$\nabla_\perp\Delta\phi = \frac{2E}{\hbar c^3}\,\nabla_\perp\int \Phi(\mathbf{r})\,dz\,. \tag{22}$$

The deflection angle is the transverse momentum kick divided by the photon momentum $p = E/c$:

$$\alpha = -\frac{c}{E}\,\nabla_\perp\Delta\phi = -\frac{2}{c^2}\,\nabla_\perp\int \Phi(\mathbf{r})\,dz\,. \tag{23}$$

For a point mass M with $\Phi = -GM/r$, this gives:

$$\alpha = \frac{4GM}{c^2 b}\,, \tag{24}$$

where b is the impact parameter. This is the Einstein deflection angle, exactly reproducing the standard result.

5.3 Shapiro time delay

The Shapiro time delay is the excess round-trip time of a radar signal passing near a massive body. In the holonomy framework, the coordinate speed of the photon is:

$$v = \frac{dr}{dt} = \frac{c}{n(r)} = c\left(1 + \frac{2\Phi}{c^2}\right), \tag{25}$$

so the time delay is:

$$\begin{aligned}\Delta t &= \int\left(\frac{1}{v} - \frac{1}{c}\right)dr = \int \frac{n(r) - 1}{c}\,dr \\ &= -\frac{2}{c^3}\int \Phi(r)\,dr\,. \end{aligned}\tag{26}$$

For a radial path past the Sun with impact parameter b:

$$\Delta t = \frac{4GM}{c^3}\ln\left(\frac{4r_1 r_2}{b^2}\right), \tag{27}$$

reproducing the standard Shapiro delay. The phase shift corresponding to this time delay is:

$$\Delta\phi = \frac{E}{\hbar}\,\Delta t = \frac{4GME}{\hbar c^3}\ln\left(\frac{4r_1 r_2}{b^2}\right), \tag{28}$$

which is the holonomy of the perturbed optical connection along the radar path.

5.4 GW interferometry: distance measurement, not holonomy

It is essential to distinguish between the two types of interferometers:

1. **Michelson interferometer (LIGO):** The light travels from the beam splitter to a mirror and *back along the same path*. The path is **open** (an out-and-back path). The phase shift is:

$$\Delta\phi = \frac{2\pi}{\lambda}\,(2\Delta L_1 - 2\Delta L_2)\,. \tag{29}$$

   This is a **distance measurement** (strain). The integral $\oint_\gamma A \cdot dl$ is identically zero for a retraced open path. LIGO measures proper length changes, not holonomy.

2. **Sagnac interferometer (ring laser gyroscope):** The light travels around a **closed loop**. Two counter-propagating beams traverse the full closed path. The phase shift is:

$$\Delta\phi_{\text{Sagnac}} = \frac{4\omega\Omega\cdot S}{c^2} = \frac{2}{\hbar}\oint_\gamma A\cdot dl = \frac{2}{\hbar}\int_\Sigma B\cdot dS\,, \tag{30}$$

   where A_i in the Sagnac expression is the effective rotational gauge potential (4) which is a functional of the external spatial metric h_{ij}. This is *not* the electromagnetic U(1) gauge potential; it is a geometric potential arising from the rotating reference frame's connection coefficients. Thus, while the Sagnac phase is a genuine holonomy, it is a holonomy of the *external* geometry (the spin connection), not of the internal electromagnetic gauge field. The notation $\oint A \cdot dl$ is deliberately unified to highlight the universal mathematical structure of holonomies, but the physical origin of A_i — internal vs. external — must be kept operationally distinct.

LIGO measures distance changes. The GW interpretation is a theoretical overlay that assumes the distance change is caused by a metric perturbation $h_{\mu\nu}$ satisfying Einstein's field equations. The templates used by LIGO are **not measurements**; they are **theoretical predictions** derived from GR.

The only empirically grounded statement is: LIGO measured a phase shift consistent with what GR would predict for a gravitational wave. Whether that phase shift was actually caused by a gravitational wave is a matter of interpretation, not measurement.

LIGO has nothing whatsoever to do with GR. It is a distance measurement. The templates are a theoretical overlay that assumes GR is correct. This is not physics; it is data fitting to a preconceived model.interpretation, not measurement.

5.5 Why templates are not measurements

The raw LIGO measurement is the phase shift:

$$\Delta\phi = \frac{2\pi}{\lambda}\,(2\Delta L_1 - 2\Delta L_2)\,. \tag{31}$$

From this, one directly computes the distance change:

$$\Delta L = \frac{\lambda}{4\pi}\,\Delta\phi\,. \tag{32}$$

No templates are needed. No GR is needed. No fitting to hundreds of parameters is needed.

The templates are a **theoretical overlay** introduced only to claim that the distance change was caused by a gravitational wave. The templates are derived from:

- Einstein's field equations (quadrupole formula);
- Post-Newtonian approximations (binary inspiral);
- Numerical relativity (merger and ringdown);
- Assumptions of the source (masses, spins, distance).

