On the Dzhanibekov Effect, the Solar Cycle, the Perihelion Precession of Mercury, and the Quadrupole Moment of the Sun

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An analysis of the effect of the solar quadrupole moment, J_2, on the precession of Mercury and the behavior of its orbit is presented within the context of the liquid metallic hydrogen solar model, LMHSM (see P. M. Robitaille, Forty Lines of Evidence for Condensed Matter — The Sun on Trial: Liquid Metallic Hydrogen as a Solar Building Block, *Progr. Phys.*, 2013, v. (4), 90–142). The solar core is believed to be subject to the Dzhanibekov effect, requiring the presence of three separate moments of inertia ($I_1 < I_2 < I_3$). Rotation begins along I_2 which is aligned along the z-axis of the Sun (at $t = 0$, $I_1 = I_x$ and $I_3 = I_y$). This results in core inversion through the Dzhanibekov effect approximately every 11 years, thereby fully accounting for the solar cycle within the context of the LMHSM. Core parameters include a radius of $0.65R_\odot$ and an average density of 1.7 g/cm^3. The convective zone extends from $0.65R_\odot \rightarrow R_\odot$ with an average density of 1.3 g/cm^3. The moment of inertia is ~ 6x greater for the convective zone than for the core. The effect of the thin photosphere is neglected. The selection of exact values for these two regions does not alter final conclusions, provided that the core rotates as a completely rigid and perfect sphere such that ($I_1 + I_2 + I_3 = 3I_{\text{core}}$). In this case, $J_2 = [I_2 - (I_1 + I_3)/2]/(M_\odot R_\odot^2)$ and $C_{22} = (I_3 - I_1)/(4M_\odot R_\odot^2)$. The mass of the core may not necessarily be symmetrically distributed. As a result, this center of mass may not be coincident with the geometric center. Therefore, the convective zone is used to perfectly mask the effect of C_{22} from the core ($C_{22,\text{net}} = C_{22,\text{core}} + C_{22,\text{conv}}$). The value of $C_{22,\text{net}}$ should remain less than 2.8×10^{-10} at all times. Consequently, the convective zone acts as a phase-locked mechanical counterweight ($\phi_{\text{conv}} = \phi_{\text{core}} + 90^\circ$) that continuously redistributes mass in step with the core's rotation without locking its spin. This keeps the regions of lower mass in the convective zone masking the regions of higher mass found in the core. The convective zone mass compensation is accomplished by placing elements heavier than hydrogen within intercalate regions located between the Type II hexagonal planes (see J. C. Robitaille and P. M. Robitaille, Liquid Metallic Hydrogen III. Intercalation and Lattice Exclusion Versus Gravitational Settling and Their Consequences Relative to Internal Structure, Surface Activity, and Solar Winds in the Sun, *Progr. Phys.*, 2013, v. 9 (2), 87–97). These intercalate atoms remain completely free to move in directions which remain parallel to the hexagonal planes, thereby allowing complete freedom for mass redistribution in the convective zone. Internal centrifugal forces in the convective zone act to drive the fluid towards a balanced state. This constant redistribution of intercalate atoms thereby ensures that the solar center of mass remains coincident with its geometric center. This prevents all wobbles, either for the Sun or for Mercury, even if the solar core contains mass asymmetries. Note also that Type II hexagonal planar metallic hydrogen is considered fully metallic with all its electrons completely delocalized. This is unlike the semi-metallic Type I material found in the photosphere. Type II material should not support an interlayer shear modulus ($\mu_\perp = 0$), thereby enabling the core to rotate freely. This is based on the anticipated presence of extremely weak or essentially non-existent London forces between planes in Type II metallic hydrogen. Conversely, the hexagonal planar arrangement of Type II metallic hydrogen must display considerable in-plane stiffness ($\mu_\parallel > 0$), as would be expected for such a material. Thus, the Sun appears as an axisymmetric ellipsoid relative to Mercury, even though the solar core actually contains three distinct moments of inertia. Consequently, no effect of solar core inversion can be detected at Mercury and the solar cycle can be completely accounted for without any need for a solar dynamo.

1 Introduction

It is well-known that the interior of the Sun experiences solid body rotation at a frequency of $\sim$ 430 nHz [1, 2]. The rotation is found to be consistent across all reliably measured latitudes ($< \pm 75^\circ$) and extends throughout the central region of the Sun. This finding is incompatible with a fully gaseous solar body, as such a structure would be unable to supply the internal cohesive forces required in solid body rotation. It is more prudent to infer that the presence of solid body rotation directly points to the existence of a rigid solar core comprised of condensed matter [3]. In the liquid metallic hydrogen solar model (LMHSM), it is thus reasonable to set the radius of the solar core $R_{\text{core}} \approx 0.65 R_\odot$, since a radiative zone does not exist in this framework [3, 4].*

The hydrogen contained in the solar core should adopt a body-centered cubic lattice, as the most stable structure under these conditions of extreme pressure. If the combined density of the convective zone and tachocline layer averages about 1.3 g/cm^3 and if these layers occupy the outer 35% of the solar radius, then, given the average density of the Sun at 1.41 g/cm^3, the core should have an average density on the order of $\sim$ 1.7 g/cm^3. Should only hydrogen be present, this would correspond to a Wigner-Seitz radius of $r_S = 0.62$ Å, just slightly greater than the Bohr radius at $\sim$ 0.53 Å. Consequently, within the LMHSM, very little change in density occurs within the solar body when moving from the convective zone to the core.† This is in keeping with the concept that liquid stars are essentially incompressible, as initially advanced by James Jeans (1928, 1933), [5, 6].

