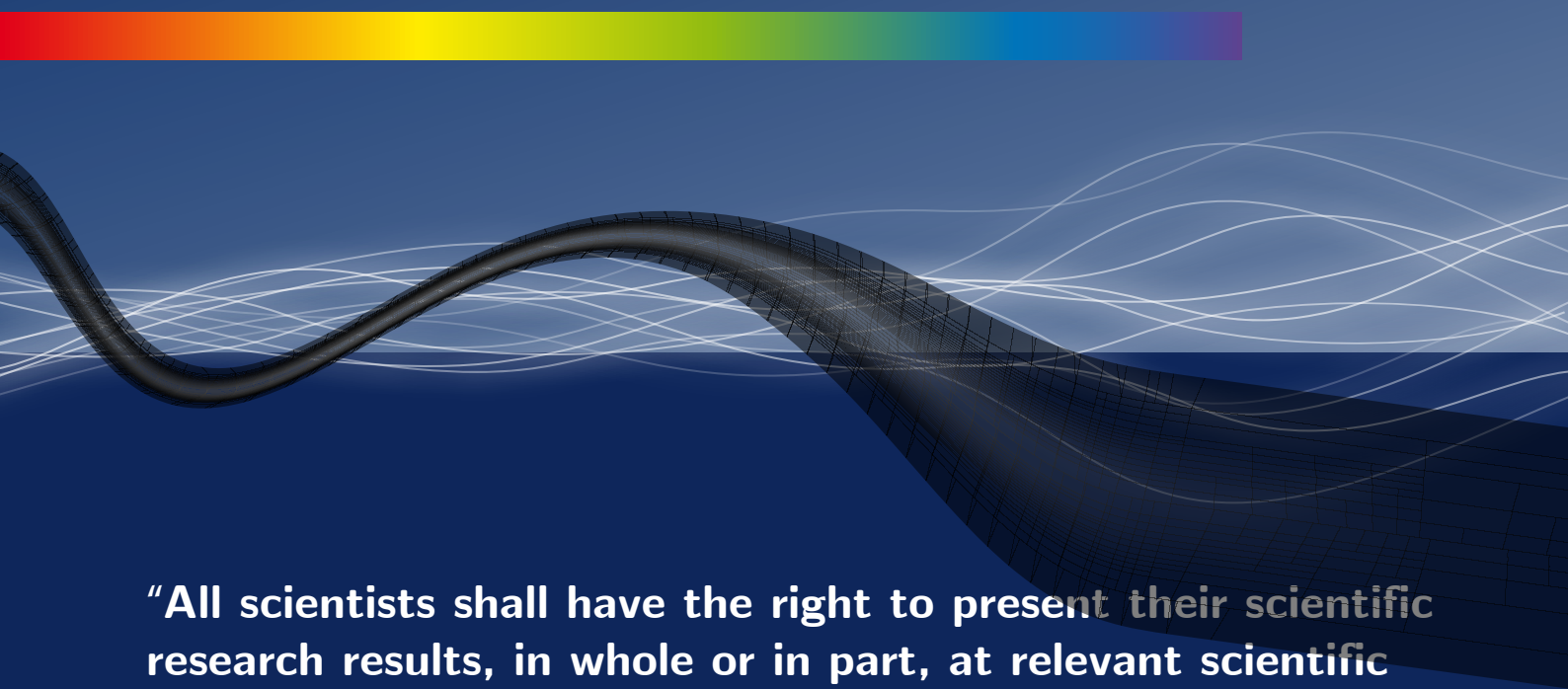


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The Energy Spectra of Cosmic Ray Protons, the Origin of Gluons, and the Mechanism of Baryon Generation

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For the past sixty years, the generation of hadrons has been dealt with through a framework of theories devised to describe the so-called Strong interactions. About two years ago, the author put forward an essentially quantum electrodynamical model for the same purpose. The present paper contains the latest development in the interpretation of those results, and we reached a point where a bridge can be extended to existing theories. The main result of our previous work has been the determination of an energetic interval of 2.7 GeV between a “vacuum” parent state and the proton rest-energy. The full interpretation of this finding is that this is the energy advantage (calculated from a Regularization procedure) that stabilizes charge (the baryons) confined in the shape of loops by correlating EM excitations at 3.7 GeV. That is, we have been able to establish that these EM excitations are in fact the Gluons of high-energy physics, and they come straight from relativistic quantum electrodynamics through the Regularization procedure of loop energies. The value 2.7 GeV obtained from Regularization is of the correct magnitude to explain the difference between the strength of Strong and EM interactions (15 versus 1/137). The size of a proton can also be approximately deduced from our arguments.

1 Introduction

The present paper contains the main results of investigations which have directly addressed the long-standing problem of describing the genesis of particles. In particular, the issue of the origin of mass is considered [1, 2]. Many of the ideas and concepts in this work have previously been advanced by Barut [3], Bostick [4], and Jehle [5]. In particular, the starting point in this treatment, is that magnetic moments are fundamental properties of leptons and baryons, and that the presence of magnetic moments in particles can be modelled by the introduction of an intrinsic closed electrical current loop of finite (rather than point-like) size. It might be argued that such hypothesis should be incompatible with QED and that electrons behave experimentally as point-like objects. However, the present treatment may be regarded as describing the earliest stages of a particle condensation process taking place in an extremely dense medium at 10^{13} K. Present day experiments take place under completely different conditions.

The current-loop model refers more properly to the EM fields in this *embryonic* stage, and no specific mass/charge distribution for baryons is explicitly needed or introduced. A blend of fermion/EM fields in loop form act to correlate and confine baryon fields inside the loop. A multiply-connected current path should arise, whose possible topological forms were the object of intense discussions by Bostick and Jehle, but in the absence of more concrete evidence a simple circular loop path is adopted in this treatment. The confinement of magnetic flux within such paths was initially [1] assumed as occurring in numbers of flux quanta determined by the magnetic moments in magneton units, a property easily derived

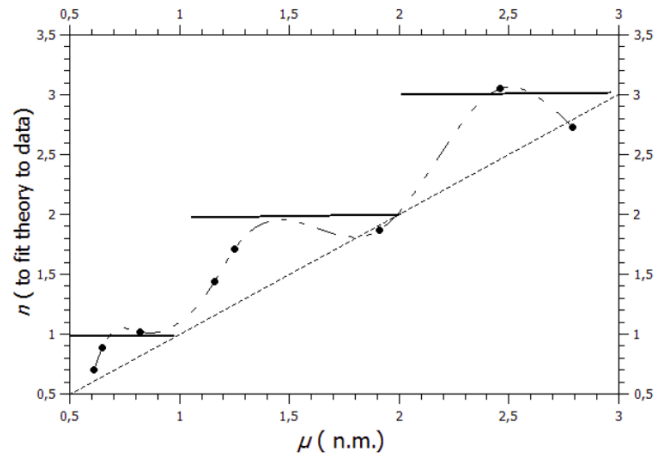


Fig. 1: Plot of n against the magnetic moment for the baryons octet (points) from the definition $n = (2c^2\alpha/e^3)\mu m$. The diagonal line is the classical prediction of one flux quantum per nuclear magneton (n.m.). Nucleons are on the line. The data display undulations, and a tendency to reach for the steps (traced line as guide) [2].

from Barut's semiclassical spinning particle-model, but such assumption is later adjusted to better fit data.

In paper [1], we have shown that it is possible to describe the masses m of all the baryons of the octet and decuplet in terms of a single formula, involving the magnetic moments μ and corresponding numbers of confined flux quanta n . One might otherwise use this relation to define n from the experimental masses and moments [2]:

$$n = (2fc^2\alpha/e^3)\mu m$$

where $f = 1$ for spin $1/2$ and $\approx 1/\sqrt{3}$ for spin $3/2$, and α is the fine-structure constant (one immediately recovers the often-mentioned inverse relation of mass with the constant α , since n and μ are approximately proportional to each other). The treatment that produces this equation is essentially heuristic, but precise enough for instance to highlight the dependence of mass upon the square-root of the spin angular momentum, as reported in the literature (note that the phenomenological factor f that corrects for spin is related to kinetic energies rather than to magnetic contributions) (cf. Fig. 1 of [1]).

In paper [2], whose main results are reproduced in the following section for the sake of clarity, we took much further the treatment presented in [1]. A key parameter in this analysis is the number of flux quanta n arrested inside a current loop. In particular, we obtain in [2] a very revealing result which has previously been reported mainly through Condensed Matter physics investigations, which is that the energy of currents (here regarded as a particle's rest mass) is a *periodic function* of the confined magnetic flux in multiply-connected structures. Consistently with these results from Condensed Matter systems, the periodic dependence of baryons masses (and confined flux) with the magnetic moments (see Fig. 1) can be regarded as a demonstration that the initial hypotheses of the present investigations are sound. That is, indeed mass is a manifestation of magnetodynamic energies (related to currents) confined in a multiply-connected region. Such hypothesis is therefore consistent with experimental data. With this evidence in hand, the next step clearly was to advance beyond the initial phenomenological-heuristic argumentation and propose a field-theoretical treatment that would describe the observed mass-energy relations for actual particles.

Such kind of treatment has previously been applied for fermion fields flowing around a closed loop containing magnetic flux (see references in [2]). Starting from a Lagrangian suitable to these fields (assumed as built upon a proton “substrate”, following Barut), we then obtain an energy spectrum for the possible traveling wave-states around a closed path. To simulate the perturbations coming from the vacuum background which will be added to the proton state, a sum over the states in the energy spectrum of kinetic energies for the EM/fermion quasiparticles is necessary. An Epstein-Riemann Zeta function Regularization procedure previously adopted for the Casimir Effect problem is applied to eliminate divergences when the sum over the energy spectrum states is carried out, and the periodic behavior of the baryon masses with magnetic flux is quantitatively reproduced with no further forms of energies required besides the magnetodynamic terms. A new result of this treatment [2], is the prediction of a parent state at $U_0 = 3.7 \text{ GeV}$, which should be identified with a dense medium (opposite to what we usually qualify as “vacuum”), whose fluctuation instabilities would give origin to baryons. The present work goes beyond [2] in the

search for evidence for the existence of this state as well as the source of the correlations. The calculated value of U_0 immediately indicates that protons (of rest mass 0.94 GeV) should become unstable if accelerated to kinetic energies beyond 2.7 GeV if their structure were not strong enough and capable to radiate excess energy. We found out that a very good way to investigate this point is through the analysis of the spectra of protons in cosmic rays, whose energy flux profile peaks at 2.7 GeV kinetic energy (Fig. 3 below) for reasons we will discuss.

In the following sections, we firstly present the field-theoretical model introduced in [2], alongside the comparison with experimental data for mass and magnetic moments for baryons. In the analysis in Section 3, we test the hypothesis of the existence of an energy level for vacuum by examining data collected for protons in cosmic rays and discuss the relation between this energy level and gluons. In Section 4, we show that an estimate for the proton size can be obtained from the theory.

2 Field-theoretical model for generation of baryons

For the developments that led to this field-theoretical treatment, we make reference also to the *Annales* paper [1] (see also references therein and in [2]). Let's consider a fermion field confined by EM energy inside a circular path of length L , enclosing an amount of self induced magnetic flux φ , in a potential A . We need to show that such an EM/fermion packet corresponds to a state detached from a higher state associated with a sea of excitations in equilibrium, and therefore might be used to represent a “quasiparticle”. The relativistic Lagrangian for such an object can be modelled through the dressing of a proton of mass m_p (as once proposed by A. Barut) in view of the presence of magnetodynamic terms [2]:

$$L = \bar{\Psi} \left\{ i\alpha_\mu \left(\hbar \partial_\mu - i \frac{e}{c} A_\mu \right) - \alpha_4 m_p c \right\} \Psi \quad (1)$$

where the α_μ are Dirac matrices. This Lagrangian can readily be transformed into a Hamiltonian form. Assuming a constant potential A around the ring path, the spectrum of possible energies for a confined fermion becomes:

$$\epsilon_k = c \left\{ (p_k - eA/c)^2 + m_p^2 c^2 \right\}^{1/2} \quad (2)$$

which comes straight from the orthonormalized definition of the Dirac matrices and diagonalization of the Hamiltonian. We now definitely impose a circular closed path. If one takes the Bohr-Sommerfeld quantization conditions, the field momentum p_k (for integer k) is quantized in discrete values $2\pi \hbar k/L$. We start from this assumption, but the true boundary conditions to close the wave loop might impose corrections to this rule in the form of a phase factor (a phase factor is introduced in the fit to the data in Fig.3 below). The potential A can be replaced by φ/L . Environmental (vacuum) fluctuation effects on the kinetic energy are accounted for in

a way similar to that applied in the analysis of the Casimir Effect, by summing over all possible integer values of k in (2) [2]. This summation diverges. According to the theory of functions of a complex variable, the removal of such divergences requires that the analytic continuation of the terms be taken, which reveals the diverging parts which are thus considered as contributions from the infinite vacuum reservoir. What remains plays the important role of energetically stabilizing the loops (in a way that resembles the role of phonons in the formation of Cooper pairs). It is necessary to rewrite (2) in terms of Epstein-Riemann Zeta functions [2], including the summation over k from minus to plus infinity integers, and making a Regularization (Reg) transformation. Here $M(\varphi)$ is the flux-dependent dressed mass of a baryon, and $s \rightarrow -1$:

$$M c^2 = U_0 + \text{Reg} \sum_k c \left\{ (p_k - e\varphi/Lc)^2 + m_p^2 c^2 \right\}^{-s/2} \quad (3)$$

where we have allowed for the existence of a finite energy U_0 to represent an hypothetical state from which the individual baryons would condense, since they would correspond to lower energy states. Such particles should be characterized as states of energy lower than U_0 . It is convenient to define from L a parameter with units of mass $m_0 = 2\pi\hbar/cL$, which will be used to define a scale in the fit to the data. We notice that m_0 is related to the parameter L in the same way field theories regard mass as created from broken symmetries of fields, establishing a range for an otherwise boundless field distribution (e.g. as happens with the London penetration depth at the establishment of a superconductor state, which is related to an electromagnetic field “mass” by a similar expression). For convenience, we define the ratios $m' = m_p/m_0$ and $u_0 = U_0/m_p c^2$. For comparison with the data analysis in our previous work [2], we must introduce also the number of flux quanta n (integer or not) associated to φ , such that $n = \varphi/\varphi_0$. In terms of these parameters one may write (3) in the form:

$$M(n)/m_p = u_0 + (1/m') \text{Reg} \sum_k \left\{ (k - n)^2 + m'^2 \right\}^{-s/2} . \quad (4)$$

In the analysis of data, the experimental values of M/m_p for baryons will be plotted against n . The sum on the right side of (4) is a particular case of an Epstein Zeta function $Z(s)$, and becomes a Riemann Zeta function, since the summation is over one parameter k only. The summation diverges but it can be analytically continued over the complex plane, since the Epstein Zeta function displays the property of reflection. It has been shown that after the application of reflection, the resulting sum is already regularized, with the divergences eliminated. The reflection formula is [2]:

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) Z(s) = \pi^{\frac{s-1}{2}} \Gamma\left(\frac{1-s}{2}\right) Z(1-s) . \quad (5)$$

This replaces the diverging $Z(s)$ straight away by the regularized $Z(1-s)$, which is a convergent sum (since $\Gamma(-1/2) =$

$-2\sqrt{\pi}$ we see that the regularized sums are negative, like in the Casimir Effect solution). The Regularization of (4) is carried out as follows (note that $s \rightarrow -1$, and the “reflected” exponent $-(1-s)/2$ replaces $-s/2$ of (4)). $Z(1-s)$ is given as [2]:

$$\begin{aligned} \sum_k \left\{ (k - n)^2 + m'^2 \right\}^{-(1-s)/2} = \\ \frac{2}{\Gamma\left(\frac{1-s}{2}\right)} \int_0^\infty t^{\frac{1-s}{2}-1} \left(\sum_k e^{-(k-n)^2 t - m'^2 t} \right) dt = \\ \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \int_0^\infty t^{-\frac{s}{2}-1} \left(\sum_k e^{-2\pi i k n} e^{-\frac{\pi^2 k^2}{t} - m'^2 t} \right) dt = \\ \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \left(\frac{\Gamma\left(-\frac{s}{2}\right)}{m'^{-s}} + 2\pi^{-s/2} \sum_{k \neq 0} \left(\frac{k}{m'} \right)^{\frac{-s}{2}} K_{\frac{s}{2}}(2\pi m' k) e^{-2\pi i k n} \right) \end{aligned} \quad (6)$$

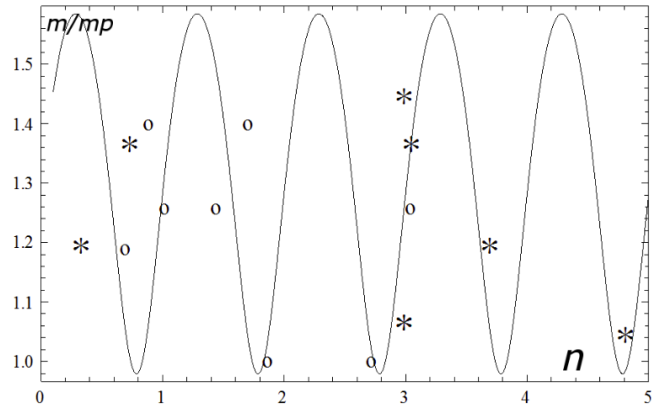


Fig. 2: Comparison of baryons masses calculated from (6) (line) as a function of confined flux n , with data points from Tables I and II of [2] for octet (open circles) and decuplet particles (m_i used [2], stars). Nucleons are on the basis of the figure. The points come from the heuristic/phenomenological equation $n = (2c^2\alpha/e^3)\mu m$. The fit produces $U_0 = 3710$ MeV as the vacuum/environment parent level.