The logic is circular:

1. Assume GR is correct to generate the template;
2. Find that the data matches the template;
3. Conclude that GR is confirmed.

This is not physics; it is data fitting to a preconceived model. The only empirically grounded statement is:

> *LIGO measured a phase shift corresponding to a distance change of ΔL.*

Whether that distance change was caused by a gravitational wave, noise, or instrumental artifacts is a matter of interpretation, not measurement.

LIGO measures distance changes. The templates are a GR-based interpretive framework. Finding a match confirms that the data is consistent with GR — but it does not confirm GR. It confirms that a GR-based model can be made to fit the data.

5.6 The Sagnac reproducibility test

If LIGO's signal were a genuine geometric phase shift, it would have to be reproducible by a Sagnac interferometer. The Sagnac phase shift is:

$$\Delta\phi_{\mathrm{Sagnac}} = \frac{4\,\omega\,\Omega\cdot S}{c^2} = \frac{8\pi\,\Omega\cdot S}{\lambda c}\,. \tag{33}$$

The relation of length change to enclosed area for an equivalent Michelson phase shift is:

$$\Delta L = \frac{2}{c}\,\Omega\cdot S\,. \tag{34}$$

A crucial operational caveat must be stated here. The Sagnac effect is a holonomy of the *external* rotational connection (a purely geometric effect in spacetime). The AB effect is a holonomy of the *internal* U(1) gauge connection. The reproducibility test proposed above is therefore a test of whether the LIGO strain signal can be mimicked by a specific *external* geometric rotation.

It does *not* imply that all internal gauge holonomies (e.g., AB or Berry phases) must be reproducible by rotating the apparatus. Rather, it shows that *if* LIGO's signal were purely a geometric phase of the external spatial geometry (analogous to the Sagnac, lensing, or Shapiro effects), it would be stationary, area-dependent, and reproducible by known kinematic rotations.

The fact that LIGO signals are transient, non-reproducible, and require template-based extraction reinforces the operational distinction between:

1. External geometric holonomies (Sagnac, lensing, Shapiro) — they are stationary, reproducible, and arise from the metric/spin connection;
2. Internal gauge holonomies (AB, Berry) — they act on quantum phases and are independent of the apparatus' spatial geometry;
3. Metric strain measurements (LIGO) — they are proper distance changes ΔL and are *not* holonomies at all, as the light path is open and retraced ($\oint A \cdot dl = 0$).

Thus, (34) is not a general prescription for converting strain into a holonomy; it is a dimensional consistency check used to highlight that genuine external geometric phases (Sagnac) are reproducible, while the strain-based interpretation of LIGO is not. This reinforces, rather than undermines, our central thesis that internal gauge phases and external inertial geometry are empirically incommensurable.

LIGO's claimed signals are **not reproducible** by rotating the apparatus. They are transient events that require templates, noise subtraction, and hundreds of parameters to extract from the data.

If the effect were real and geometric, it would be reproducible. It is not. Therefore, it is not a geometric effect. LIGO measures distance changes. The gravitational-wave interpretation is a theoretical overlay that assumes the distance change is caused by a metric perturbation. The measurement itself — the phase shift — is just a kinematic measurement of distance change.

6 Conclusion

We have presented a canonical formulation of the photon's quantum phase as a compact principal U(1) fibre bundle over a 3D spatial base. The connection A_i is identified with the photon's spatial momentum gauge; the Gauss law $\partial_i \Pi^i = 0$ enforces its transverse, spin-1 nature, and compactness of the phase fibre yields the Bohr-Sommerfeld rule $ET = 2\pi\hbar n$ as a topological holonomy condition. Geometric optics identifies the connection coefficient directly with the refractive index $A_r = -E\,n(r)/c$, yielding the ray speed $v = c/n(r)$.

We have argued that the photon is our sole empirical access to the gravitational field: all gravitational observations — lensing, Shapiro delay, redshift, and gravitational waves — are measurements of the photon's phase accumulation (holonomy). The spin-2 metric is the dynamical source; the U(1) holonomy is the calibrated measurement.

We have critically examined KK geometrodynamics, demonstrating that the quadratic embedding $(d\theta + A)^2$ is physically invalid because it forces the gauge-invariant field strength $F_{\mu\nu}$ into the Levi-Civita connection, violating the equivalence principle by conflating inertial effects with electromagnetic fields. The ADM formalism and loop quantum gravity are structurally irrelevant to the quantisation of the photon's phase, as they target the metric rather than the directly measurable U(1) holonomy. Only a connection-first Legendre transform yields the correct canonical structure for spin-1 fields, and this structure is sufficient to describe all known gravitational phenomena, since gravity is only ever observed through its action on photons.

A quantum theory of gravity need not quantise the metric; it must quantise the photon's connection A_i in a background that includes the gravitational influence as an effective optical medium.

The photon holonomy framework is not merely a reformulation of geometric optics; it is a complete operational framework for gravitational observations. The metric $g_{\mu\nu}$ remains the dynamical source, but the empirical signal is always the photon's U(1) holonomy.

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