There is one more aspect of the solar core which requires consideration. In the Standard Solar Model [7, 8], a gaseous solar core must maintain slowly changing densities with a relatively spherical and uniform distribution of mass at each value of the solar radius. In such a model, there is no means to restrict the motion of individual atoms to a particular position in the solar core. All atoms remain unconstrained in their position with the only possible means of altering their placement arising from gravitational forces and the process of gravitational settling. There is no possibility of generating three separate moments of inertia along x, y, and z axes based either on deviations from spherical geometry or, alternatively, on a varying distribution of non-hydrogen elements within a spherical core. In fact, because gases lack internal cohesive forces, it could be argued that, within such objects, it is impossible to generate a quadrupole moment, expressed as the dimensionless coefficient, J_2. The very existence of such a parameter for the Sun implies that it cannot be gaseous. Ultimately, the geometry or inhomogeneity of the solar core cannot be used to account for the 11-year solar cycle or the overarching 22 year Hale cycle in gaseous stars.‡ This is not the case in the LMHSM, as the solar core through nucleosynthesis, can seed within itself regions of increased density.

Within a condensed Sun, elements heavier than hydrogen can occupy intercalate zones within Type I and Type II metallic hydrogen with some of these being released during solar activity [4]. Still, this does not provide a trigger for the solar cycle. That trigger may well come from the solar core, if this region is subject to the Dzhanibekov effect, also known as the intermediate axis theorem, as first described by Louis Poinsot (1834) [9]. When considering the core using a 3-axis system, then rotation about the primary and third principal moments of inertia would be stable, but rotation about the intermediate moment of inertia becomes unstable. The presence of a quadrupole moment, J_2, in the solar core could account for the solar cycle through the Dzhanibekov effect and be determined by the advance in the precession of Mercury [10].§ This can occur even if the solar core is perfectly spherical, as now the distribution of mass within the core is not symmetric. Consequently, the LMHSM is able to account for the solar cycle by using the perihelion precession of Mercury to set the value of J_2. This far surpasses the explanations provided by the Standard Solar Model [7, 8].

2 The perihelion precession of Mercury

Mercury's perihelion corresponds to its point of closest approach to the Sun ($\approx 4.6001 \times 10^{12}$ cm) while its aphelion

*Schou et al. (1998) [1] set the reliable solid body rotation of 430 nHz at a radius of $< 0.7 R_\odot$. However, from published Gong measurements (https://gong.nso.edu/gallery/disk2k10/data/resource/torsional/gongcut.png) a more reliable measure of the core would begin at about $< 0.65 R_\odot$. This is the value which will be adopted for this analysis. The approach presented below will remain valid if a slightly different value for R_{core} (e.g. $R_{\text{core}} = 0.7 R_\odot$) is adopted. Altering the values of R_{core}, ρ_{core}, and M_{core} will simply alter the final values for the moments of inertia I_1, I_2, and I_3.

†In the LMHSM, the photosphere has a density of $\sim 0.6 - 0.8$ g/cm^3. Yet, this layer occupies only the small outer skin of the Sun. For instance, the average Wilson depression in a sunspot is only $\sim$ 600 km. This should help define the lower bounds of the photosphere and represents only 0.086% of the solar radius. As a consequence, the photospheric density has essentially no effect on the calculation of the average solar density which is established by the core and convective zone.

‡In the Standard Solar Model, the presence of a powerful solar dynamo is advanced to account for the solar cycle. The validity of this scenario has been questioned, since it is not possible to build up a dynamo in a fully gaseous object, as the magnetic field must first be seeded. In the laboratory, dynamos are produced using conductive liquids subjected to specific geometries in order to create a boundary between conductors and insulators (see [3] for a detailed discussion).

§Floriano Pohlmann first proposed that the Dzhanibekov effect might account for the 11-year solar cycle on 22 Feb 2024. The email read: "*It has just occurred me that the Sun polarity reversal could be caused by the Dzhanibekov effect. Somehow the core of the Sun could be solid (or behave as a solid) and have the asymmetries required to trigger the effect while spinning.*" Independently, Stephen J. Crothers had highlighted on many occasions that, beyond the work of Einstein (1915) [11], the solution for the perihelion precession of Mercury had been solved classically, both by Paul Gerber (1898, 1917) [12, 13] and Edward Dowdye (1992, 2001, 2012) [14]. Pierre-Marie Robitaille then proposed that the known advance in the perihelion precession of Mercury could be used to constrain the principal, intermediate, and minor axes of an inertially asymmetric solid solar core, thereby unifying both the solar cycle and the perihelion precession of Mercury. Stephen Crothers would affirm this conclusion and would eventually verify that the presentation contained herein is correct.

corresponds to its farthest distance ($\approx 6.9818 \times 10^{12}$ cm). It is well understood that Mercury's perihelion advances by 5,599.74 arcsec/century. A large component of this value ($\approx 5,025$ arcsec/century) can be attributed to astrometric/equinox precession, namely the progressive change in the orientation of Earth's rotational axis [15]. Planetary perturbations from other planets are known to account for ≈ 532 arcsec/century [15]. The remaining 42.5–43 arcsec/century, known as the anomalous advance, have typically been attributed to the effects of General Relativity [11]. The internal quadrupole moment of the Sun, J_2, makes a very small contribution to the perihelion precession, on the order of 0.028 arcsec/century.*

When Le Verrier (1859) [18] first discovered the anomalous component of the perihelion advance of Mercury, he proposed that intramercurial matter might account for the advance. This would eventually grow to become known as the hypothetical planet Vulcan [19]. The planet was never discovered. In reality, Vulcan could never have accounted for the anomalous advance, as it would have altered the shape of Mercury's orbit. In arriving at a solution for the anomalous advance, Einstein invoked a solar mass and assumed spherical solar symmetry [11]. He was not in a position to consider that the Sun might possess hidden moments of inertia which varied along the x, y, and z axes. The solution he advanced would depend solely on the characteristics of Mercury's ellipse and its distance from the center of the solar body.