The “Reg” summation in (4) then becomes

$$\left(\pi^{\frac{2s-1}{2}} / \Gamma\left(\frac{s}{2}\right) \right) \Gamma\left(\frac{1-s}{2}\right) Z(1-s) ,$$

and the exponential produces a cosine term.

Since $\Gamma(-1/2) = -2\sqrt{\pi}$ we see that the regularized sum is negative, corresponding to energies lower than U_0 . In the fitting to the data, we will admit that both m' and u_0 are adjustable parameters.

Fig. 2 shows the data for all baryons in Tables I and II of [2], and the plot of mass in (4) regularized by (6), for $u_0 = 3.96$ and $m' = 0.347$ (corresponding to $m_0 = 2.88 m_p$ and $U_0 = 3710$ MeV). The energy 3710 MeV would represent the environment (“vacuum”) energy (state) from which

the baryons would evolve. By comparison with Yukawa's theory of the meson, one may interpret $m_0 c^2 = 2710 \text{ MeV}$ as the mass of a particle that provides an internal correlation and keeps the dressed proton stable, a task usually attributed to gluons in particle theory. Such particle is essentially EM energy confined in a loop [4], and the correlated system would follow the behavior of an harmonic oscillator in resonance with the particle motion.

3 Evidence from the energy-flux profile of cosmic rays

This model produced an entirely new result, which is the proposal of a parent vacuum/environment energy state at 3.71 GeV. Flux profiles of cosmic ray (CR) protons display important features [6] that seem related to the existence of a "correlation" energy that keeps the loops dressing of protons, corresponding to the difference between 3.71 GeV and the proton's rest energy of 0.94 GeV.

It is worthwhile to examine some available experimental data, well gathered in [6]. Fig. 3 shows the energy flux profile of protons as detected from interstellar outer space by a space probe. The symmetry of this figure clearly gives an average energy per proton of about 2.7 GeV. Tsallis and collaborators [7] carried out the integration of a related set of data to obtain an averaged energy of about 2.88 GeV, with the comment "Any connection of this value with other cosmological or astrophysical quantities is of course very welcome". Statistical mechanics has several famous similar cases. For instance, in the Maxwell kinetic theory of velocities distribution in a gas, the average energy of a molecule matches the energy provided by the environment under equilibrium conditions, which is measured in terms of the absolute temperature as $3/2 kT$. According to the theory in [2], in the case of the proton, equilibrium is reached against a vacuum at 3.7 GeV, which is 2.7 GeV higher in energy than the proton's rest energy, corresponding to a local temperature of about 10^{13} K . Therefore, the CR protons, similar to the classical gas case, display an averaged energy consistent with an equilibrium reached against the environment, at the predicted level at 3.7 GeV energy. It must be stressed that such equilibrium does not follow the classical formalism of Maxwell-Gibbs statistics, and requires relativistic effects to be included [7, 8].

4 About the size of the proton

The structure of fully developed protons is known to be formed by (the entanglement of) at least three major quark constituents, each of them with $1/3$ of the proton rest mass (see topological considerations in [4, 5]), with charges of opposite signs. This is a very important detail, since the same external electric fields that accelerate the proton as a whole will stretch this structure with a similar force. Therefore, the proton can be regarded as a stressed/strained ensemble of charged objects strongly connected (entangled) together, and thus its elastic response behavior should be considered.

Excited by external forces, a three-dimensional elastic structure will vibrate at its natural frequencies. The proton might be represented by a three-dimensional quantum harmonic oscillator. Following the considerations at the end of Section 2, we shall take 2.7 GeV as the ground state energy of an isolated oscillator [9]. This is the share of the 3.7 GeV that lays beyond the rest-energy. Extremely energetic quasiparticles from the original "vacuum" reservoir at 3.7 GeV would dress the proton fields leading to stabilization of the structure in the form of oscillators, in an energy state lower than the original "vacuum", establishing in this case the rest energy of a proton (in a way probably similar to how low-energy lattice phonons promote electron correlations and make the Cooper pairs stable in the superconductor ground state, which is lower in energy than the Fermi level by a small gap). This stable dressed-proton structure behaves like a vibrating system. The model developed in [2] and Section 2 actually deals with the fields of this correlations calculation. We now treat the elastic response of the particle in equilibrium with those fields.

The natural frequency of three-dimensional oscillations ω is given as: $3/2 \hbar \omega = 2.7 \text{ GeV}$. One obtains $\omega = 2.7 \times 10^{24} \text{ rad/s}$, an extremely high figure, within the gamma-ray range of photons in the EM spectrum. What makes such oscillations regime stable?

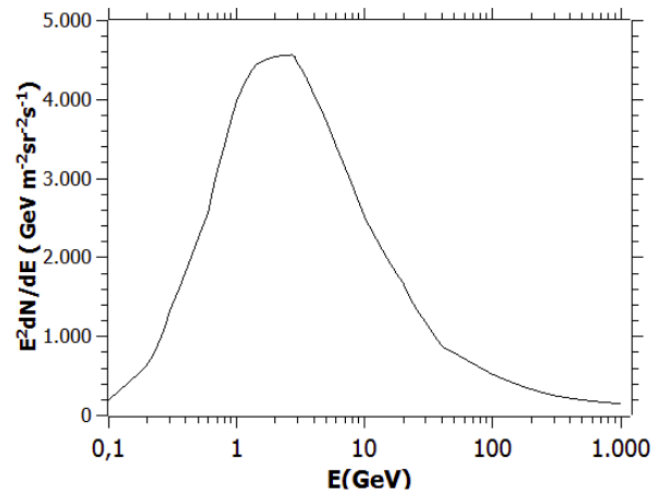


Fig. 3: Interstellar energy-flux profile of protons in CR, which peak at, and have an average energy of 2.7 GeV kinetic energy [3].

The diameter of a proton determined by scattering experiments is $\approx 1.8 \text{ fm}$. This should be taken as the maximum spacing between constituents in a "relaxed" proton structure, but such spacing is deformed by oscillations. Criteria have been developed to evaluate whether the deformation of interatomic spacing in a substance might provoke a change of state. A range of deformation between 5 and 10% of the relaxed "inter-constituent" spacing is usually recognized as within a typical limit for a structure to remain stable. The

maximum possible oscillating displacement is

$$x_m = (2/3 E/(m\omega^2))^{1/2},$$

where E is 2.7 GeV. We obtain $x_m = 0.16$ fm, which is obtained independently of the knowledge of the proton size. In view of the stability criteria mentioned earlier, this would independently establish a proton size of at least 3 fm at 3.7 GeV conditions. When cooling took place, the structure shrunk to the measured 1.8 fm. One might even conjecture that the observed size of the proton cannot be smaller since smaller particles with same constituents simply break apart as soon as formed due to inelastic strains.

5 Conclusions

It is then possible to summarize all the results in this work: The observed size of the proton, 1.8 fm would be a consequence of its origins in an environment at about 3.7 GeV. According to the model, the particle condenses due to the provision of a 2.7 GeV correlation energy from fields confinement in the form of loops, as calculated by the Regularization procedure. These confined fields play the role of quasiparticles (“vacuum-dynamics” quasiparticles), which provide strongly-binding correlations, which join constituents like in a harmonic oscillator, promoting a stable structure. There is a clear potential association between these energetic quasiparticles and what is called gluons. The magnitude of 2.7 GeV is in a proportion consistent with the ratio of 15 to 1/137, to EM coupling energies, which is the accepted relation between Strong and EM interactions in the range of tenths of a femtometer.

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On the Quantification of Relativistic Trajectories

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Solving the geodesic equation on a relativistic manifold is possible numerically step by step. This process can be transposed into a quantisation. We study here the effect of this quantisation on the Schwarzschild spacetime, more precisely in the Kruskal-Szekeres map.

1 From digitization to quantification

The geodesics are obtained using the Euler-Lagrange variational method, with the Lagrangian $L = g_{\mu\nu}x'^{\mu}x'^{\nu}$ which leads to the well-known equation [1, 8.26]

$$x''^{\alpha} + \Gamma_{\mu\nu}^{\alpha}x'^{\mu}x'^{\nu} = 0.$$

The goal is to obtain the extremal solutions for

$$\tau = \int_{\lambda_0}^{\lambda_1} \sqrt{L} d\lambda$$

which happens to be the proper time for a test particle subjected to the field g . Except for the mass of the particle, which is in fact an energy, this proper time is an action.

Finding solutions digitally is extremely simple. Given a digitisation step $\delta\lambda$ and an initial state (x, x') of the mobile, the position x is incremented by $x'\delta\lambda$. The geodesic equation gives $x'' = -\Gamma_{\mu\nu}^{\alpha}x'^{\mu}x'^{\nu}$ and the velocity x' is incremented by $x''\delta\lambda$. The process is then iterated.

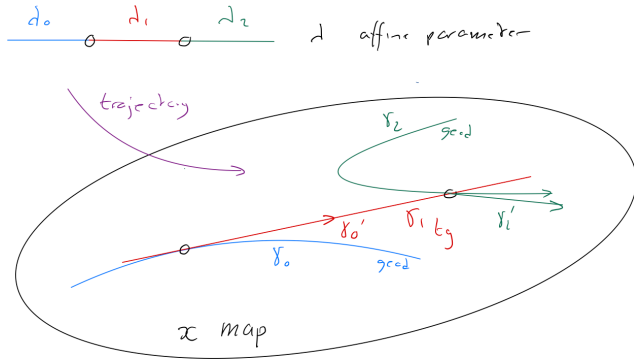


Fig. 1: A test-particle moves from a geodesic $\gamma_0(\lambda_0)$ to a geodesic $\gamma_2(\lambda_2)$ by a trajectory element $\gamma_1(\lambda_1)$ on an interval $\delta\lambda$. This is a straight line in the tangent space. We use the fact that the tangent spaces $T_x\mathbb{R}^n$ are in fact canonically included in \mathbb{R}^n .

The choice of the affine step will be made here by keeping the time step constant $\delta\tau$ which gives

$$\delta\lambda = \delta\tau / \sqrt{L}.$$

This time step can be physically equated with the quantum of action in the following interpretation.

At each step, the mobile requests a quantum according to the chosen coordinate system x . It uses this quantum to continue its trajectory in its local context, which is the tangent space to the space-time manifold at the current point. Then the new state is considered as such in global space-time. An observer placed on the particle moves during the quantum of time according to a trajectory linearised by the choice of its map.

Some remarkable facts emerge.

First, the coordinate system selected by the observer is essential. The linearisation of the trajectory during $\delta\tau$ depends on the map x and makes the interaction between space-time and the observer contextual. There is an effect of the observation on the trajectory.

Second, it cannot be excluded that the quantisation step involves speeds higher than those of light. This phenomenon can be related to certain quantum effects, such as the possibility for a particle to tunnel through a potential barrier, or to violate the conservation of energy law for a time short enough to be allowed by Heisenberg's uncertainty relations.

Third, in the particular case of the Schwarzschild model with a radius r_s it becomes possible to be in the forbidden zone beyond the naked singularity described below.

2 Reminder on Schwarzschild, Kruskal and Szekeres

Karl Schwarzschild was one of the first to find a solution to the gravitational equations of Einstein's general relativity in 1916. This solution, which describes the field created by a point mass, is expressed by the following metric in polar coordinates, with a speed of light $c = 1$ and a Schwarzschild radius r_s :

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).$$

Two peculiar radii were observed immediately. The first one, $r = r_s$, gives the horizon beyond which a particle cannot escape, giving the name of a black hole to this zone. The second one, $r = 0$, is a singularity of the metric, known as naked, where any particle entering the black hole ends its trajectory in a finite time.

The Kruskal-Szekeres coordinate transformation leads to a formulation in terms of the variables (T, X, θ, ϕ) [2]:

$$d\tau^2 = \frac{4r_s^3}{r} \exp\left(-\frac{r}{r_s}\right) (dT^2 - dX^2) - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).$$

The parameter $r = r_S \left(\mathcal{W}_0 \left(\frac{1}{e} (X^2 - T^2) \right) + 1 \right)$ is given by the branch 0 of the Lambert function \mathcal{W} .

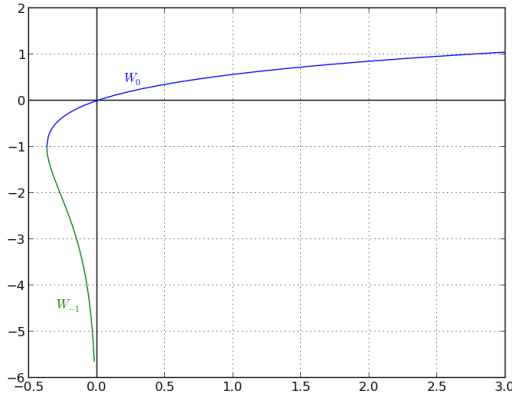


Fig. 2: Real branches of the Lambert function.

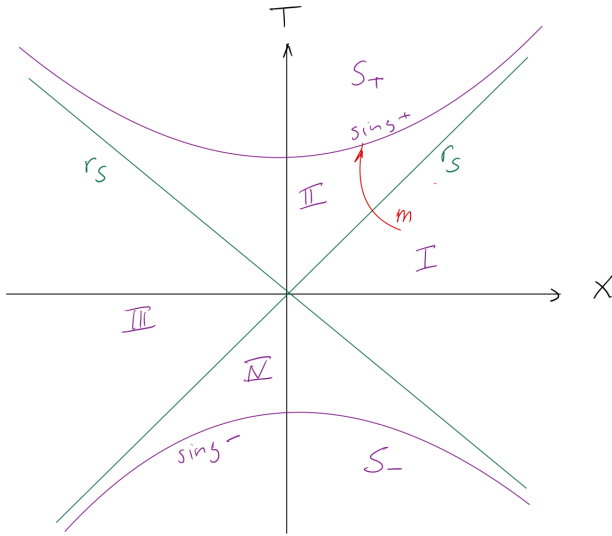


Fig. 3: Kruskal-Szekeres map.