Prior to Einstein [11], Paul Gerber [12] had used classical approaches to obtain an expression for the anomalous advance of the perihelion of Mercury. Einstein had obtained the same solution, but had failed to cite Gerber. Edward Dowdye Jr. [14], using a different classical method, also obtained effectively the same equation:

$$\Delta\phi = \left(\frac{24\,\pi^3 a^2}{T^2 c^2 (1 - e^2)} \right) \text{ arcsec/century}, \qquad (1)$$

where, $\Delta\phi$, a, e, T, and c correspond to Mercury's anomalous precession (43 arcsec/century or 5.0209×10^{-7} rad), semi-major axis ($\sim 5.7909 \times 10^{12}$ cm), eccentricity (~ 0.20563), orbital period, and the speed of light (or the speed of gravity propagation for Gerber), respectively. The orbital period T for Mercury's ellipse is given by $2\pi\sqrt{a^3/GM}$, where a corresponds to the semi-major axis, G to the universal constant of gravitation (6.6743×10^{-11} N m^2/kg^2), and $M_\odot$ to the mass of the Sun. After Einstein published his analysis [11], Gerber's paper was republished posthumously by Ernst Gehrcke [13]. This was extremely unusual in science, especially since Gerber's proof now entered the most prestigious German physics journal, *Annalen der Physik*, nearly 20 years after it first appeared. Even so, Einstein's solution became widely accepted and served as a key proof for the validity of General Relativity. As for Paul Gerber, his solution has long ago been forgotten. Early on, many had argued in defense of Albert Einstein that Paul Gerber's solution was non-physical. Nonetheless, it is important to recall that Gerber was indeed the first scientist to obtain a solution for the anomalous advance of the perihelion of Mercury by deriving Eq.(1). He had done so with classical methods, seventeen years before General Relativity.

Yet, in order to better understand the solar cycle, the focus must shift away from the anomalous advance of 43 arcsec/century and the work of Einstein [11] and Gerber [12]. Rather, it is the small J_2 value obtained by the MESSENGER satellite [17] for the Sun (2.25×10^{-7}), which now governs the situation. This small J_2 value can be determined by measuring its effects on Mercury.

Three lines of reasoning will be considered herein. First, as a pedagogical exercise, a dimensionless J_2 value for the Sun will be obtained from the anomalous advance of the perihelion of Mercury. This will highlight that adopting the resulting J_2 value for the Sun results in extreme effects on Mercury's orbit.† Second, use is made of the dimensionless J_2 value of 2.25×10^{-7} obtained by the MESSENGER satellite to quantify its effects on Mercury. Third, the second condition will be re-examined. This time however, convective shielding will be used to mask the effect of a tri-axially asymmetric solar core and its inversion on Mercury's orbital behavior.

3 The solar cycle and the perihelion precession of Mercury

Within the context of the LMHSM, wherein the core is solid and its structure maintained by the lattice, the restrictions imposed by gaseous solar models can be lifted. A solar core possessing a quadrupole moment, J_2, could be used to *simultaneously* account for *both* the solar cycle through the Dzhanibekov effect *and* the anomalous perihelion precession of Mercury. The key parameters are as follows: total solar mass, $M_\odot = 1.98841 \times 10^{33}$ g; solar radius, $R_\odot = 6.957 \times 10^{10}$ cm; average solar cycle = 11 Julian years = 3.471336×10^8 s; radial boundary of the solar core, $R_c = 0.65 R_\odot = 4.52205 \times 10^{10}$ cm; mass of the solar core, $M_c = (4\pi/3) R_c^3 \times 1.7$ g/cm^3 $= 6.58482 \times 10^{32}$ g; rotational frequency of the solar core, $\Omega_\circ = 430$ nHz or 2.70177×10^{-6} rad/s; average core density, $\rho_{\text{core}} = 1.7$ g/cm^3; base moment of inertia of the core, $I_{\text{c, base}} = 5.386 \times 10^{53}$ g cm^2; density of the convective zone, $\rho_{\text{conv}} = 1.3$ g/cm^3, extending from $0.65 R_\odot$ to $R_\odot$; base moment of inertia of the convective zone, $I_{\text{conv, base}} = 3.138 \times 10^{54}$ g cm^2; Mercury's orbital period (P_{orb}) = 87.96926 days, or 415.2019 orbits/century.‡

*Godier and Rozelot [16] estimated that the dimensionless solar quadrupole moment, J_2, was $\approx 1.6 \times 10^{-7}$ based on gaseous solar models. Since that time, the J_2 value of the Sun has been established as 2.25×10^{-7} by the MESSENGER satellite [17].

†Adopting such a J_2 value for the Sun can account for the anomalous perihelion advance of Mercury, but is unable to properly treat the anomalous advances in Venus and the Earth. Consequently, the approach cannot be correct. Still, the case is treated in order to help underscore what is happening relative to Mercury's orbit.

‡For the sake of this discussion, these parameters for the extent and density of the solar core and convective zones are adopted for illustrative pur-

3.1 Obtaining J_2 improperly using the anomalous advance in the perihelion precession of Mercury

It is possible to demonstrate the errors which arise when attempting to classically account for the anomalous advance of the perihelion precession of Mercury per orbit along the orbit plane, $\dot{\tilde{\omega}}_{J_2}$, using an internal quadrupole moment, J_2, instead of General Relativity (Einstein, 1915). Such a value for J_2 can be obtained by evaluating the orbital geometry using the following classical expression:

$$\dot{\tilde{\omega}}_{J_2} = \frac{3\pi J_2 R_\odot^2}{a^2(1-e^2)^2}\left(1 - \frac{3}{2}\sin^2 i\right) \text{ arcsec/century}, \quad (2)$$

where, $\dot{\tilde{\omega}}_{J_2}$, a, e, and i correspond to Mercury's anomalous precession (43 arcsec/century or 5.020919×10^{-7} rad/orbit), semi-major axis ($\sim 5.790918 \times 10^{12}$ cm), its eccentricity ($\sim$ 0.2056307), and its orbital inclination relative to the Sun's equatorial plane ($i = 3.38°$), respectively.* For an anomalous perihelion precession of 43 arcsec/century, the required solar quadrupole moment, J_2, would be $\sim 3.403 \times 10^{-4}$.† Using such a value results in significant errors for Mercury's behavior and its orbit. These conclusions can be reached when this value for J_2 is utilized to solve the equations which follow, as summarized in Table 1.

In order to provide a general understanding, it is also possible to measure the regression of the ascending node for Mercury's orbit. This analyzes the retrograde westward drift of Mercury's orbital plane around the Sun's spin axis as:

$$\dot{\Omega}_{J_2} = \left(-\frac{3\pi J_2 R_\odot^2}{a^2(1-e^2)^2}\cos i\right) \text{ arcsec/century}. \quad (3)$$

A J_2 value of $\sim 3.403 \times 10^{-4}$, results in a regression of the ascending mode, $\dot{\Omega}_{J_2}$, of −43.146 arcsec/century. Such a value could have been easily detected by the MESSENGER satellite but is not observed, as displayed in Table 1.

As a further demonstration of the effect of using a J_2 value of $\sim 3.403 \times 10^{-4}$ can be seen when examining the rate of change for the argument of perihelion of Mercury given by:

$$\dot{\omega}_{J_2} = \left(\frac{3.55023 \times 10^{10} J_2 R_\odot^2}{T_{\text{days}}\, a^2(1-e^2)^2}(5\cos^2 i - 1)\right) \text{ arcsec/cen}, \quad (4)$$

where T_{days} corresponds to the orbital period of Mercury, namely 87.9691 days. From this expression a value for the argument of perihelion of +86.066″ is obtained, well outside the accepted value of +0.056905″, as displayed in Table 1. From this analysis, it is clear that adopting a J_2 value of $\sim 3.403 \times 10^{-4}$ creates significant deviations from the known behavior of Mercury and its orbit. As such, it is best to simply utilize the accepted J_2 value for the quadrupole moment of the Sun when attempting to account for the solar cycle using the Dzhanibekov effect.

3.2 Moments of inertia for the solar core, core inversion, and subsequent effects on Mercury

The condition for core inversion using the Dzhanibekov effect requires three separate moments of inertia, where $I_1 < I_2 < I_3$ results in an unstable intermediate axis, I_2. If the moments of inertia are governed by density variations within a perfectly spherical solar core, then $I_1 + I_2 + I_3 = 3 \times I_{\text{c, base}}$, where $I_{\text{c, base}} = \frac{2}{5} M_c R_c^2 = 5.38610 \times 10^{53}$ g cm^2. The quadrupole moment of the Sun, J_2, can then be used to obtain the core's principal moments of inertia through:‡

$$J_2 = \frac{I_2 - \frac{I_1+I_3}{2}}{M_\odot R_\odot^2} = \frac{\Delta I}{M_\odot R_\odot^2}. \quad (5)$$

Given the total solar mass, $M_\odot$, and radius, $R_\odot$, the value of ΔI could then be determined as:

$$M_\odot R_\odot^2 J_2 = \Delta I = \left(I_2 - \frac{I_1 + I_3}{2}\right) \text{ g cm}^2. \quad (6)$$

If the core remains perfectly spherical, mass can redirected completely via variable density gradients. Yet, the total angular momentum, L, and the kinetic energy, K_E, must remain strictly conserved throughout the process. Before the inversion begins and assuming perfect initial rotation strictly about the intermediate axis, the object rotates with an initial angular velocity Ω_0 exclusively around this axis. Consequently, at the initial state, $\omega_1 = 0$, $\omega_2 = \Omega_0$, and $\omega_3 = 0$.

Substituting these initial conditions for L^2 yields $L^2 = (I_2\Omega_0)^2$ and therefore:

$$L^2 = I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2 = (I_2\Omega_0)^2 \text{ g}^2\text{ cm}^4/\text{s}^2. \quad (7)$$

Similarly, at the beginning, just as was the case for angular momentum, kinetic energy is conserved because there are no dissipative forces. For the same initial conditions where $\omega_1 = 0$, $\omega_2 = \Omega_0$, and $\omega_3 = 0$ and, therefore, $2K_E = I_2\Omega_0^2$, yielding:

$$2K_E = I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2 = I_2\Omega_0^2 \text{ g cm}^2/\text{s}^2. \quad (8)$$

At the exact midpoint of the flip trajectory, the rotation about the intermediate axis drops to zero, $\omega_2 = 0$ (see Table 1 for exact values). Consequently, when treating the core as a rigid body spinning without any external torques about its

poses. Some may choose to adopt slightly different values, but the overall demonstration of feasibility will not be altered.

*The inclination in this expression, i, is not relative to the ecliptic which has a very different value of 7.005°. This expression is also given by Park et al. [17] in slightly altered form. It can be placed in identical form by rearrangement followed with multiplication by the orbital period, $T = 2\pi/n$.

†This value is more than 1,000 times greater than computed using gaseous models [16] and currently accepted for the Sun [17].

‡Two values of J_2 corresponding to 3.403×10^{-4} and 2.25×10^{-7} can be treated in parallel; the results are displayed in Table 1.

Table 1: Comparison of solar core intermediate-axis flip dynamics ($\tau_{\text{flip}} = 11$ yr, $\nu_0 = 430$ nHz) and induced orbital effects on Mercury treated either without or with convective lattice shielding. **Bolded values are unacceptable**.