The diagram in Fig. 3 shows the following regions:

- I** space-time outside the black hole
- II** black hole
- III** other component of space-time
- IV** white hole
- S+** inside of the naked singularity
- S-** other component inside the naked singularity.

This map shows that the Schwarzschild horizon is not a physical singularity, but only an artefact due to the choice of the map.

The diagonal lines represent the Schwarzschild horizon, and the two boundary branches of the **sing+** and **sing-** hyperbola the entrance and exit of the naked singularity.

A particle from region **II** ends its trajectory on **sing+**, without being able to exit. Conversely, a particle in region **IV** cannot do anything else, but exit; hence the name of the white hole. One also finds the expressions *sink* and *source* for these two regions.

S- and **S+** are inaccessible, or forbidden, because they are outside the map domain. These two regions and their boundaries are associated with a single point, the zero of the polar coordinates, and can be considered as collapsed. At least in the hypothesis of a strictly continuous world.

3 Appearance of tachyons

Traditionally, the term *tachyon* has been applied to a hypothetical particle with a speed greater than the speed of light. The exit of the speed of the future light cone is identified by the fact that $L < 0$ and thus an imaginary quantisation step. Here, we propose using a complex proper time:

$$\tau = \tau_r + i\tau_i \in \mathbb{C}.$$

This time is measured by two clocks, one real and the other imaginary. The increase in the affine parameter becomes $\delta\lambda = \delta\tau_r / \sqrt{L}$ if $L > 0$ or $\delta\lambda = i\delta\tau_i / \sqrt{-L}$ if $L < 0$. In this way, the trajectory remains real in the map x . For a tachyon, it is the imaginary clock that works, the other one remains fixed, and the opposite is true for a standard particle.

For any coordinate system on space-time, the notions of time and space are found locally by placing an orthonormal basis in the tangent space which diagonalizes the metric. Afterwards, thanks to a possible permutation of the axes and a calibration of the units, we can obtain the diagonal metric of Minkowski $\text{Diag}(1, -1, -1, -1)$. The zero coordinate is then time and the others define the space. The base obtained in this manner is generally referred to as a *tetrad*.

The proper speed $\dot{x} = \frac{\delta\lambda}{\delta\tau} x'$ is transformed into a quad-speed $u = \gamma \begin{pmatrix} 1 \\ v \end{pmatrix}$ where v is the space velocity of the mobile. Let v be its Euclidean norm and n_G be the unit vector v/v , the so-called *slip* vector. We easily obtain $\gamma = (1 - v^2)^{-1/2}$.

If $v < 1$, it is possible to put the mobile at rest with a Lorentz boost $\Lambda(v)$ such as

$$\Lambda(v)u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

If $v > 1$, γ becomes purely imaginary. Nevertheless, it is possible to extend this boost by

$$\Lambda(v) = \Lambda\left(\frac{n_G}{v}\right) R\left(n_G, \frac{\pi}{2}\right)$$

where $R(n, \theta)$ is the rotation of angle θ and axis n . It can be seen that $\Lambda(v)u = \begin{pmatrix} 0 \\ in_G \end{pmatrix}$. The “putting at rest” with this extended boost makes a particle appear in the direction n_G with a proper time marked by its imaginary clock. As $1/v <$

1, this transformation is physically feasible for an external observer, and the tachyon could be visible. One can notice that the factor i in front of n_G is consistent, as it implies a quadrivector of Minkowskian norm one.

4 Transition from black hole to white hole

The appearance of a state in a zone forbidden by the singularity poses a more delicate problem. Indeed, the Christoffel coefficients involve the parameter

$$r = r_s \left(\mathcal{W}_0 \left(\frac{1}{e} (X^2 - T^2) \right) + 1 \right).$$

This critical zone is defined by $X^2 - T^2 < -1$ which is outside the domain of \mathcal{W}_0 .

The solution proposed here is to use the other part of this function on the real line, namely

$$r = r_s \left(\mathcal{W}_{-1} \left(\frac{1}{e} (X^2 - T^2)^{-1} \right) + 1 \right)$$

by reversing the term $X^2 - T^2$ which enters the domain of \mathcal{W}_{-1} . The trajectory is then continued by changing the signs of T and X , which moves the mobile from the black hole to the white hole. This idea is supported by the hyperbolic character of the Kruskal map.

5 Cost of quantification

The evolution of the trajectory during the time quantum is no longer geodesic, and therefore requires some work. The force that appears during this displacement is given by

$$f^\alpha = x''^\alpha + \Gamma_{\mu\nu}^\alpha x'^\mu x'^\nu$$

and its work on the affine segment $\delta\lambda$ is given by

$$\delta W = \int_{\lambda_0}^{\lambda_1} g_{\mu\nu} f^\mu x'^\nu d\lambda.$$

A quick calculation shows that

$$\begin{aligned} \delta W &= \frac{1}{2} \delta\lambda x'^\mu x'^\nu x'^\rho \int_0^1 \frac{\partial g_{\mu\nu}}{\partial x^\rho} (x + x' \xi \delta\lambda) d\xi \\ &= \delta\lambda x'^\mu x'^\nu x'^\rho \int_0^1 \Gamma_{\rho\mu\nu} (x + x' \xi \delta\lambda) d\xi. \end{aligned}$$

This expression makes it possible to estimate the energy needed to quantify the movement.

6 Refutability of the model

Given a time quantum, one can ask which mass M_0 corresponds to a quantum of action equal to Planck's constant. Thus, $M_0 c^2 \delta\tau = \hbar$.

Clearly, the finer the digitisation, the closer the trajectories to the unquantized geodesics, thus deferring the quantum effects mentioned above.

The smaller the quantum, the later the effect, the longer the calculation time. The calculations carried out here allowed us to aim for a time quantum of approximately 10^{-13} s which corresponds to a mass of 10^{-2} eV/c². For example, reaching the mass of the neutrino, which is currently estimated at 1.1 eV/c², would require a temporal resolution two orders of magnitude lower, resulting in calculation times that are approximately 100 times longer. As the calculations performed here require several days, it is not impossible to think that an optimisation could be achieved up to the level of actually observable particles.

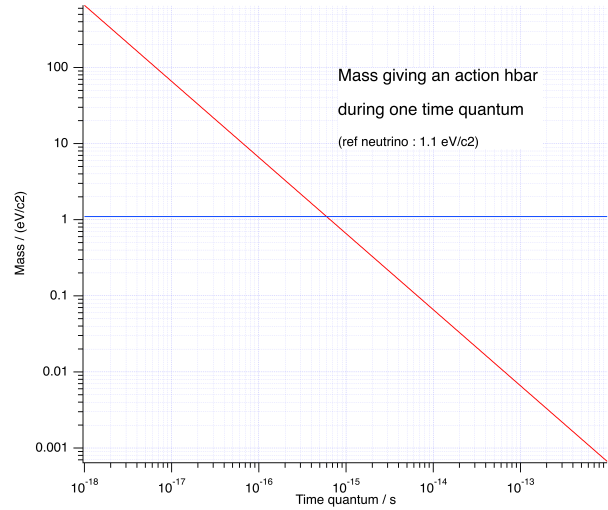


Fig. 4: Mass-time quantum relationship.

7 Two typical trajectories

In general, the trajectories end either with the limiting velocity 1 or at the singularity. Tachyons are short-lived, and return to standard space-time with a final velocity of 1. Two examples are given in Fig. 5 and Fig. 6.

8 Calculation tools

The digital tracking of trajectories requires over several million steps. The standard precision of the current computers (double precision) is 53 bits, which is totally insufficient. The MPFR library [4] implements the calculation with an arbitrary precision, which is only limited by the machine's memory. An interface written by P. Holoborodko [3] then allows the use of the Eigen vector calculation library [6]. The very complete study of F. Johansson [5] on the Lambert function finally makes it possible to carry out the calculation of trajectories, which becomes stable with a precision of 4096 bits (approximately 1200 decimal places).

The exploration of the various trajectories is programmed in C++ and uses a 128-processors machine running in the *Gnu-Linux Ubuntu 20.4* environment.

The trajectories presented here generally require several days of parallel CPU.

9 Analogy with quantum measurement

As we have seen, some of the effects emerging in a time quantum $\delta\tau$ of a relativistic motion are due to the presence of the observer. In summary, the motion naturally follows a geodesic; then during the time of observation, it follows a tangent, and it resumes its natural trajectory, but on another, neighbouring geodesic.

This sequence is similar to the Copenhagen version of quantum measurement, in which two types of evolution co-exist in a quantum system. The first, known as unitary (U-type), is governed by the Schrödinger or Dirac equation. The second, which appears when the system is measured, is called wave packet reduction (type R), and consists of projecting the wave function onto an eigenspace associated with the observable to be measured.

Let \hat{A} be the self-adjoint operator translating an observable. To measure A according to Geneva's school [8], the observer asks a series of questions whose answers are *yes* or *no*. A question about A is for example: "Will the value of A appear in a certain interval Δ of the real line?".

Let $\text{Sp } \hat{A}$ be the spectrum of the operator \hat{A} . This question is represented by the projection operator $J_\Delta = \sum_{a \in \Delta \cap \text{Sp } \hat{A}} J_a^A$ where J_a^A is the projector onto the eigenspace of eigenvalue a . The result of the measurement, i.e. the answer to the question, will be *yes* with probability $p_1 = \langle \psi | J_\Delta | \psi \rangle$ and the system will then be in the state $|1\rangle = J_\Delta \psi / \|J_\Delta \psi\|$. The answer *no* is treated in the same way, but with the projector $J_{\complement\Delta}$ and gives the final state $|0\rangle$.

One can imagine that the measurement lasts for a time interval $\delta\tau$ and, after the response has been randomly chosen, the wave function evolves "linearly" towards its final state.

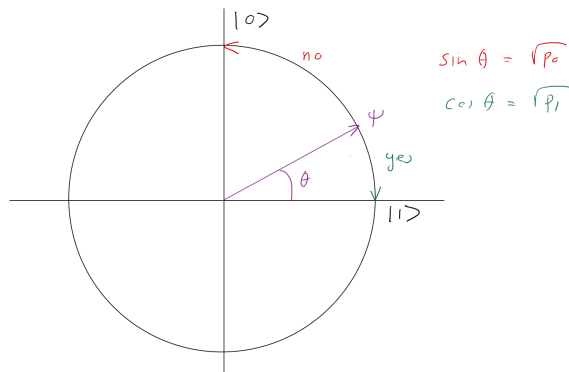


Fig. 7: Evolution of the quantum probability amplitude in R mode.

For example, the path in Fig. 7

$$t \mapsto \psi_t = \cos\left(\theta\left(1 - \frac{t}{\delta\tau}\right)\right)|1\rangle + \sin\left(\theta\left(1 - \frac{t}{\delta\tau}\right)\right)|0\rangle$$

where $\cos \theta = \sqrt{p_1}$, moves in a uniform and unitary manner from ψ to $|1\rangle$ in case of a *yes* answer.

For the Schrödinger equation, this evolution is governed in the $(|1\rangle, |0\rangle)$ basis by the Hamiltonian operator

$$\hat{H}_1 = \frac{\theta\hbar}{\delta\tau} \sigma_2 \quad \theta = \arccos \sqrt{p_1}$$

where σ_2 is the second Pauli matrix. It can be seen that $\langle \hat{H}_1 \rangle = 0$, and that we have

$$\psi_t = \exp \frac{\theta t}{i\delta\tau} \sigma_2 \psi = \exp \frac{\theta t}{\delta\tau} (|1\rangle\langle 0| - |0\rangle\langle 1|) \psi.$$

Initially, the wave function follows a trajectory U given by a Hamiltonian \hat{H} . During the measurement, the reduction R is replaced by a trajectory U with a Hamiltonian proportional to σ_2 . It then resumes the trajectory U given by \hat{H} .

10 From the quantum to the infinitesimal

The infinitesimals of Leibnitz and Newton were only recently given a consistent axiomatic basis. They have been used systematically by mathematicians such as Euler, Lagrange or Wallis with success and without rigorous justification. Physicists use these devices without further ado on a daily basis. The axiomatization of continuity by d'Alembert, Cauchy and Weierstrass almost sounded the death knell of these quantities, as small as one likes, but nonzero.

Nevertheless, they have made a surprising reappearance through topos, equipped with their not necessarily Boolean logic. For smooth infinitesimal analysis, for example, they are defined by the subset of the line $\Delta = \{\varepsilon | \varepsilon^2 = 0\}$ which is no longer reduced to $\{0\}$. One then speaks of nilpotent real numbers. This has the effect of eliminating all powers greater than or equal to 2 in the Taylor developments on this set. In other words, any function becomes linear on Δ or: Δ is a representation of the tangent space in zero which is included in the real line.

The above analysis performs this integration with the idea that the time quantum could ideally be understood from the nilpotents.

The quantum effects in the vicinity of singularities are reminiscent of John Lane Bell's formula:

$$\text{Vale } i\epsilon, \text{ ave } i\epsilon !* [7]$$

as an extension of the discussion of the introduction of imaginary time by Minkowski [1, Box 2.1 p. 51].

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*Goodbye $i\epsilon$, welcome $i\varepsilon$!

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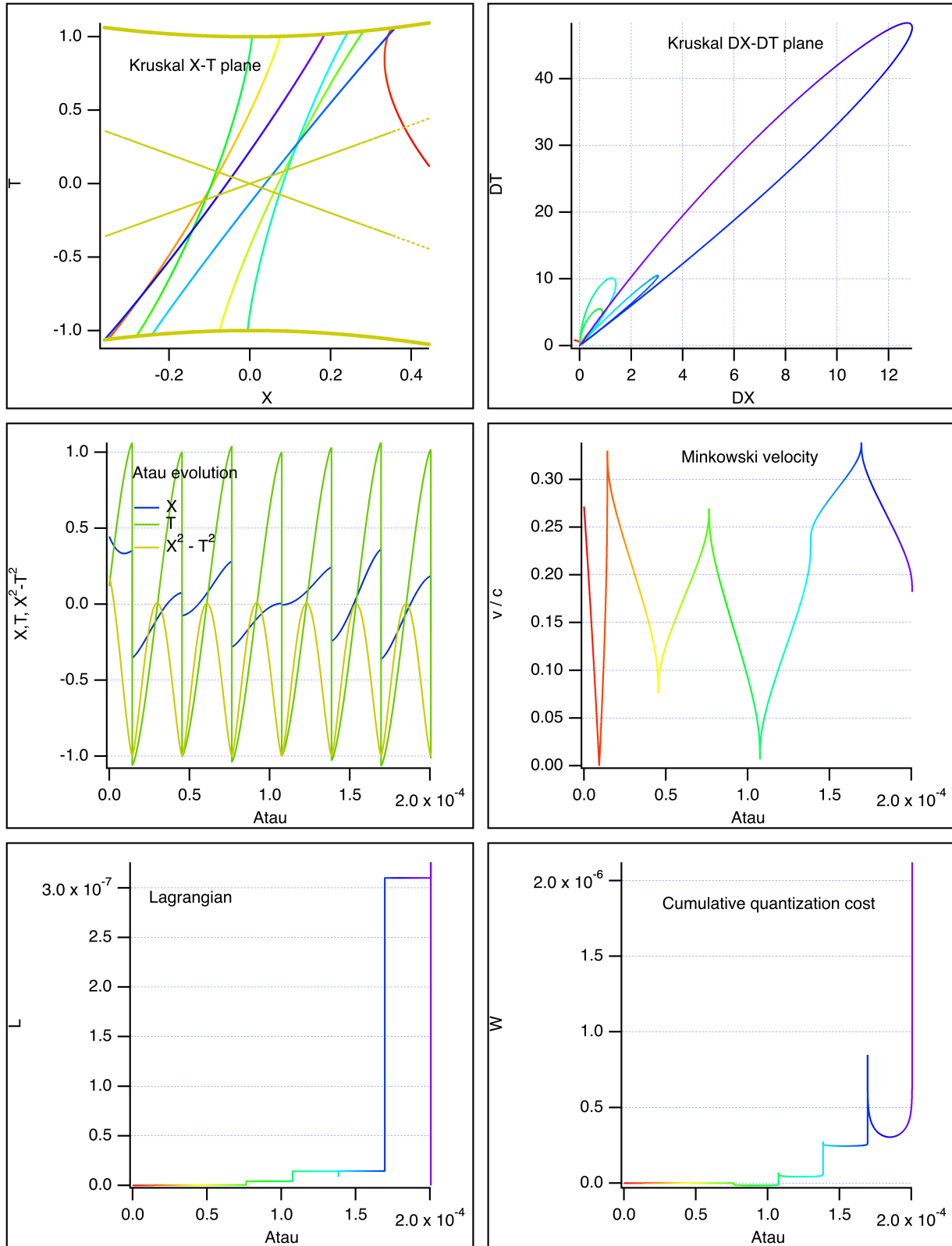


Fig. 5: Trajectory evolving towards the singularity. The variable $Atau$ is simply the addition of the two real and imaginary clocks. The imaginary time is identified by a negative Lagrangian. The start of the trajectory is in red and its end in dark blue. The passages through the singularity are located at the points where $X^2 - T^2 < -1$.

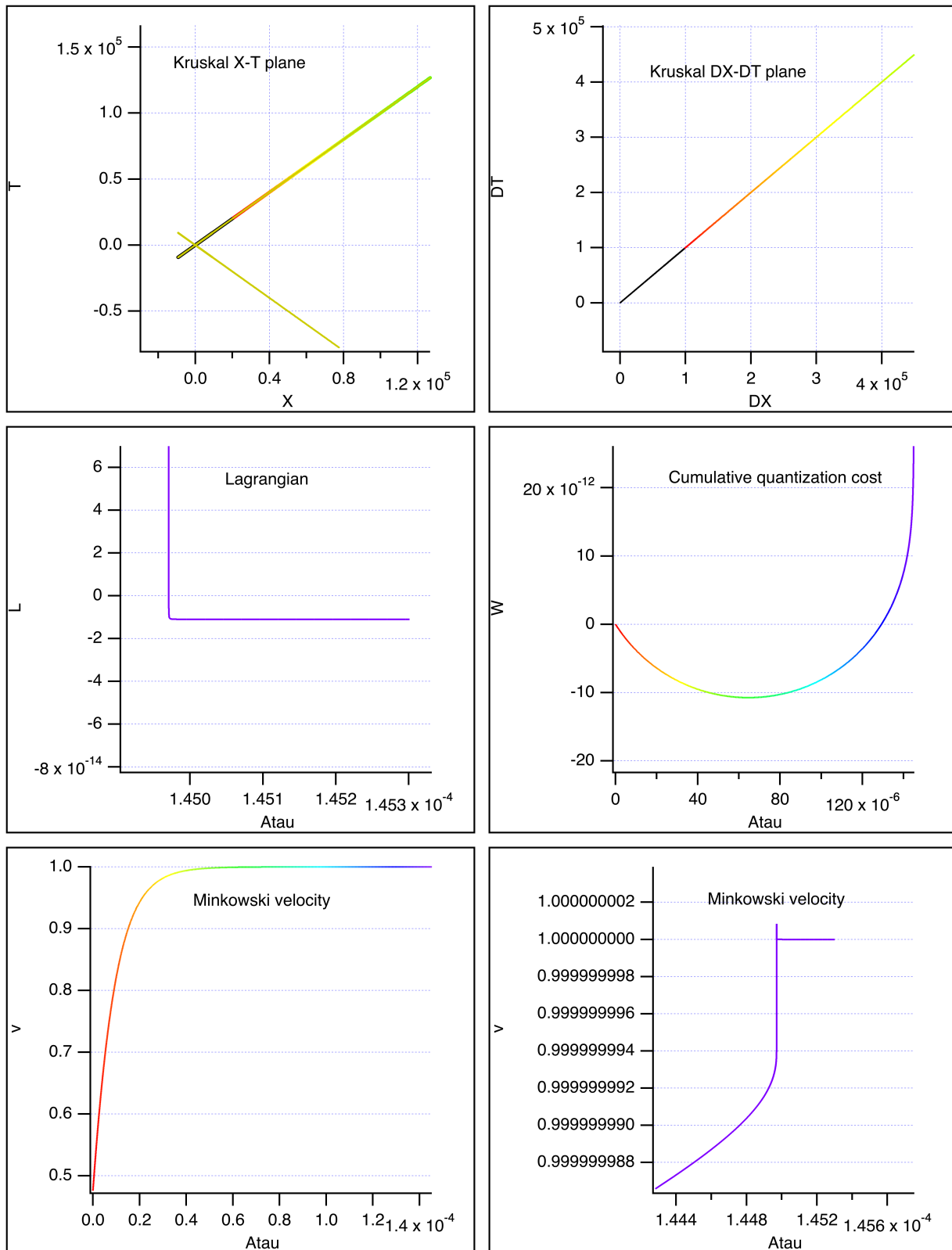


Fig. 6: Trajectory leading to a tachyon, before ending on the singularity. The colouring of the top two graphs is given by the imaginary clock from the black part. The calculation was redone by increasing the precision from 4096 to 8192 bits, with no significant difference.

Combinatorics and Frequency Distributions as the Determining Factors of Electron and Nuclear Spectra

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It has been established that electronic and nuclear spectra can be calculated and formed using combinatorics and frequency distributions (FD) provided that electrons, nucleons and other elementary particles in the composition of an atom are represented as unit structureless elements. The examples given show a good match between the calculated spectra and the experimental ones. The program for calculating spectrograms has been compiled.

1 Introduction

Electronic and nuclear spectra are characterized by a set of emission (absorption) spectral frequency lines arising from the excitation of atoms or by the energy spectrum of split off nucleons in the nuclear decay process. In electronic spectra the spectral lines position for hydrogen and for hydrogen-like atoms is determined by the Balmer-Rydberg formula for the radiation wavelength

$$\lambda_e = \frac{n^2 m^2}{m^2 - n^2} \frac{1}{R_\infty Z^2}, \quad (1)$$

where n and m are the quantum numbers or orbit numbers, R_∞ is the Rydberg constant, Z is the element atomic number.

For other spectral transitions in multielectron atoms the Rydberg formula gives incorrect results, since the internal electrons screening varies, and for external electrons transitions it is not possible to make a similar correction in the formula to compensate for the nuclear charge weakening, as described above. Therefore, in the general case, to find the position of spectral lines, the Ritz combination principle [1], which has become the basis of modern spectroscopy, is used. Its validity has been confirmed by numerous experimental data. But it is not clear what regularities underlie it, what processes exactly exist, and how the atom internal structure is rearranged in order to cause the waves emission with a frequency corresponding to any spectral line.

Nuclear spectra arise when a nucleus is exposed to hard radiation or high-energy electrons. The nucleons split off in this case have the energy of tens of MeV and form the giant dipole resonance (GDR) [2]. The giant resonance is inherent in all nuclei, it has been studied since 1947 and it manifests itself so brightly and universally that, perhaps, not a single nuclear "event" can compare with it. The giant resonance nature is believed to lie in the nucleus dipole oscillations (displacement of all nucleus protons relative to all its neutrons) under the action of long-wavelength γ -radiation. When irradiated with electrons having an energy of more than 200 MeV, along with dipole vibrations, other types of vibrations can also be excited in the nucleus. These vibrations are of a collective nature and form giant multipole resonances (GMRs) [3]. The

photonucleons energy spectra are not described by smooth curves, and when studying the cross sections for the (γ, n) reactions, maxima of the first and subsequent orders are found, forming the GDR structure of three types: rough (gross), intermediate, and fine.

There are several GDR theories, the most detailed being the multiparticle shell model [4]. Its development proceeds through a unified description of various collective motions (rotations, surface oscillations, nucleus dipole oscillations), as well as interactions between them. At present, theories are not yet able to give a good quantitative description of the width and fine structure of the giant resonance and the entire spectrum of nucleons separated during the nuclei decay, since there are large computational difficulties and a lack of reliable information about a number of important parameters of the theory.

2 Initial conditions

In this article, as in previous works, in accordance with the mechanistic interpretation of J. Wheeler's idea, charged particles are considered to be the singular points on the three-dimensional surface of our world (conditionally this is the X-region), connected by vortex tubes (current lines of force) through an additional dimension (conditionally this is the Y-region), which is responsible for the electromagnetic forces and the "hidden" mass of the microparticles [5, 6].

If, as is commonly believed, the microparticles are oscillators, then the atom itself can be considered as a collective oscillator, which consists of the "oscillator-electrons" (X-region) and the "oscillators-protons" (Y-region), and these oscillators are elastically connected to each other by the vortex current tubes. At that, according to [7], the electrons located at the more distant orbits are associated with the protons located at the deeper nucleus levels; thus the layers or envelopes are formed in the nucleus that similarly to the electronic shells.

The multielectron atoms protons number's increasing in proportion to the atomic number Z increases the bonds inflexibility, as if "stretching" the vortex tubes, which reduces the oscillators-electrons wavelength in the X-region in accor-

dance with the formula (1). At the same time, the protons mass's increasing reduces the system as a whole inflexibility, therefore in the same proportion increases the oscillator-protons wavelength in the Y-region.

For multielectronic atoms, the numbers m and n lose their meaning of the electron shell number, and n must be taken equal to Z , since in the limit, when $m \rightarrow \infty$ and $n = Z$, in accordance with (1), $\lambda \rightarrow 1/R_\infty$, and the atom becomes hydrogen-like one. For the radii smaller than $1/R_\infty$, i.e. when there is “sinking” into the Y-region, quantum numbers formally become inverses of n and m , and the formula (1) for oscillator-protons takes the form

$$\lambda_p = \frac{1}{m^2 - n^2} \frac{Z^2}{R_\infty}. \quad (2)$$

The dependence of wavelengths on Z^2 is understandable, since, unlike a simple one-dimensional oscillator, where the oscillation period depends on its inflexibility and on its mass to the power of $1/2$, the atom (taking into account the additional degree of freedom in Y) is a four-dimensional oscillator and $(Z^{1/2})^4 = Z^2$.

3 Formation of the electronic spectra

The spectra revealed in physical experiments is obvious to be as a joint result of the electronic and proton oscillators oscillations superposition; it is clear that in this case, as a result of interference, both damping and amplification of certain frequencies of the spectrum occur. Therefore, to obtain the spectrum, it is necessary to calculate all possible wavelengths of oscillators-electrons according to (1), as well as all possible wavelengths of oscillators-protons according to (2) for all combinations of n and m , and multiply the results logically.

For this purpose, a calculation program has been drawn up (see Appendix). The essence of the program is as follows: to divide a certain spectrum region into intervals, to calculate the *frequency distributions* (FD) of all functions values according to (1) and (2) in the spectrum selected region, write them into the corresponding arrays and multiply these arrays. The type of the spectra obtained by this program depends on the number of values λ_i falling into the i -th interval (i.e., on the parameter q value in the program) and can have the histogram form of different detail (q -large) or the line spectra form (q -small) [8]. Moreover, the type of histograms can reflect some additional spectrum parameters, since the histogram peaks height is proportional to the probability (intensity) of the corresponding spectral parameter along the Y-ordinate.

Fig. 1 shows the experimental spectrum of the holmium liquid filter (240–650 nm) which is a solution of holmium dissolved in perchloric acid for checking the wavelength accuracy [9], and Fig. 2 shows the calculated histogram for ${}_{67}\text{Ho}$. Here and below, the intervals for substituting variables a and b in the calculation program are indicated.

Fig. 3 shows part of the ${}_{80}\text{Hg}$ spectrum as a line spectrum. Above it, the spectral lines experimental values are shown

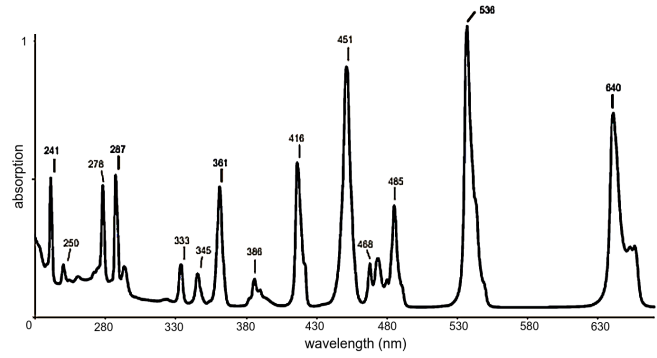


Fig. 1: Typical spectrum of holmium liquid filter (240–650 nm) [9] consists of a solution of holmium dissolved in perchloric acid.

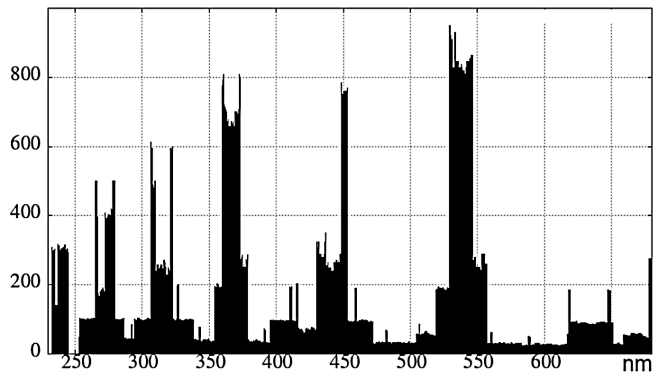


Fig. 2: Calculated spectrum for ${}_{67}\text{Ho}$: $q = 0.026$, $\lambda_i(a, b) = 57\text{--}67$, $\lambda_p(a, b) = 1\text{--}67$.

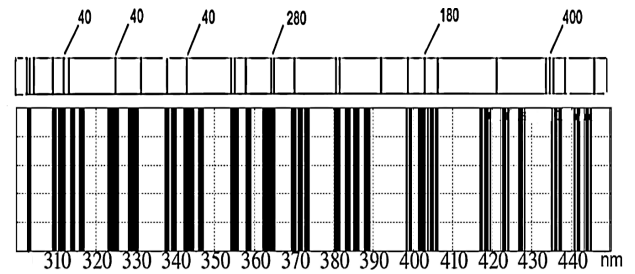


Fig. 3: Calculated spectrum for ${}_{80}\text{Hg}$: $q = 0.0021$, $\lambda_e(a, b) = 50\text{--}80$, $\lambda_p(a, b) = 1\text{--}80$.

and the brightness values of some of them are given. Note, when changing the interval of substitution of variables the histograms shape changes; in the case of a line spectrum it affects the spectral lines intensity, the presence or absence of some lines, but their position in the spectrum does not change.

Obviously, the constructed spectra are in good agreement with the experimental ones. Of course, one cannot expect the calculated spectra to match exactly with the experimental ones, since the latter are influenced by various factors: the methods of excitation of atoms, the degree of their ionization, the presence of forbidden transitions, the medium which the element is located in, etc.