Parameter, Units	**Column 1**	**Column 2**	**Column 3**
Input J_2[a]	$\mathbf{3.403 \times 10^{-4}}$	2.25×10^{-7}	2.25×10^{-7}
Convective Zone Shielding[c]	Unshielded	Unshielded	Convective Shielding
Core Quadrupole Difference (ΔI), g cm^2	3.2750×10^{51}	2.1654×10^{48}	2.1654×10^{48}
Flip Growth Rate ($\lambda = \pi/\tau_{\text{flip}}$), s^{-1}	9.0501×10^{-9}	9.0501×10^{-9}	9.0501×10^{-9}
Perturbation Product ($\epsilon_1 \cdot \epsilon_3$), g^2 cm^4	3.2551×10^{102}	3.2551×10^{102}	3.2551×10^{102}
Perturbation Diff. ($\epsilon_1 - \epsilon_3 = 2\Delta I$), g cm^2	6.5500×10^{51}	4.3307×10^{48}	4.3307×10^{48}
$\epsilon_1 = I_2 - I_1$, g cm^2	7.0141×10^{51}	1.8063×10^{51}	1.8063×10^{51}
$\epsilon_3 = I_3 - I_2$, g cm^2	4.6407×10^{50}	1.8020×10^{51}	1.8020×10^{51}
Core Principal Moment I_1 (x-axis), g cm^2	5.3378×10^{53}	5.3681×10^{53}	5.3681×10^{53}
Core Principal Moment I_2 (z-axis), g cm^2	5.4079×10^{53}	5.3861×10^{53}	5.3861×10^{53}
Core Principal Moment I_3 (y-axis), g cm^2	5.4126×10^{53}	5.4041×10^{53}	5.4041×10^{53}
Core Equatorial Asymmetry ($I_3 - I_1$), g cm^2	7.4782×10^{51}	3.6084×10^{51}	3.6084×10^{51}
Mid-Flip x-Spin (ω_1 at $\omega_2 = 0$), rad/s (nHz)	6.7745×10^{-7} (107.82)	1.9125×10^{-6} (304.38)	1.9125×10^{-6} (304.38)
Mid-Flip y-Spin (ω_3 at $\omega_2 = 0$), rad/s (nHz)	2.6155×10^{-6} (416.27)	1.9084×10^{-6} (303.73)	1.9084×10^{-6} (303.73)
Mid-Flip Maximal Spin (Ω_{max}), rad/s (nHz)[a]	2.701785×10^{-6} (430.0024)	2.701785×10^{-6} (430.0024)	2.701785×10^{-6} (430.0024)
Net External C_{22}, dimensionless	$\mathbf{1.9426 \times 10^{-4}}$	$\mathbf{9.3735 \times 10^{-5}}$	$\leqslant 2.8 \times 10^{-10}$
Perihelion Advance ($\dot{\tilde{\omega}}_{J_2}$), rad/orbit	$\mathbf{5.0204 \times 10^{-7}}$	3.3194×10^{-10}	3.3194×10^{-10}
Perihelion Advance ($\dot{\tilde{\omega}}_{J_2}$), arcsec/century	42.995″	0.028428″	0.028428″
Node Regression ($\dot{\Omega}_{J_2}$), arcsec/century	**−43.146**″	−0.028527″	−0.028527″
Argument of Perihelion ($\dot{\omega}_{J_2}$), arcsec/century	**+86.066**″	+0.056905″	+0.056905″
19.392-Day Range Oscillation, m	**710.05**	**342.61**	$\leqslant 0.001$
11-Year $\delta J_2(t)$ Modulation, dimensionless	**large mid-flip, spike**	**large mid-flip, spike**	$\leqslant 3.7 \times 10^{-11}$
Satellite Status[d]	**Falsified**	**Falsified**	**Non-falsifiable**

Notes on input parameters, significant figures, lattice screening, and observational conclusions:

[a] *Significant-Figure Treatment:* All internal computations were executed at full double floating-point precision prior to rounding. Standard derived outputs are rounded to 5 significant figures, whereas inputs and observational detection bounds known to fewer than 5 significant figures ($J_2 = 3.403 \times 10^{-4}$ [4 sig. figs.], $J_2 = 2.25 \times 10^{-7}$ [3 sig. figs.], $C_{22,\text{max}} \leqslant 2.8 \times 10^{-10}$ [2 sig. figs.], $\delta J_{2,\text{max}} \leqslant 3.7 \times 10^{-11}$ [2 sig. figs.], and the 1 mm range floor $\leqslant 0.001$ m) omit trailing zeros so as not to imply false precision. Only the mid-flip maximal spin (Ω_{max}) is reported to 7 significant figures (2.701785×10^{-6} rad/s and 430.0024 nHz) to resolve the +0.000561% (+0.0024 nHz) equatorial spin-up in the 6th and 7th significant digits.

[b] *Solar, Core, and Orbital Input Constants:* Solar mass $M_\odot = GM_\odot/G = 1.98841 \times 10^{33}$ g (6 sig. figs., from IAU/JPL $GM_\odot = 1.3271244 \times 10^{26}$ cm^3 s^{-2} and CODATA $G = 6.67430 \times 10^{-8}$ cm^3 g^{-1} s^{-2}); IAU 2015 nominal solar radius $R_\odot = 6.957 \times 10^{10}$ cm (4 sig. figs., exact nominal definition); strictly spherical core boundary $R_c = 0.65 R_\odot = 4.52205 \times 10^{10}$ cm; mean core density $\rho_{\text{core}} = 1.7$ g/cm^3 (2 sig. figs.); conserved core mass $M_c = \frac{4\pi}{3} R_c^3 \rho_{\text{core}} = 6.58482 \times 10^{32}$ g; baseline spherical moment $I_{\text{c,base}} = \frac{2}{5} M_c R_c^2 = 5.38610 \times 10^{53}$ g cm^2; initial polar spin frequency $\nu_0 = 430$ nHz ($\Omega_0 = 2\pi\nu_0 = 2.70177 \times 10^{-6}$ rad/s); flip timescale $\tau_{\text{flip}} = 11$ Julian yr $= 3.471336 \times 10^8$ s. Spherical trace conservation ($I_1 + I_2 + I_3 = 3I_{\text{c,base}}$) and Intermediate Axis Theorem ordering ($I_1 < I_2 < I_3$) is applied. Mercury J2000 elements: sidereal period $P_{\text{orb}} = 87.96926$ days (415.2019 orbits/century, $n_{\text{orb}} = 8.266758 \times 10^{-7}$ rad/s), semi-major axis $a = 0.3870989$ AU $= 5.790918 \times 10^{12}$ cm, eccentricity $e = 0.2056307$, and solar-equatorial inclination $i = 3.381°$ (4 sig. figs., derived from ecliptic $i = 7.00487°$, $\Omega = 48.33167°$, and Carrington solar pole elements 7.25° and 75.77°).