Nevertheless, in a number of cases, for some sets of variables, even the external shape of the calculated non-line spectra (the histograms shape) is very similar to the real non-line spectra (see Figs. 1 and 2), which, apparently, corresponds to certain physical conditions. Thus, limiting the function λ_e to variables within 57–67 means that when calculating the spectrum only the electrons in the outer shells of $_{67}\text{Ho}$ (10 units) are taken into account. Indeed, it is known the inner shells electrons do not take part in the formation of the visible spectrum range for atoms having high numbers Z . For the $_{80}\text{Hg}$ spectrum 30 electrons are taken into account. This turned out to be sufficient to form the spectrum. With a full set of variables, the short-wavelength spectrum part is enhanced and, in general, the spectrum detail is enhanced.

4 Formation of the nuclear spectra

The region of the giant dipole resonance extends within the energy range of tens of MeV, and its shape and structure are extremely diverse. When the nucleus is exposed to gamma radiation or high-energy electrons, there is both protons and neutrons's splitting off. Thus, the nucleus as an oscillator should contain the maximum number of unit elements (oscillators-nucleons) equal to its mass number A . On the other hand, by analogy with oscillators-electrons, one can imagine that there are oscillators-pions or other mesons, which, as expected, exist in the proton close environment in the form of a virtual meson “coat” [10].

So, to build a nuclear spectrum one should use the same formulas (1) and (2), replacing the element number Z with the mass number A , but at the same time, as it were, “going deeper” along the Y-axis, that is, moving to smaller sizes and higher energies. The transition coefficient, as it turned out, is equal to a^3 — the fine structure constant in the cube $(1/137)^3$. Thus, for the nuclear resonance wavelengths, denoting them λ_π and λ_n , we have

$$\lambda_\pi = \frac{a^3 n^2 m^2}{m^2 - n^2} \frac{1}{A^2 R_\infty}, \quad (3)$$

$$\lambda_n = a^3 \frac{1}{m^2 - n^2} \frac{A^2}{R_\infty}. \quad (4)$$

These formulas, passing to the frequencies and further to the energies in MeV, are written as

$$E_\pi = \frac{m^2 - n^2}{n^2 m^2} \frac{A^2 R_\infty c h}{a^3 k}, \quad (5)$$

$$E_n = (m^2 - n^2) \frac{R_\infty c h}{a^3 k A^2}, \quad (6)$$

where c is the light speed, h is the Planck's constant, k is the conversion factor 1.602×10^{-13} [J/MeV]. Calculating the constants, we get

$$E_\pi = 35.02 A^2 \frac{m^2 - n^2}{n^2 m^2}, \quad (7)$$

$$E_n = 35.02 \frac{m^2 - n^2}{A^2}. \quad (8)$$

The general view of the giant resonance for light and heavy nuclei, obtained from the calculation in accordance with (7) and (8), is shown in Fig. 4, which generally agrees with the experimental results. Indeed, in the experiments with irradiation of the nuclei with low mass numbers, even at low resolution, maxima are found in the giant resonance, in contrast to the heavy nuclei, where numerous weak peaks are detected only at high resolution.

The giant resonance has been established to be formed in the heavy nuclei with the participation of nucleons from the two outermost nuclear shells, while the main nucleon core lying under the outer shells is not affected at all by the photo disintegration process. At that, with an increase in the mass number A , the neutron fraction knocked out of the nucleus increases, while the proton fraction decreases, reaching only about 1% in a nucleus with $A \approx 200$ [11].

Therefore, if one takes into account only those nucleons (neutrons) that are not included in clusters and therefore easily splitting off from the nucleus (for Pb^{207} , as indicated in [7], there are 65 units), then when forming a spectrogram for Pb^{207} , one should limit the variables range for E_n within 142–207. In this case, the maximum of the spectrogram shifts to the range of 11–12 MeV and it takes the form close to the Poisson distribution; this is generally match to the experimental data. As in the case of the electronic spectrum, this is, as it were, the formal restriction on the range of variables, coincides with the physical meaning of the phenomenon, otherwise the GDR peak would be shifted towards higher energies.

Fig. 5 shows the experimental cross sections for reactions $\text{Al}^{27}(p, \gamma_0)\text{Si}^{28}$ according to [12], and Fig. 6 shows the calcu-

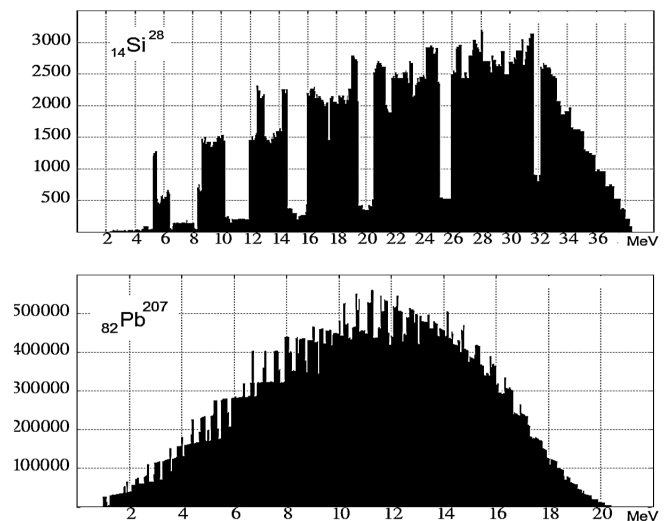


Fig. 4: General view of the GDR for light and heavy nuclei : $q = 0.1$, $A = 28$: $E_\pi, E_n(a, b) = 1-28$, $A = 207$: $E_\pi(a, b) = 1-207$, $E_n(a, b) = 142-207$.

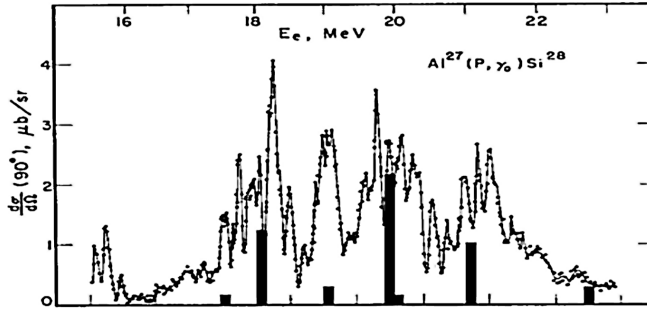


Fig. 5: Reaction cross sections $\text{Al}^{27}(p, \gamma_0)\text{Si}^{28}$ [12] and data of theoretical calculations photodisintegration within the multiparticle shell model.

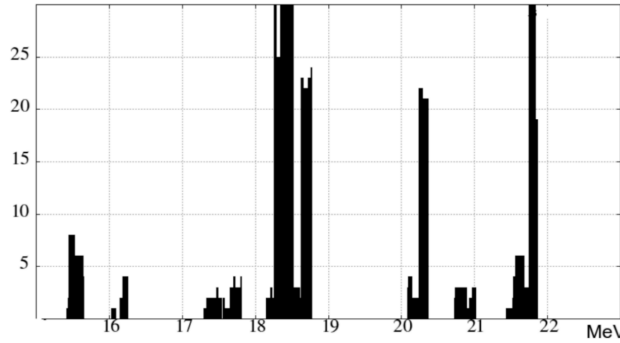


Fig. 6: Calculated spectrum for $\text{Al}^{27}\text{Si}^{28}$: $A = 27.5$, $q = 0.007$, $E_\pi, E_n(a, b) = 1-27$.

lated spectrogram in the histogram form. Obviously, the main peaks of the reaction cross section (p, γ_0) coincide with those in the calculated spectrogram. At the same time, a large number of narrow peaks with a width of 50–100 keV are observed in the reaction cross section against the intermediate structure background (resonances with a width of 0.4–1.0 MeV). The existing theory does not explain the nature of these peaks. But in the calculated spectrograms they are revealed as the parameter q decreases. These coincidences point to the manifestation of combinatorics, to the fact that any maximum is not the result of any particular resonance, but the superposition of many single events.

Fig. 7 shows the experimental spectrum for Ca^{40} [13], and Fig. 8 shows the calculated spectrogram. It also demonstrates good agreement with the experimental data both in terms of the peaks number and the peaks positions for Ca^{40} .

In the process of the studying the atomic nuclei structure by the method of scattering of electrons with energies up to 225 MeV new giant multipole resonances (GMR) were discovered. These resonances go beyond the GDR, which arise during photodisintegration. They have a much more complex structure than that obtained from photonuclear experiments and theoretical predictions. To explain them, quadrupole, octupole, and other types of oscillations was assumed can be excited in the nucleus.

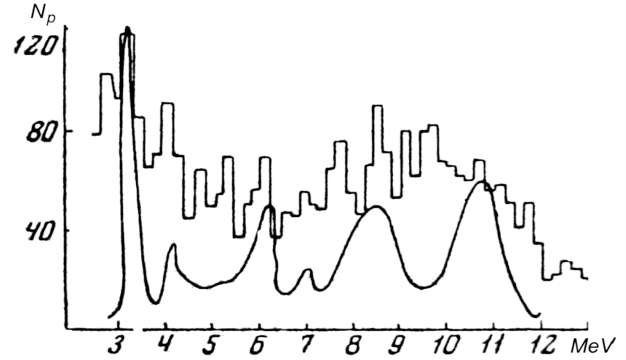


Fig. 7: Spectrum of photoprotons from Ca^{40} upon irradiation with the bremsstrahlung spectrum of γ -quanta with $E_{\gamma \max} = 25$ MeV and calculated spectrum in the shell model (smooth curve).

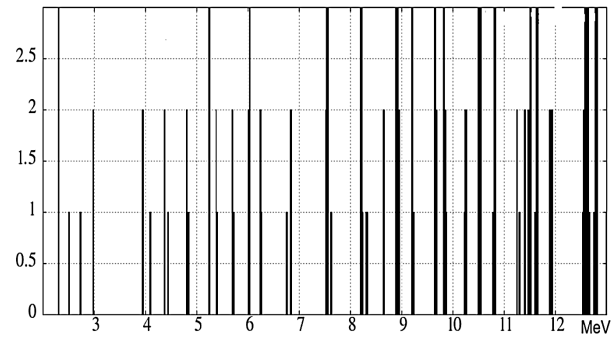


Fig. 8: Calculated spectrum for Ca^{40} : $q = 0.003$, $E_\pi, E_n(a, b) = 1-40$.

The parameters of the giant resonance in Fe^{56} E_{res} , MeV are given in [3, p. 142] (the plus or minus errors in absolute value are shown in parentheses):

| | | | |
|------------|------------|------------|------------|
| 9.5 (0.1) | 10.1 (0.1) | 10.3 (0.3) | 11.3 (0.5) |
| 11.9 (0.9) | 13.0 (0.3) | 13.0 (0.9) | 13.1 (0.1) |
| 14.6 (0.3) | 15.0 (0.4) | 15.6 (1.2) | 16.0 (0.2) |
| 16.1 (0.5) | 16.3 (0.1) | 16.9 (0.1) | 17.3 (0.1) |
| 17.9 (0.2) | 18.2 (0.1) | 18.3 (0.1) | 19.0 (0.5) |
| 19.8 (0.3) | 23.9 (0.3) | | |

On Fig. 9 the calculated spectrogram for Fe^{56} is shown. It can be seen that almost all of the above energy values, within the limits of errors, coincide with the peaks of the first (most) and second orders in the calculated spectrogram.

On Fig. 10 a part of the spectrogram for Fe^{54} in the line spectrum form is shown. Here is a good agreement with experimental data [3, p. 149] too:

Of course, as in the case of electronic spectra, the calculated nuclear spectrograms cannot completely coincide with the real ones, because in addition, there is an incomplete agreement between the data of different experiments. The

| | | | | |
|------------|------------|------------|------------|------------|
| 9.7 (0.1) | 12.4 (0.5) | 12.6 (0.4) | 13.4 (0.2) | 13.8 (0.2) |
| 15.0 (0.9) | 15.0 (1.3) | 17.5 (0.2) | 17.9 (0.2) | 19.2 (0.1) |
| 20.2 (0.1) | 20.3 (0.1) | 23.9 (0.3) | 25.4 (0.4) | |

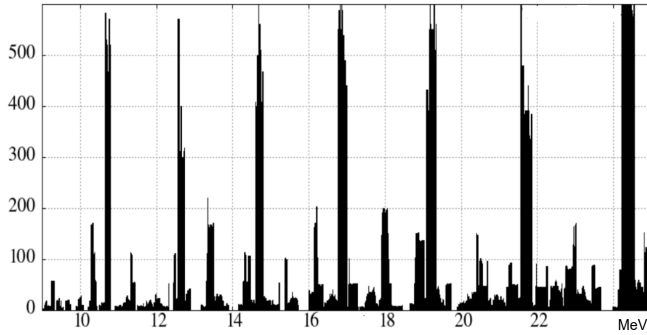


Fig. 9: Calculated spectrum for Fe^{56} : $q = 0.005$, $E_\pi, E_n(a, b) = 1-56$.

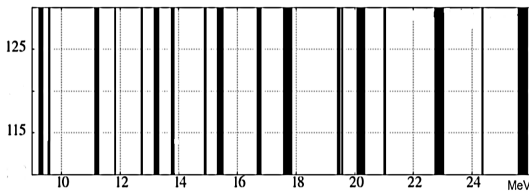


Fig. 10: Calculated spectrum for Fe^{54} : $q = 0.007$, $E_\pi, E_n(a, b) = 1-54$.

instrumental functions of the experiments have very complex shapes, so that the determined cross sections differ in all the main parameters (shape, size, and energy position). It should also be noted that in the above method for calculating spectrograms, the mass number A is the sum of neutrons and protons, and the spectrograms do not differ for nuclei with the same A . The accuracy of the calculated spectra can be improved by introducing additional restrictions or additions to the set of variables.

Nevertheless, the obtained results show this method of analysis can be used as an addition to the instrumental spectrography methods, since it makes it possible to quickly, almost instantly find the statistically most probable form of electronic and nuclear spectra.

5 Conclusions and generalization

The main conclusion from the foregoing is not so much the fact of the emergence of a new analytical method, but that it is possible to obtain results close to reality by considering the complex structure of electronic and nuclear shells as a set of structureless single uniform elements. This contradicts the quantum provisions, according to which elementary particles differ in a set of quantum numbers. On the other hand, each element acquires individuality, since it (and all of them at the same time) moves in the space of variables $n_i - m_i$, and any movement of any element is accompanied by the release of an individual portion of energy E_i , which, being mutually superimposed, eventually form the spectrum. But this again contradicts the quantum principle, this time it contradicts the

principle of indistinguishability of identical particles.

Applying a FD to an array of values of the functions E_π and E_n , i.e. to just a set of numbers, gives physically reliable results, but this fact should not be surprising. So in the work of S. E. Shnoll [13], when processing the FD of the experimental data array of the various physical processes, obtained initially in the normal distributions form, these distributions was found are discrete and depend on the algorithms that determine these processes.

The fact that simple formulas for the i -th wavelength or the energy value give results, otherwise obtained through the laborious experiments and complex calculations, leads to the question — do dipole and other resonances affect the nuclear spectra and do they exist in the nucleus at all? Is it really necessary to calculate the nuclear spectra consider nuclei and their components to be the sources of oscillations, or is a set of statistical methods sufficient? This question can only be answered by further wide application of the method described both to the electronic and nuclear spectra and to other physical phenomena in those cases, where it is possible to apply the FD to the functions describing these phenomena.

But now, summing up the above, we can conclude:

- the electronic spectra are reproduced at a deeper level of matter in the nuclear spectra form,
- the type of spectrograms is mainly determined by combinatorics and the frequency distributions of elementary particles, considered as structureless unit elements in the range of their atomic numbers or their mass numbers.

Appendix

A C++ program is written to calculate the wavelength in nanometers. When calculating nuclear spectra, the atomic number Z is replaced by the mass number A , and M is a dimensional coefficient.

```
#include <iomanip>
#include <stdlib.h>
#include <algorithm>
#include <stdio.h>

using namespace std;
struct preobr: binary_function<double, double, double> {
    double operator()(double x, double y) const {return x*y;} };

float R, M;
float f1(float x, float y, float z) { return M*(x*x*y*y)
/(y*y - x*x)/z/R ;};
float f2(float x, float y, float z) { return M*z*z/(y*y - x*x)/R ;};

FILE *fp = fopen("uuu", "w");

int main() {
```



```

int n=1000000; float*m1 = new float[n]; float*my1 = new float[n];
float*my2 = new float[n]; float*my12 = new float[n];

for (int c=0; c<n; c++) m1[c]=my1[c]=my2[c]=my12[c]= 0;

float z=80, xn=300, xm=500, q=0.002, t=0, c=0; int j=0;
R=1.0974e+7, M=1e+9;
// xn and xm — range limits

for(t=xn; t<xm; t=t+0.001*t) {
j++, m1[j] = t;
// dividing the range into segments
// proportional to its current value

for(int a=1; a<=80; a++) {
for(int b=1; b<=80; b++) {

if(b>a) c = f1(a,b,z);

if(fabs(m1[j] - c) < fabs(q*c)) my1[j]++;
// recording the number of values in intervals for f1
}
}
for(int a=1; a<=80; a++) {
for(int b=1; b<=80; b++) {

if(b>a) c = f2(a,b,z);

if(fabs(m1[j] - c) < fabs(q*c)) my2[j]++;
// recording the number of values in intervals for f2
}
}
}
transform(my1, my1+j, my2, my12, preobr());

for(int i=0; i<j; i++) {
if(m1[i]!=0)

fprintf(fp, "%20f %20f\n", m1[i], my12[i]);
// writing results to the file "uuu"

std::cout << m1[i] <<" " << my12[i] << std::endl;
// outputting results to the terminal

}
}

```

Submitted on February 18, 2022

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Apparent Unsettled Value of the Recently Measured Muon Magnetic Moment

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In this paper, we point out that the anomalous value of the muon magnetic moment recently measured at FERMILAB appears to be unsettled due to the experimentally unresolved behavior of the mean life of muons with speed caused by non-local internal effects as well as the irreversibility of the muon decay.

1 Introduction

In the preceding paper [1], we outlined:

1) Historical and recent verifications of the Einstein-Podolsky-Rosen argument that "*Quantum mechanics is not a complete theory*" (EPR argument) [2];

2) Mathematical, theoretical and experimental foundations of the completion of quantum mechanics into *hadronic mechanics* (hm) for the representation of the extended, thus deformable, and hyperdense hadrons which representation is achieved via the symmetries and physical laws of the completed invariant

$$\hat{x}^2 = \frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}, \quad (1)$$

where $n_k^2 > 0$, $k = 1, 2, 3$ represents the dimension and shape of hadrons normalized to the values $n_k^2 = 1$ for the perfect sphere, and $n_4^2 > 0$ represents the density of hadrons normalized to the value $n_4^2 = 1$ for the vacuum;

3) The representation of *all* characteristics of the muons, including the recently measured difference between the experimental value of the *muon* g-factor, g_μ^{EXP} , and its prediction via quantum electrodynamics, g_μ^{QED} ,

$$\begin{aligned} g_\mu^{EXP} - g_\mu^{QED} &= \\ &= 2.00233184122 - 2.00233183620 \\ &= 0.00000000502 > 0, \end{aligned} \quad (2)$$

which representation is achieved via the hadronic structure model of the muons

$$\mu^\pm = (e_\downarrow^-, e_\uparrow^+, e_\downarrow^+)_{hm}, \quad (3)$$

with physical constituents produced free in the spontaneous decay with the lowest mode, $\mu^\pm \rightarrow e^- + e^\pm + e^+$, $\%10^{-12}$, while the presence of an electron-positron pair in the muon structure, which is confirmed by the additional spontaneous decay $\mu^\pm \rightarrow e^\pm + 2\gamma$, $\%10^{-11}$, allows the understanding of the instability of the muons as well as a numeric representation of its mean life. In particular, thanks to the use of hadronic mechanics, [1] has achieved the following numeric values of the n -characteristic quantities of the muons

$$n_1^2 = n_2^2 \approx 0.4926, \quad n_3^2 \approx 0.0149, \quad n_4^2 \approx 0.0149, \quad (4)$$

with the following EPR completion of the muon g-factor

$$\hat{g}_\mu^{EXP} = \frac{n_4}{n_3} g_\mu^{QED}, \quad \frac{n_4}{n_3} = 1.00000000502. \quad (5)$$

In this paper, we use the preceding results to indicate that, despite the accuracy of measurements [6], the anomalous magnetic moment of the muons appears to remain unsettled due to deviations from the relativistic behavior of mean lives of unstable hadrons with speed that are predicted by internal non-local effects, the time irreversibility of spontaneous decays, and other aspects.

2 Apparent unsettled aspects in the muon magnetic moment

To implement due scientific process on anomalous values (2), we should recall P. A. M. Dirac's [7] and other authoritative doubts on the final character of the numeric values obtained from quantum electrodynamics due to the divergence of Feynman's and other series (see [8] for a recent account on QED divergences).

Additionally, measurements [6] have been done via the assumption that the mean life of muons behaves with speed according to the time dilation law of special relativity

$$t = t_o \sqrt{1 - \frac{v^2}{c^2}}. \quad (6)$$

The exact validity of the above law for electrons and other *point-like* particles in vacuum can be considered, nowadays, to be beyond scientific doubt.

However, at this writing there exist unresolved aspects in regard to the experimental behavior of law (6) for the behavior of the mean life with speed (or, equivalently, with energy) of *unstable, thus composite particles*.

In 1965, D. I. Blokhintsev [9] pointed out the expected inapplicability (rather than the violation) of special relativity for the interior of hadrons due to non-local effects in their hyperdense structure and suggested that deviations due to internal effects could be measured in the outside via deviations from time dilation law (6).

In 1983, R. M. Santilli [10, 11] (see also notes [12] from lectures delivered in 1991 by Santilli at the ICTP, Trieste, Italy, and Section 8 of the recent update [4]) showed that the

axioms of special relativity remain valid for time reversible processes of extended particles with invariant (1) when realized via the use of isomathematics with axiom-preserving EPR completion of law (6)

$$t = t_o \sqrt{1 - \frac{v^2/n_3^2}{c^2/n_4^2}}. \quad (7)$$

Additionally, Santilli [13, 14] pointed the inapplicability (rather than the violation) of special relativity and relativistic quantum mechanics for time irreversible processes such as the spontaneous decays, due to the known reversibility of said theories. Its origin was identified in the invariance of Lie's theory under anti-Hermiticity, by therefore suggesting the completion of Lie and Lie-isotopic methods into the broader Lie-admissible methods [16, 17] (see [3] for detailed treatments and [4] for a recent update).

These studies triggered a number of generalizations of time dilation law (6), such as those by L. B. Redei [18], D. Y. Kim [19] and others.

In 1989, A. K. Aringazin [20] proved that all preceding generalizations of law (6) are particular cases of the isotopic law (7) because they can be obtained via different expansions of the latter law in terms of different parameters and with different truncation, thus restricting the experiments to the test of law (7).

In 1983, S. H. Aronson *et al* [21] reported the outcome of experiments conducted at FERMILAB showing apparent *deviations* from law (6) for the $K^0 - \bar{K}^0$ system in the energy range from 0 to 100 GeV.

In 1987, N. Grossman *et al* [22] reported counter-experiments also conducted at FERMILAB showing an apparent *confirmation* of law (6), but in the *different energy range* from 100 to 250 GeV.

In 1992, F. Cardone *et al* [23] indicated that counter-measurements [22] from 100 to 350 GeV leave basically unresolved the deviations of law (6) from 0 to 100 GeV [21], and that the isotopic law (7) provides an exact fit for both measurements [21, 22] (Fig. 1).

Finally, in 1998, Yu. Arestov *et al* [24] pointed out apparent flaws in the theoretical elaboration of the experimental data of measurements [22].

3 Concluding remarks

The above results appear to confirm the lack of exact character of time evolution law (6) for the behavior of the mean life of unstable particles with speed. In fact, under the assumption in first approximation that the muon spontaneous decay is time-reversible, isotopic time dilation law (7) with values (5) predicts the *increase* of anomalous value (2)

$$t = t_o \sqrt{1 - 1.00000000502 \frac{v^2}{c^2}}, \quad (8)$$

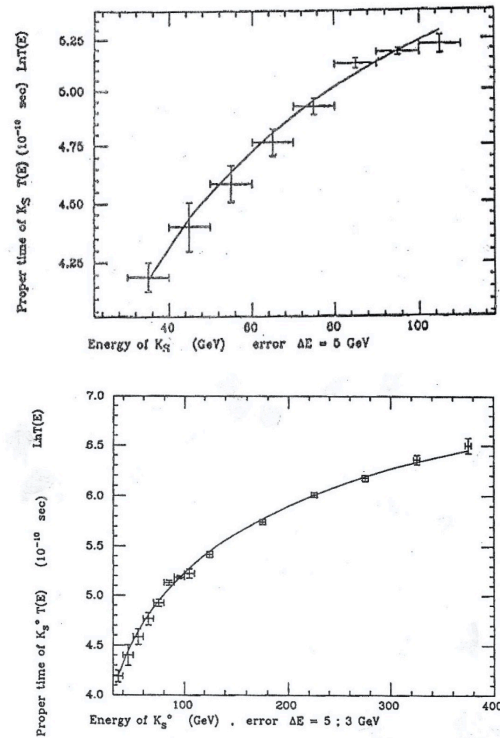


Fig. 1: In this figure, we reproduce the exact fits of isotopic time dilation law (7) obtained by F. Cardone *et al* [23] of: 1) Deviations [21] from the time dilation law (6) for the behavior of the $K^0 - \bar{K}^0$ -system from 0 to 100 GeV (top view); 2) The exact fit of both deviations 0 to 100 GeV [21] and apparent verification in the range from 100 to 350 GeV [22] (bottom view).

with the expectation of bigger deviations for a full time irreversible treatment.

In conclusion, it seems plausible to expect that, in the event deviations [21] from time dilation (6) are confirmed, experimental value (2) of the muon magnetic moment should be correspondently revised.

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Are Tensorial Affinities Possible?

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This short paper is an extraction from our previous work [1], the purpose of which is to make clear that it is very much possible to use Weyl's idea [2] of a conformal metric to achieve tensorial affinities. We are of the strong view that this is very important as it is predominantly assumed that this is not possible. We want to dispel this myth once and for all.

Nature is pleased with simplicity.

Sir Isaac Newton (1642-1727)

1 Introduction

The purpose of this article is to present in a much simpler and succinct form, the ideas presented in our first installation [1] on an attempt to bring the gravitational force and the other forces of Nature (the Electromagnetic, Weak and the Strong Nuclear force) into unity with all the other forces and, as well, unity of all the forces with Quantum Mechanics. For clarity's sake, we have herein removed most of the intricate mathematical and philosophical details found in [1]. We hope this abridged version will make clear to our readers what it is we have done in paper [1].

Further, the purpose and motivation of the present paper has been propelled by one of our favourite Weylian blogger and American physicist – Dr. William O. Straub. He posted on his blog-site* on 28 February 2022 an interesting article entitled: *I'm Still Rooting for the Underdog*. In his article, Dr. Straub expresses his justified frustration on the lack of progress in the search for darkmatter and wonders if it is not time for physicists to abandon this idea/concept and seriously consider much more seriously already existing alternative theories to darkmatter – e.g. Milgrom's *Modified Newtonian Gravity* (MoND) [3–5]. Dr. Straub's frustration is not his alone, it is shared by a plethora of physicists.

To prepare his reader(s) for the conclusion that he seeks, in the introduction of his article, Dr. Straub talks of *perpetual motion machines* and the *luminiferous aether* – i.e. concepts that were once thought to have a direct relation with reality but were eventually found to be worthless/non-physical and were thus abandoned by mainstream science and these ideas are not expected to re-appear anytime soon in mainstream science.

Amongst the many alternative ideas to darkmatter, Dr. Straub considers the subtle flaws in Einstein [6]'s General Theory of Relativity (GTR) and wonders if Weyl's [2] supposed failed unified theory of gravitation and electromagnet-

ism holds any hope as an alternative theory to darkmatter. In the penultimate of his article: of Weyl's [2] theory, Dr. Straub had this to say:

To me, there is one glaring flaw in Einstein's theory, which is its noninvariance with respect to conformal transformations. Weyl also saw this as a flaw, and he showed us a possible way to fix it.

After reading Dr. Straub's article on the morning of 1 March 2022, I was particularly struck by the first sentence in his statement. I immediately wrote to him saying

I must say, I hold the same view and like Einstein [7, 8], Schrödinger [9–11], *etc*, I believe this requires that the affinities be tensors. I have worked out a new theory that is just that – I am sure I have sent this to you before.

Rather swiftly, Dr. Straub responded to my email by saying: *... Turning the connections into true tensors will be a tough job, and I'm inclined to believe it can't be done*. His response challenged me to write a much simpler version of the idea that I used in [1], i.e. the idea of obtaining tensorial affinities. This is what we present below and I hope it is much clearer than it is presented in [1].

2 Riemann geometry

From a viewpoint of geometry, Einstein [6]'s greatest and most beautiful masterpiece, the GTR, has its rock solid foundations anchored in Riemann Geometry[†] (RG). Fundamental in RG are the affine connections (Christoffel three symbols), namely:

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\delta\lambda} (g_{\delta\mu,\nu} + g_{\nu\delta,\mu} - g_{\mu\nu,\delta}). \quad (1)$$

Their topological defect, insofar as the GTR is concerned, is that these affine connections are not tensors, as they transform in the following manner:

$$\Gamma_{\mu'\nu'}^{\lambda'} = \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \Gamma_{\mu\nu}^{\lambda} + \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\mu'} \partial x^{\nu'}}. \quad (2)$$

[†]I shall assume that the reader(s) knows very well Riemann geometry together with its symbols as commonly presented in the textbooks. Hence, we will not explain these but assume the reader(s) is/are *in sync* with us.

*<http://www.weylmann.com/aftermath.shtml>, visited on 5 Mar. 2022 @ 16h18 GMT+2

The first term on the right hand side of (2) has the characteristic transformational properties of a tensor while the second term destroy the *to-be* tensorial character of the affine. If this second term on the right hand side of (2) was not present, the affine would surely be a tensor. These affine connections present a problem when it comes to the geodesic equation of motion, namely:

$$\frac{d^2 x^\lambda}{ds^2} - \Gamma_{\mu\nu}^\lambda \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0. \quad (3)$$

Because of the nature of the non-tensorial affine connection $\Gamma_{\mu\nu}^\lambda$, this geodesic (3) of motion does not holdfast – in the truest sense – to the depth of the letter and essence of the philosophy deeply espoused and embodied in Einstein's *Principle of Relativity* (PoE) [12], namely that physical laws must require no special set of coordinates where there are to be formulated.

The non-tensorial nature of the affine connections require that the equation of motion must first be formulated in special kind of coordinate systems known as a *geodesic coordinate systems**, yet the PoE forbids this. This problem has never been adequately addressed in the GTR. In order to appreciate that this indeed is a real problem, one can for example consider the fact that affinities in the GTR represent forces. A force has no relative sense of existence either by way of a coordinate transformation or a transformation between reference systems – yet, the affine connection speaks to the construction of this seemingly non-physical scenario.