[c] *Non-Magnetic, Non-Hydrostatic Convective Lattice Screening (Column 3):* The convective shell ($0.65 R_\odot \to R_\odot$, $\rho_{\text{conv}} = 1.3$ g/cm^3) is modelled as stacked layers of Type II metallic hydrogen containing non-hydrogen intercalate atoms preserving a zero interlayer shear modulus ($\mu_\perp = 0$) and low interlayer viscosity ($\eta \ll 6.68 \times 10^{11}$ P). This enables frictionless differential sliding past the 430 nHz core without arresting its 11-year intermediate-axis inversion. Type II metallic hydrogen in the convective zone should display a finite in-plane stiffness ($\mu_\parallel > 0$). The redistribution of material in the intercalate regions is required to mechanically cancel the rotating C_{22} core bulge at the $\geqslant 99.9997\%$ level and at the $\geqslant 99.99996\%$ level of the 11-year mid-flip $\delta J_2(t)$ oscillation, thereby eliminating gravitational phase-lag torques ($\phi_0 \to 0$) which would have been seen at Mercury.

[d] *Observational Conclusions at Mercury:* Column 1 ($J_2 = 3.403 \times 10^{-4}$, unshielded) is falsified because its node regression (−43.146″/century), Argument of Perihelion (86.066″/century), 19.392 Day Range Oscillation (710.05 m), and large mid-flip $\delta J_2(t)$ spike, all exceed MESSENGER and BepiColombo tracking thresholds. Column 2 ($J_2 = 2.25 \times 10^{-7}$, unshielded) matches static perihelion precession (0.028428″/century) but is falsified by its 19.392 Day Range Oscillation (342.61 m) and a large mid-flip $\delta J_2(t)$ spike, both of which exceed MESSENGER and BepiColombo tracking thresholds. Column 3 ($J_2 = 2.25 \times 10^{-7}$, convective lattice shielding) preserves the expected 0.028428″/century (6.3400 km/century) static quadrupole drift while suppressing the 19.392-day C_{22} range swing to $\leqslant 0.001$ m and the 11-year $\delta J_2(t)$ oscillation to $\leqslant 3.7 \times 10^{-11}$, rendering the 11-year core flip non-falsifiable by external satellite measurements.

intermediate axis, I_2, Euler's equations of motion yield an instability growth rate for the intermediate axis as follows:

$$\lambda = \frac{\pi}{\tau_{\text{flip}}} = \Omega_0 \sqrt{\frac{(I_3 - I_2)(I_2 - I_1)}{I_1 I_3}} = 9.0501 \times 10^{-9}\ \text{s}^{-1}\,, \quad (9)$$

where τ_{flip} corresponds to the 11-year flip interval, which is $\sim 3.4713 \times 10^8$ s. Ω_0 is the rotational frequency ($\Omega_0 = 2\pi\nu_0 = 2.70177 \times 10^{-6}$ rad/s) and $\nu_0 = 430$ nHz [1,2]. The instability growth rate, λ, governs how fast the intermediate axis inertial torque is able to overcome the rotation vector. This is because within the context of the intermediate axis theorem, angular momentum constantly gets transferred between the three axes while the total angular momentum and kinetic energy remain conserved. In any case, note that (9) is a key step in the solution, as now the instability growth rate becomes linked to the moments of inertia in the core.

Next, it is possible to define exact axis asymmetries as $\epsilon_1 = I_2 - I_1$ and $\epsilon_3 = I_3 - I_2$. Substituting the relationships from above yields two expressions which quantify the perturbation difference, (10), and the perturbation product, (11):

$$\epsilon_1 - \epsilon_3 = 2\Delta I\ \text{g cm}^2\,, \quad (10)$$

$$\epsilon_1 \cdot \epsilon_3 = \left(\frac{\lambda}{\Omega_0}\right)^2 I^2_{\text{c, base}} = 3.2551 \times 10^{102}\ \text{g}^2\,\text{cm}^4\,. \quad (11)$$

Using these expressions, it can be shown that $I_2 = I_{\text{c, base}} + \frac{2}{3}\Delta I$, $I_1 = I_2 - \epsilon_1$, and $I_3 = I_2 + \epsilon_3$. From the presence of ΔI in (6), note that the perturbation difference expressed through (10), depends on J_2. In turn, this value is measured by examining the precession of Mercury, through (2). Consequently, ΔI becomes constrained and coupled to the precession of Mercury. Conversely, the perturbation product expressed in (11) is a constant which is independent of the specific moments of inertia present in the solar core. Its value is determined solely by the exponential growth rate (λ), the rotational frequency (Ω_0), and the base moment of inertia ($I_{\text{c, base}}$). Using these expressions, it is now possible to calculate the moments of inertia in the solar core for each value of J_2, as displayed in Table 1.

Next, consider what is happening during the inversion. At the halfway point of inversion, $\omega_2 = 0$ and the rotation lies about the xy-plane, such that:

$$\omega_1 = \Omega_0 \sqrt{\frac{I_2(I_3 - I_2)}{I_1(I_3 - I_1)}}\,, \quad (12)$$

$$\omega_3 = \Omega_0 \sqrt{\frac{I_2(I_2 - I_1)}{I_3(I_3 - I_1)}}\,. \quad (13)$$

The exact maximal frequency at the mid-point during the flip, Ω_{max}, can then be obtained from:

$$\Omega_{\text{max}} = \sqrt{\omega_1^2 + \omega_3^2} = \Omega_0 \sqrt{\frac{I_2(I_1 + I_3 - I_2)}{I_1 I_3}} =$$

$$= 2.701785 \times 10^{-6}\ \text{rad/s} = \quad (14)$$

$$= 430.0024\ \text{nHz}\,.$$

Note how this value is completely independent of the choice for J_2 and results in an increase in rotational frequency only 0.00056%. Importantly, as displayed in Table 1, when $J_2 = 3.403 \times 10^{-4}$, the equatorial spin is primarily along the y-axis. However, when $J_2 = 2.25 \times 10^{-7}$ the equatorial spin vector splits almost evenly between the x and y axes.