That is to say: if a force exists (i.e. $\Gamma_{\mu\nu}^\lambda \neq 0$) in one coordinate system, it must exist in any arbitrary coordinate system (i.e. $\Gamma_{\mu'\nu'}^\lambda \neq 0$). This surely is not the case if these affinities are to transform as spelt out in (2), because you can have $\Gamma_{\mu\nu}^\lambda = 0$ and $\Gamma_{\mu'\nu'}^\lambda \neq 0$. Against all that is expected from physical and natural reality as we have come to experience it, this literally means a force has a relative sense of existence where it can be made to come into or out of existence by a mere change of the system of coordinates. If anything, coordinates are no more than a convenient way which we use to uniquely label points in space and this should not, in any way imaginable, have any physical effect whatsoever on the resultant physics thereof.

3 Weyl (1918)'s theory

In the first such attempt to bring gravitation and electromagnetism under one mathematical scheme, in which effort one obviously hopes for a unification of these two forces in the resulting theory, Weyl [2] realised that he could forge such a scheme if he were to supplement the metric $g_{\mu\nu}$ of Riemann

geometry with a scalar function ϕ as follows:

$$\bar{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}. \quad (4)$$

The resulting affine connections from this modified Riemann metric (4) are:

$$\bar{\Gamma}_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda + W_{\mu\nu}^\lambda, \quad (5)$$

where:

$$W_{\mu\nu}^\lambda = g_{\mu}^\lambda \partial_\nu \phi + g_{\nu}^\lambda \partial_\mu \phi - g_{\mu\nu} \partial^\lambda \phi, \quad (6)$$

is the tensorial Weyl connection which results from Weyl's supplemented scalar function ϕ . Insofar as its transformation between coordinates is concerned, this new tensorial affine connection of the modified Riemann geometry (hereafter, *Weyl Geometry* (WG)) is no different from the affine connection of Riemann geometry as it transforms as follows:

$$\bar{\Gamma}_{\mu'\nu'}^\lambda = \frac{\partial x^\lambda}{\partial x^{\lambda'}} \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} \bar{\Gamma}_{\mu\nu}^\lambda + \frac{\partial x^\lambda}{\partial x^{\lambda'}} \frac{\partial^2 x^\lambda}{\partial x^{\mu'} \partial x^{\nu'}}. \quad (7)$$

So, from a viewpoint of topology, WG is the same as RG.

Now, if this Weyl scalar is chosen such that:

$$\phi = \kappa_0 \int A_\alpha dx^\alpha, \quad (8)$$

then the tensorial Weyl connection becomes:

$$W_{\mu\nu}^\lambda = \delta_\mu^\lambda A_\nu + \delta_\nu^\lambda A_\mu - \delta_{\mu\nu} A^\lambda, \quad (9)$$

where A_μ , is (here) a (dimensionless) four-vector and the δ 's are the usual Kronecker delta functions, and κ_0 is a constant with the dimensions of inverse length and this constant has been introduced for the purposes of dimensional consistency, since we here assume that the four-vector A_μ and the Weyl scalar ϕ are dimensionless physical quantities.

The versatile and agile Weyl [2] was quick to note that this new Christoffel-Weyl affine (5) is invariant under the following rescaling of the metric $g_{\mu\nu}$ and the four vector A_μ :

$$\left. \begin{array}{l} g_{\mu\nu} \mapsto e^{2\Phi} g_{\mu\nu} \\ A_\mu \mapsto A_\mu + \kappa_0^{-1} \partial_\mu \Phi \end{array} \right\} \Rightarrow \bar{\Gamma}_{\mu\nu}^\lambda \mapsto \bar{\Gamma}_{\mu\nu}^\lambda, \quad (10)$$

where $\Phi = \Phi(r, t)$ is a well-behaved, arbitrary, smooth, differentiable, integrable and uniform continuous scalar function.

Now, because Maxwell [13]'s electromagnetic theory is invariant under the same gauge transformation which the four-vector A_μ has been subjected to in (10), the great mind of Weyl seized this beautiful golden moment and identified this four-vector A_μ with the electromagnetic four-vector potential. Weyl went on to assume that the resulting theory was a unified field theory of gravitation and Maxwellian electrodynamics. Weyl's hopes were monumentally dashed, first starting with Einstein's lethal critique of the theory. Later, others joined Einstein in their critique and dismissal of Weyl's theory, where they argued that despite its irresistible grandeur and exquisite beauty, Weyl's theory can not possibly describe the measured reality of the order of the present world.

*A geodesic coordinate system is one in which the Christoffel three symbols ($\Gamma_{\mu\nu}^\lambda$) vanish at all points on the given set of coordinates – i.e. $\Gamma_{\mu\nu}^\lambda = 0$. An example is the flat rectangular (x, y, z) system of coordinates. However, when one moves from this (x, y, z) rectangular system of coordinates to, say, the spherical (r, θ, φ) , the resulting affine ($\Gamma_{\mu'\nu'}^\lambda$) is not zero – i.e. $\Gamma_{\mu'\nu'}^\lambda \neq 0$.

4 Modified Weyl theory

Now, following for example Einstein [7, 8], Eddington [14] and Schrödinger [9–11], we strongly felt that the idea of tensorial affinities is the only way to solve the aforementioned topological issues with RG and at the same time, we felt that the beautiful introduction of the four-vector into the framework of RG in WG needed to be preserved at all cost. To us, this meant modifying WG in such a manner that tensorial affinities are attained. For this, we imagined the metric of WG being modified such that it is now given by:

$$\bar{g}_{\mu\nu} = e^{2\chi} g_{\mu\nu}, \quad (11)$$

where, unlike in WG, the function: χ is no longer a scalar, but a pseudo-scalar so designed that the resulting affinities of this new geometry are true tensors.

The new metric given in (11) leads to the following affine connection:

$$\bar{\Gamma}_{\mu\nu}^{\lambda} = \Gamma_{\mu\nu}^{\lambda} + Q_{\mu\nu}^{\lambda}, \quad (12)$$

where:

$$Q_{\mu\nu}^{\lambda} = g_{\mu}^{\lambda} \partial_{\nu} \chi + g_{\nu}^{\lambda} \partial_{\mu} \chi - g_{\mu\nu} \partial^{\lambda} \chi \quad (13)$$

is a new affine connection that transforms as follows:

$$Q_{\mu'\nu'}^{\lambda'} = \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} Q_{\mu\nu}^{\lambda} - \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\mu'} \partial x^{\nu'}}. \quad (14)$$

Because of the transformational properties of the new Q -affine as spelt out in (14) above, the resultant affine $\bar{\Gamma}_{\mu\nu}^{\lambda}$ in (12) is a tensor. In order for the Q -affine to transform as desired in (14), the χ -function must transform as follows:

$$\chi' = \chi - \frac{\partial x^{\lambda}}{\partial x^{\lambda'}}. \quad (15)$$

Further, in order for the χ -function to transform as desired in (15), this function ought to be defined as follows:

$$\chi = \ln \Omega, \quad (16)$$

where the Ω -function transforms as follows:

$$\Omega' = \Omega \exp\left(-\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\right). \quad (17)$$

In this way, tensorial affinities are indeed possible.

5 Unified Field Theory

With the nagging topological defect of RG and WG now out of the way, i.e. the problem of non-tensorial affinities, we realised in [1] that Weyl [2]'s idea can be brought back to life. Instead of just supplementing the Riemann metric with the Weyl-scalar, we have to supplement it with both the Weyl-scalar ϕ and the new χ -function as follows:

$$\bar{g}_{\mu\nu} = e^{2(\phi+\chi)} g_{\mu\nu}. \quad (18)$$

This leads to the affine of the emergent geometry now being defined as follows:

$$\bar{\Gamma}_{\mu\nu}^{\lambda} = \Gamma_{\mu\nu}^{\lambda} + W_{\mu\nu}^{\lambda} + Q_{\mu\nu}^{\lambda}. \quad (19)$$

Just like the affine in the previous section defined in (12), this new affine (19) is also a tensor. From this, one can construct a unified field theory of their choice by identifying the Weyl tensor with a field of their choice. Since all our theories are designed in order to model physical and natural reality, the choice one will have to seek is obviously that which can explain physical and natural reality as we experience it and have come to know it. Our work presented in [1] makes a temerarious endeavour to that end.

6 General discussion

Without an iota of doubt, we certainly have demonstrated or shown that it is very much possible to attain tensorial affinities by simple redefining Weyl [2]'s scalar so that it is a pseudo-scalar that is, for better or for worse, forced to yield for us the desired tensorial affinities. In closing, we certainly must hasten to say that our foisting of this pseudo-scalar to yield the desired tensorial affinities has been done well within the permissible and legal confines, domains and provinces of physics, mathematics and philosophy.

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Conditions for the Riemannian Description of Maxwell's Source-Free Field Equations

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In this paper, away from the intricate mathematics and philosophy presented in our earlier work [1], we demonstrate that well within Riemann geometry, Maxwell's electrodynamic source-free field equations [2] are indeed susceptible to a geometric description by the metric tensor, provided: (1) the non-linear term of the Riemann curvature tensor is assumed to vanish identically, and (2) the electromagnetic four-vector field A_μ obeys the gauge condition $A^\alpha \partial_\alpha A_\mu$. We strongly believe that this demonstration is important for physics because if the electromagnetic force can be given a geometric description, this most certainly will lead to the opening of new pathways for incorporating the gravitational force into such a scheme.

Truth is ever to be found in simplicity, and not in the multiplicity and confusion of things.

Sir Isaac Newton (1642-1727)

1 Introduction

As far as prevailing wisdom is concerned, there is only one *Force of Nature* that is described geometrically and this is the force of gravity and its geometric description was handed down to us by Albert Einstein [3] in his intellectual masterpiece – the *General Theory of Relativity* (GTR). By geometric description, we here mean the ability of the force in question to submit to a metric description in a manner redolent or akin to the force of gravity in Einstein [3]'s GTR, where the gravitational force is described by the metric tensor $g_{\mu\nu}$. In turn, the metric tensor $g_{\mu\nu}$ evolves and is governed by the laws governing Riemann Geometry (RG).

Given our opening statement, the question naturally suggests itself: *Can Maxwell's electromagnetic force be given a geometric description?* Our answer to this question is that with the *proviso* that the:

1. Riemann curvature tensor is linearized, i.e. $\Gamma_{\delta\sigma}^\lambda \Gamma_{\mu\nu}^\delta - \Gamma_{\delta\nu}^\lambda \Gamma_{\mu\sigma}^\delta \equiv 0$, and the metric tensor $g_{\mu\nu}$ is decomposed into a product of the components of a four-vector, i.e. $g_{\mu\nu} = A_\mu A_\nu$,
2. Electromagnetic four-vector field A_μ obeys the gauge condition $A^\alpha \partial_\alpha A_\mu$,

then, one can successfully give a geometric description of Maxwell's [2] source-free field equations.

Herein, we have for clarity's sake removed most of the intricate mathematics and philosophy (found in [1]) so that our reader(s) will have a much greater appreciation of our ongoing work. We here only deal with the Riemann tensor and its identities and from that only, we demonstrate that a decomposed metric ($g_{\mu\nu} = A_\mu A_\nu$) can successfully lead one to the source-free Maxwell's equation [2]. This we believe is

something that will provoke our reader(s) into thinking further (than meet the eye) by asking about the possibility of doing the same for the source-coupled field equations. Not only will this provoke the reader(s) into thinking about the possibility of a geometrically derived source-coupled Maxwell's equation [2], but of the possibility of a unity between gravitation, electricity and possibly the other two forces of Nature – the *weak* and *strong* nuclear forces.

Lastly, this article is organised as follows: in §2, we present the Riemann tensor and in addition to this, we introduce a gauge condition that linearises this tensor. In §3, we present the metric tensor in its decomposed form and some of the necessary gauge conditions. In §4, we write down the affine connection in terms of the decomposed metric tensor and from this exercise, we show that the Maxwellian electrodynamic tensor can be harnessed. In §5, we delve onto the main task of the day whereby we derive the Maxwellian source-free field equations purely from the Riemann tensor and lastly, in §6, we present a general discussion.

2 Riemann curvature tensor

From the view point of tensors, the Riemann curvature tensor $R_{\mu\sigma\nu}^\lambda$ has two components to it – i.e. the linear and non-linear parts which are themselves tensors. That is to say:

$$R_{\mu\sigma\nu}^\lambda = \underbrace{\Gamma_{\mu\nu,\sigma}^\lambda - \Gamma_{\mu\sigma,\nu}^\lambda}_{\text{linear terms}} + \underbrace{\Gamma_{\delta\sigma}^\lambda \Gamma_{\mu\nu}^\delta - \Gamma_{\delta\nu}^\lambda \Gamma_{\mu\sigma}^\delta}_{\text{non-linear terms}} = \hat{R}_{\mu\sigma\nu}^\lambda + \check{R}_{\mu\sigma\nu}^\lambda, \quad (1)$$

where $\hat{R}_{\mu\sigma\nu}^\lambda$ and $\check{R}_{\mu\sigma\nu}^\lambda$ are the linear and non-linear components of the Riemann curvature tensor and these are defined as follows:

$$\hat{R}_{\mu\sigma\nu}^\lambda = \Gamma_{\mu\nu,\sigma}^\lambda - \Gamma_{\mu\sigma,\nu}^\lambda, \quad (2a)$$

$$\check{R}_{\mu\sigma\nu}^\lambda = \Gamma_{\delta\sigma}^\lambda \Gamma_{\mu\nu}^\delta - \Gamma_{\delta\nu}^\lambda \Gamma_{\mu\sigma}^\delta. \quad (2b)$$

Because $R_{\mu\sigma\nu}^\lambda$ and $\hat{R}_{\mu\sigma\nu}^\lambda$ are tensors, it directly follows that $\check{R}_{\mu\sigma\nu}^\lambda$ is a tensor too. If, as proposed in [1], we are to choose as a natural gauge condition on our desired spacetime the condition $\check{R}_{\mu\sigma\nu}^\lambda = 0$, then, in any subsequent system of coordinates and/or reference frame, this condition will hold because $\check{R}_{\mu\sigma\nu}^\lambda$ is a tensor. What we now have is a linear Riemann world. Insofar as computations are concerned, such a world is certainly much easier to deal with. Besides this, one is able to obtain exact solutions from the resultant field equations. Whether or not this is the world that we live in, we can only compare our final results with what obtains in Nature.

3 Decomposition of the metric tensor

As is well known, the metric tensor $g_{\mu\nu}$ of RG has a total of sixteen components and as a result of the symmetry in its $\mu\nu$ -indices, i.e. $g_{\mu\nu} = g_{\nu\mu}$, it has ten independent components. Starting in [4], we realised that the number of independent terms can be reduced from ten to four by way of casting this metric as a product of a four-vector A_μ , i.e.

$$g_{\mu\nu} = A_\mu A_\nu. \quad (3)$$

With the metric now written in this manner, we were able to write down a curved spacetime Dirac equation [4] using the same approach used by Dirac to arrive at the Dirac equation.

This four-vector A_μ is assumed to have unit magnitude throughout all of spacetime, i.e.

$$A^\alpha A_\alpha = 1. \quad (4)$$

In [1], we have called this condition (4), the *Normalization Gauge Condition* (NGC). Differentiating this NGC with respect to: x^μ , we obtain the following corollary condition:

$$A^\alpha \partial_\mu A_\alpha = 0. \quad (5)$$

As will be seen in §5, this corollary condition (5) and the NGC, are necessary for the derivation that we shall carry out.

Apart from the NGC (4) and its corollary (5), we will also need the following condition for our derivation, i.e.

$$A^\alpha \partial_\alpha A_\mu = 0. \quad (6)$$

At present, we have no ready natural justification for this condition, i.e. where it originates from, except that it is a necessary condition for our derivation.