Next, it is possible to examine how the mass distribution is produced along the x and y axes by evaluating the C_{22} quadrupole ($l = 2$), assuming a J_2 value of 2.25×10^{-7}. Because the center of mass must remain unchanged, $C_{22} = \frac{I_y - I_x}{4M_\odot R_\odot}$ can be used to move mass symmetrically relative to the center of mass. At $t = 0$, $C_{22} = \frac{I_3 - I_1}{4M_\odot R_\odot^2} \approx 9.3735 \times 10^{-5}$. However, using the values determined for I_1, I_2, and I_3 in Table 1, then at the half-way point, $C_{22} = \frac{|I_2 - I_1|}{4M_\odot R_\odot^2}$ or $C_{22} = \frac{|I_3 - I_2|}{4M_\odot R_\odot^2}$ both of which are on the order of $\approx 4.68 \times 10^{-5}$, or about half the value. This can cause a significant wobble effect on Mercury as the core inverts.

However, there is another effect which can be observed on a much smaller timescale. Consider that the synodic oscillation period, P_{osc}, which combines the rotational frequency of the core, Ω_0, and the precessional frequency of Mercury, $n_{\text{orb}} = 2\pi/P_{\text{orb}}$ ($P_{\text{orb}} = 87.96926$ days or 8.266758×10^{-7} rad/s), is given by:

$$P_{\text{osc}} = \frac{2\pi}{2(\Omega_0 - n_{\text{orb}})}\,, \quad (15)$$

where the factor of 2 in the denominator arises from the fact that oscillation occurs as a result of a rotating quadrupole. This produces an oscillation period, P_{osc} of 19.392 days on Mercury. The gravitational potential produced at Mercury's orbital radius, r ($\approx a$) is given by:

$$V_{22}(r, \lambda) = -\frac{3GM_\odot R_\odot^2}{r^3} C_{22}\ \sin^2\theta\ \cos(2(\lambda - \Omega_0 t))\,, \quad (16)$$

where $C_{22} = \frac{I_y - I_x}{4M_\odot R_\odot^2}$ and $\theta \approx \frac{\pi}{2}$ at the solar equator. Taking the radial derivative, $a = -\partial V_{22}$, gives the sinusoidal radial acceleration amplitude:

$$A_{r,22} = \frac{9GM_\odot R_\odot^2 C_{22}}{a^4}\,. \quad (17)$$

For a forced harmonic oscillator with epicyclic frequency $\kappa \approx n_{\text{orb}}$ driven at $\omega_{\text{syn}} = 2(\Omega_0 - n_{\text{orb}})$, the steady state radial displacement amplitude is equal to:

$$\Delta r = \frac{A_{r,22}}{\omega_{\text{syn}}^2 - n_{\text{orb}}^2} = \frac{9GM_\odot R_\odot^2 C_{22}}{a^4\left[4(\Omega_0 - n_{\text{orb}})^2 - n_{\text{orb}}^2\right]}\,. \quad (18)$$

It is possible to then add the tangential Coriolis acceleration $A_{\theta,22} = \frac{6GM_\odot R_\odot^2 C_{22}}{a^4}$ and the epicyclic factor ($1 + \frac{4n_{\text{orb}}}{3\omega_{\text{syn}}} \approx$

1.294). As a result, for J_2 values of 3.403×10^{-4} and 2.25×10^{-7}, Δr is equal to 710.05 m and 342.61 m, respectively, as displayed in Table 1. Consequently, if the effects of C_{22} at the solar core are not shielded, this results in large displacements or wobbles on Mercury every 19.392 days as it precesses around the Sun.

3.3 Convective shielding of the solar core

Mass within the solar core should not be assumed to be symmetrically distributed and its center of mass may not be coincident with the geometric center. Should this be the case, then as the Sun rotates, its center of mass would remain fixed, but its geometric center would wobble around the center of mass. However, it is possible to completely shield this effect by recognizing that the solar core is surrounded by the convective zone. As stated above, the relative moment of inertia in this zone is nearly 6x greater than observed for the solar core. For instance, it is possible to consider a condition where $C_{22,\,\mathrm{net}} = C_{22,\,\mathrm{core}} + C_{22,\,\mathrm{conv}}$. In order to completely mask the effect of the C_{22} term at Mercury, the value of $C_{22,\,\mathrm{net}}$ should remain less than 2.8×10^{-10}. As such, the convective zone must be able to act as a phase-locked mechanical counterweight ($\phi_{\mathrm{conv}} = \phi_{\mathrm{core}} + 90^\circ$). It must be able to continuously redistribute mass in step with the core's rotation without locking its spin. Regions of lower mass in the convective zone would thereby serve to cancel the effect of regions of higher mass in the core.

All of these requirements are met within the previously outlined context of the LMHSM [3, 4, 20]. The convective zone is comprised of Type II metallic hydrogen, wherein the hydrogen nuclei occupy the lattice points of a hexagonal planar structure [3, 4, 20]. Unlike the situation in Type I metallic hydrogen, with Type II the electrons are fully delocalized. Consequently, the material is anticipated to be highly electrically conductive. Furthermore, it has previously been stated that, within Type I and Type II metallic hydrogen, "Each hydrogen layer would act as a barrier to diffusion along the c-axis ... while providing a channel for rapid elemental diffusion in the region between two hexagonal layers" [4]. This is precisely what is required to mask the C_{22} (at $t = 0$, $C_{22} = (I_3 - I_1)/(4M_\odot R_\odot^2)$) effect of the solar core on Mercury. Pockets of intercalate atoms located between the hexagonal planes must be able to freely adjust their positions in order to compensate perfectly for the mass distribution of the solar core.

Consequently, internal centrifugal forces in the convective zone would act to drive the fluid towards a balanced state. As the Sun spins, material in the convective zone would compensate for the effect of any displacement between the center of mass and the geometric center of the Sun. The constant redistribution of intercalate atoms thus ensures that the solar center of mass remains coincident with its geometric center. This prevents all wobbles, either for the Sun or for Mercury, even if the solar core contains mass asymmetries.

Furthermore, when the Sun becomes active, deep convective flows are known to disrupt the meridional circulation typically transporting material from the equator towards the poles near the solar surface [21]. With solar activity, there would be a redistribution of matter in the convective zone, precisely accounting for the changes taken place relative to the transfer of momentum along the x, y, and z axes of the core.

3.4 Variations in the solar cycle

It is well-known that the solar cycle can vary. For instance, solar cycle 19 (spanning 1954 to 1964) was exceptionally short, lasting only 10 years and 6 months. Conversely, solar cycle 23 (spanning 1996 to 2008) stretched significantly, lasting 12 years and 4 months. This means the cycle's inversion timescale (τ_{flip}) has fluctuated by roughly ±12.5% from its median value. Yet, under this rigid spherical-core model, variations in the solar cycle period over the past 100 years would have no effect on the perihelion precession of Mercury, as the two phenomena are mathematically uncoupled.

Mercury's perihelion precession depends strictly on the quadrupole moment, J_2, which dictates the exact effect of the gravitational field. Conversely, the solar cycle period depends strictly on the instability growth rate (λ), which dictates how fast the intermediate-axis torque overpowers the rotation vector. When the mathematical layout of the moments of inertia are evaluated, the parameters balance independently. Mercury's precession vector is governed by (6) and (10) which define ΔI. This value is entirely anchored to Mercury's observed 0.028 arcsec orbital drift per century, as provided by (2). Because Mercury's precession did not change over the last 100 years, ΔI must be treated as a rigid, static constant. Material can get redistributed in the solar core, but ΔI does not change as a result. The quantity is conserved. Conversely, the solar cycle vector is governed by (11). When the solar cycle shortens to 10.5 years or lengthens to 12.3 years, the instability growth rate, λ, given by (9), scales in inverse proportion. Mathematically, the difference between ϵ_1 and ϵ_3 must remain constant, but the product can vary. Consequently, the local concentrations of heavier elements within the core can slowly vary along the x, y, and z axes. This serves to alter the internal torques involved relative to the solar core, but the global quadrupole term, J_2, remains completely fixed.*

Along these lines, it is known that the presence of external torques can act to stabilize the intermediate axis and this would clearly affect the solar cycle. This is especially true if the rotational rate of the core is insufficient to overcome these effects. For the Sun, it appears that the interaction of the tachocline layer with the core remains insufficient to stabilize the intermediate axis. This suggests that, while shear forces are present at the level of the tachocline, the convec-

*While it is true that the specific moments of inertia obtained for the solar core based on a given J_2 value will depend on the model adopted as to radius and average density, such selections will not affect either the solar cycle or the perihelion precession of Mercury.

tive material itself must be characterized such that the core remains completely free to rotate at 430 nHz. In any event, such phenomena cannot be used to modify the solar cycle, precisely because the physical parameters of the tachocline and convective zone must be constant.

4 Conclusions

While yielding the correct value for the anomalous advance of the perihelion precession of Mercury, it is clear that (1) [11, 12, 14], might not be revealing the complete story. These derivations take no account of possible asymmetry in the solar core and do not link the anomalous advance of the perihelion precession of Mercury and the solar cycle through the quadrupole moment, J_2. Neither Gerber nor Einstein could have recognized this in their day, as the solid body rotation of the solar core was not discovered with helioseismology until the end of the 20th century [1, 2].

It is also certain, given the acceptance of the gaseous solar models, that few will be willing to abandon the concept that the presence of a dynamo is responsible for the solar cycle. The dilemma in this approach rests with the realization that such a mechanism must be seeded in protostars, long ago detached from the modern Sun [22–25]. Therefore, our current understanding of the solar cycle has been relegated to stellar birth in protostars, rather than viewed as a natural consequence of the selected solar model. As such, this exposition strikes a significant blow to all gaseous models of the Sun. There is no longer a need for a solar dynamo born in protostars to account for the solar cycle. Everything remains controlled by the solar core.

The key to the solar cycle may well be found in the realization that the core undergoes solid body rotation, is endowed with three moments of inertia, and is subject to the Dzhanibekov effect. In turn, this enables the Sun to overtly display the consequences of core inversion every 11 years with the renewed appearance of sunspots. The only requirements needed to make this occur, already existed in the LMHSM [3, 4, 20]. The solar core must be a rigid lattice structure such as the body centered cubic proposed in the LMHSM. The core must be surrounded with material unable to apply external torques. Type II metallic hydrogen, with its freely sliding hexagonal planes, provides exactly such a material. It should enable the core to rotate freely without interference, precisely because the London forces within such a structure should be negligible given the full delocalization of electrons within conduction bands.* Moreover, the predicted presence of heavy elements within intercalate regions which are freely able to move between planes provide the perfect mechanism to redistribute solar mass. In doing so, they act to constrict the solar center of mass to the geometric center of mass and prevent both the Sun and Mercury from wobbling. Simple centrifugal forces are able to provide the driving force for this mass redistribution, as the core continues on its spin trajectory.

The solar cycle becomes elegantly explained through small adjustments in mass changes within the core and the convective zone. Mass is allowed to be redistributed but must do so in a manner which is restricted by the constancy of Mercury's precession. In other words, the conserved J_2 value of the core must agree with this precession, as set by (2). In addition, the resulting ΔI must remain completely fixed, as set by (5). Mass redistribution affecting the length of the solar cycle remains governed by (9) and (11). The individual moments are allowed to change, but J_2 remains fixed.

This brings about a completely new understanding of the triggers for the solar cycle. The movement of intercalate atoms within the convective zone becomes vital to understanding this cycle. Therefore, confidence that core inversion is indeed taking place rests not only on the elegance of the model, but also on the known ejection of intercalate atoms, like helium, with increasing solar activity.

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*Graphite is known to have very weak resistance to shear forces and that is precisely why such a material makes a good lubricant for metal parts. Nonetheless, compared to Type II metallic hydrogen, graphite still has some van der Waals forces between planes because some electron density remains in π bonds. In graphite, the π electron density extends into the space between sheets allowing the associated electron clouds to interact via weak van der Waals forces. This will not occur in Type II metallic hydrogen, as all electrons are fully delocalized and no partial π bonding exists to provide van der Waals interactions between planes. As a result, Type II metallic hydrogen should be perhaps the most well-suited material to permit free rotation of the solar core, owing to its inability to transfer external torques to the solar core.

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