4 Recomposition of the affine connection

The Christoffel three-symbol [5] (affine connection) is given by:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} (\partial_\nu g_{\mu}^\lambda + \partial_\mu g_{\nu}^\lambda - \partial^\lambda g_{\mu\nu}). \quad (7)$$

Under the new decomposition of the metric given in (3), this affine connection can be recomposed or redefined by substituting the decomposed metric tensor. So doing, we obtain:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} [\partial_\nu (A^\lambda A_\mu) + \partial_\mu (A^\lambda A_\nu) - \partial^\lambda (A_\mu A_\nu)]. \quad (8)$$

Differentiating the terms of the metric in (8), we will have:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} \left(\underbrace{A^\lambda \partial_\nu A_\mu}_{\text{Term I}} + \underbrace{A_\mu \partial_\nu A^\lambda}_{\text{Term II}} + \underbrace{A^\lambda \partial_\mu A_\nu}_{\text{Term III}} + \underbrace{A_\nu \partial_\mu A^\lambda}_{\text{Term IV}} - \underbrace{A_\nu \partial^\lambda A_\mu}_{\text{Term V}} - \underbrace{A_\mu \partial^\lambda A_\nu}_{\text{Term VII}} \right). \quad (9)$$

Rearranging the differentiated terms of the metric tensor labelled in (9) above as: Term I, II, III, etc, we will have:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} \left[\left(\underbrace{A_\mu \partial_\nu A^\lambda}_{\text{Term II}} - \underbrace{A_\mu \partial^\lambda A_\nu}_{\text{Term VII}} \right) + \left(\underbrace{A_\nu \partial_\mu A^\lambda}_{\text{Term IV}} - \underbrace{A_\nu \partial^\lambda A_\mu}_{\text{Term V}} \right) + \left(\underbrace{A^\lambda \partial_\nu A_\mu}_{\text{Term I}} + \underbrace{A^\lambda \partial_\mu A_\nu}_{\text{Term III}} \right) \right]. \quad (10)$$

From (10), we can now write the Christoffel as follows:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} (A_\mu F_{\nu}^\lambda + A_\nu F_{\mu}^\lambda + A^\lambda H_{\mu\nu}), \quad (11)$$

where:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (12a)$$

$$H_{\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu. \quad (12b)$$

The object $F_{\mu\nu}$ in (12a) can easily be identified with Maxwell's electromagnetic field tensor [2] while the object $H_{\mu\nu}$ is a new object which may appear to be unrelated to Maxwell's electromagnetic field tensor [2]. As will be seen in the next section where we are going to derive the electrodynamic source-free field equations, this seemingly unrelated object $H_{\mu\nu}$ is what shall lead us to our desideratum.

Now, on the corollary to the NGC, i.e. (5) and (6), an application of these to (12), leads to the following corollary gauge conditions:

$$A^\alpha F_{\alpha\nu} = A^\alpha F_{\nu\alpha} = 0, \quad (13a)$$

$$A^\alpha H_{\alpha\nu} = A^\alpha H_{\nu\alpha} = 0. \quad (13b)$$

The above completes the necessary package of conditions needed to derive Maxwell's source-free field equations.

We shall now make a further reduction in the symbols by writing the affine connection as follows:

$$\Gamma_{\mu\nu}^\lambda = F_{\mu\nu}^\lambda + H_{\mu\nu}^\lambda, \quad (14)$$

where:

$$F_{\mu\nu}^\lambda = \frac{1}{2} (A_\mu F_{\nu}^\lambda + A_\nu F_{\mu}^\lambda), \quad (15a)$$

$$H_{\mu\nu}^\lambda = \frac{1}{2} A^\lambda H_{\mu\nu}. \quad (15b)$$

With the affine connection written as we have written it in (14), we can now write the linear Riemann tensor as follows:

$$R_{\mu\sigma\nu}^\lambda = F_{\mu\sigma\nu}^\lambda + H_{\mu\sigma\nu}^\lambda, \quad (16)$$

where the new curvature tensors $F_{\mu\sigma\nu}^\lambda$ and $H_{\mu\sigma\nu}^\lambda$ are such that:

$$F_{\mu\sigma\nu}^\lambda = \partial_\sigma F_{\mu\nu}^\lambda - \partial_\nu F_{\mu\sigma}^\lambda, \quad (17a)$$

$$H_{\mu\sigma\nu}^\lambda = \partial_\sigma H_{\mu\nu}^\lambda - \partial_\nu H_{\mu\sigma}^\lambda. \quad (17b)$$

We are now ready to demonstrate that deeply embedded in the Riemann metric under the present metric decomposition (3), are the Maxwell source-free field equations [2].

5 Derivation

We know that the Riemann tensor satisfies the following first Bianchi identity:

$$R_{\mu\sigma\nu}^\lambda + R_{\nu\mu\sigma}^\lambda + R_{\sigma\nu\mu}^\lambda \equiv 0. \quad (18)$$

Multiplying this identity (18) throughout by A_γ , and then contracting the $\gamma\lambda$ -indices of the resulting tensor, i.e. $\gamma = \lambda = \alpha$, (19) will reduce to:

$$A_\alpha R_{\mu\sigma\nu}^\alpha + A_\alpha R_{\nu\mu\sigma}^\alpha + A_\alpha R_{\sigma\nu\mu}^\alpha \equiv 0. \quad (19)$$

From the decomposition of $R_{\mu\sigma\nu}^\lambda$ into the curvature tensors $F_{\mu\sigma\nu}^\lambda$ and $H_{\mu\sigma\nu}^\lambda$ given in (16), it follows that we can decompose (19) into two corresponding parts as follows:

$$\begin{aligned} & (A_\alpha F_{\mu\sigma\nu}^\alpha + A_\alpha F_{\nu\mu\sigma}^\alpha + A_\alpha F_{\sigma\nu\mu}^\alpha) + \\ & + (A_\alpha H_{\mu\sigma\nu}^\alpha + A_\alpha H_{\nu\mu\sigma}^\alpha + A_\alpha H_{\sigma\nu\mu}^\alpha) \equiv 0. \end{aligned} \quad (20)$$

In our calculation of (19), we shall first compute:

$$A_\alpha F_{\mu\sigma\nu}^\alpha + A_\alpha F_{\nu\mu\sigma}^\alpha + A_\alpha F_{\sigma\nu\mu}^\alpha,$$

followed by:

$$A_\alpha H_{\mu\sigma\nu}^\alpha + A_\alpha H_{\nu\mu\sigma}^\alpha + A_\alpha H_{\sigma\nu\mu}^\alpha.$$

5.1 Part I

We know that:

$$\begin{aligned} 2F_{\mu\sigma\nu}^\lambda &= (\partial_\sigma A_\mu F_{\nu}^\lambda + A_\mu \partial_\sigma F_{\nu}^\lambda + \partial_\sigma A_\nu F_{\mu}^\lambda + A_\nu \partial_\sigma F_{\mu}^\lambda) \\ &- (\partial_\nu A_\mu F_{\sigma}^\lambda + A_\mu \partial_\nu F_{\sigma}^\lambda + \partial_\nu A_\sigma F_{\mu}^\lambda + A_\sigma \partial_\nu F_{\mu}^\lambda). \end{aligned} \quad (21)$$

Multiplying $F_{\mu\sigma\nu}^\lambda$ by A_γ , and then contracting the $\gamma\lambda$ -indices of the resulting tensor, i.e. $\gamma = \lambda = \alpha$, and taking into account the gauge condition $A_\alpha F_{\mu}^\alpha = 0$, (21) will reduce to:

$$\begin{aligned} 2A_\alpha F_{\mu\sigma\nu}^\alpha &= (A_\alpha A_\mu \partial_\sigma F_{\nu}^\alpha - A_\alpha A_\mu \partial_\nu F_{\sigma}^\alpha) + \\ &+ (A_\alpha A_\nu \partial_\sigma F_{\mu}^\alpha - A_\alpha A_\sigma \partial_\nu F_{\mu}^\alpha). \end{aligned} \quad (22)$$

Writing $A_\alpha A_\mu = g_{\alpha\mu}$, $A_\alpha A_\nu = g_{\alpha\nu}$ and $A_\alpha A_\sigma = g_{\alpha\sigma}$, we will have:

$$\begin{aligned} 2A_\alpha F_{\mu\sigma\nu}^\alpha &= (g_{\alpha\mu} \partial_\sigma F_{\nu}^\alpha - g_{\alpha\mu} \partial_\nu F_{\sigma}^\alpha) + \\ &+ (g_{\alpha\nu} \partial_\sigma F_{\mu}^\alpha - g_{\alpha\sigma} \partial_\nu F_{\mu}^\alpha), \end{aligned} \quad (23)$$

hence, lowering the indices in (23) where applicable, we will have:

$$2A_\alpha F_{\mu\sigma\nu}^\alpha = (\partial_\sigma F_{\mu\nu} - \partial_\nu F_{\mu\sigma}) + (\partial_\sigma F_{\nu\mu} - \partial_\nu F_{\sigma\mu}). \quad (24)$$

Using in (24) the antisymmetry property of the electromagnetic field tensor, namely $F_{\nu\mu} = -F_{\mu\nu}$ and $F_{\sigma\mu} = -F_{\mu\sigma}$, we will have:

$$2A_\alpha F_{\mu\sigma\nu}^\alpha = 0 \Rightarrow A_\alpha F_{\mu\sigma\nu}^\alpha = 0, \quad (25)$$

hence:

$$A_\alpha F_{\mu\sigma\nu}^\alpha + A_\alpha F_{\nu\mu\sigma}^\alpha + A_\alpha F_{\sigma\nu\mu}^\alpha = 0. \quad (26)$$

Next, we need to calculate $A_\alpha H_{\mu\sigma\nu}^\alpha + A_\alpha H_{\nu\mu\sigma}^\alpha + A_\alpha H_{\sigma\nu\mu}^\alpha$.

5.2 Part II

We know that:

$$\begin{aligned} 2H_{\mu\sigma\nu}^\lambda &= (A^\lambda \partial_\sigma H_{\mu\nu} - A^\lambda \partial_\nu H_{\mu\sigma}) + \\ &+ (H_{\mu\nu} \partial_\sigma A^\lambda - H_{\mu\sigma} \partial_\nu A^\lambda). \end{aligned} \quad (27)$$

Multiplying $H_{\mu\sigma\nu}^\lambda$ by A_γ , and then contracting the $\gamma\lambda$ -indices of the resulting tensor, i.e. $\gamma = \lambda = \alpha$, (27) will reduce to:

$$\begin{aligned} 2A_\alpha H_{\mu\sigma\nu}^\alpha &= (A_\alpha A^\alpha \partial_\sigma H_{\mu\nu} - A_\alpha A^\alpha \partial_\nu H_{\mu\sigma}) + \\ &+ (H_{\mu\nu} A_\alpha \partial_\sigma A^\alpha - H_{\mu\sigma} A_\alpha \partial_\nu A^\alpha). \end{aligned} \quad (28)$$

From the normalization gauge ($A_\alpha A^\alpha = 1$), and the corollary of this gauge, namely $A_\alpha \partial_\mu A^\alpha = 0$, (28) reduces to:

$$2A_\alpha H_{\mu\sigma\nu}^\alpha = \partial_\sigma H_{\mu\nu} - \partial_\nu H_{\mu\sigma} = \partial_\sigma F_{\nu\mu} + \partial_\nu F_{\mu\sigma}, \quad (29)$$

hence:

$$2A_\alpha H_{\mu\sigma\nu}^\alpha = \partial_\sigma F_{\nu\mu} + \partial_\nu F_{\mu\sigma} \neq 0, \quad (30a)$$

$$2A_\alpha H_{\nu\mu\sigma}^\alpha = \partial_\mu F_{\sigma\nu} + \partial_\sigma F_{\nu\mu} \neq 0, \quad (30b)$$

$$2A_\alpha H_{\sigma\nu\mu}^\alpha = \partial_\nu F_{\mu\sigma} + \partial_\mu F_{\sigma\nu} \neq 0. \quad (30c)$$

From (30), it is clear that:

$$\begin{aligned} A_\alpha H_{\mu\sigma\nu}^\alpha + A_\alpha H_{\nu\mu\sigma}^\alpha + A_\alpha H_{\sigma\nu\mu}^\alpha &= \\ &= \partial_\mu F_{\sigma\nu} + \partial_\nu F_{\mu\sigma} + \partial_\sigma F_{\nu\mu}. \end{aligned} \quad (31)$$

Now, we can put everything together.

5.3 Summary

Putting everything together, i.e. (19), (26) and (31), we will have:

$$\begin{aligned} A_\alpha R_{\mu\sigma\nu}^\alpha + A_\alpha R_{\nu\mu\sigma}^\alpha + A_\alpha R_{\sigma\nu\mu}^\alpha &= \\ &= 0 + (\partial_\mu F_{\sigma\nu} + \partial_\nu F_{\mu\sigma} + \partial_\sigma F_{\nu\mu}) \equiv 0, \end{aligned} \quad (32)$$

hence:

$$\partial_\mu F_{\sigma\nu} + \partial_\nu F_{\mu\sigma} + \partial_\sigma F_{\nu\mu} \equiv 0. \quad (33)$$

Of course, (33) is indeed Maxwell's source-free field equations [2].

6 Discussion

Given a linearized Riemann curvature tensor, we have herein demonstrated, in clear and no uncertain terms, the conditions under which Maxwell's electrodynamic source-free field equations [2] are readily susceptible to a geometric description by a metric tensor in much the same way that the force of gravity is described by the metric tensor in Einstein's GTR [3]. This description has come at the following cost:

1. A decomposed metric $g_{\mu\nu} = A_\mu A_\nu$. This reduces the number of independent fields from ten to four. In accordance with *Occam's Razor*, this is a welcome development in any theory, especially if the new theory does not destroy the old but enriches and engenders it.
2. A linearised (i.e. $\Gamma_{\delta\sigma}^\lambda \Gamma_{\mu\nu}^\delta - \Gamma_{\delta\nu}^\lambda \Gamma_{\mu\sigma}^\delta \equiv 0$) Riemann curvature tensor. This eliminates the computational complexity that ensues from these non-linear terms.
3. A normalization (i.e. $A^\alpha A_\alpha = 1$) gauge on the four-vector. A corollary to this normalization gauge is that $A^\alpha \partial_\mu A_\alpha = 0$.
4. Introduction of an extra exo-gauge condition to the metric four-vector, i.e. $A^\alpha \partial_\alpha A_\mu = 0$.

Having demonstrated the susceptibility of Maxwell [2]'s source-free field equation to a geometric description, the value of this work is that it indicates that Maxwell's equations [2] may very well be embedded deep inside the labyrinth of Riemann geometry. In addition to this, we strongly believe that this work is important for physics because if the electromagnetic force can be given a geometric description, this most certainly will lead to the opening of new pathways for incorporating the gravitational force into such a scheme.

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Non-Quantum Teleportation in a Rotating Space With a Strong Electromagnetic Field

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In 1991 we derived the physical conditions opening the gate to a fully degenerate space-time, where from the point of view of a regular observer the observable spatial and time intervals are equal to zero. The one of the conditions, under which the observable interval of time is zero, enables instant displacement (non-quantum teleportation) of physical bodies at any distance. In this article, we derive the teleportation condition for Schwarzschild's mass-point metric, Schwarzschild's metric inside a sphere filled with an incompressible liquid and de Sitter's metric of a space filled with the physical vacuum. We also introduce the modifications of the above three metrics, which contain rotation due to the space-time non-holonomy (non-orthogonality of the time lines to the three-dimensional spatial section) and derive the teleportation condition in each of these spaces. The obtained teleportation condition requires either a near-light-speed rotation or a super-strong gravitational field (depending on the particular space metric), which is very problematic if not impossible in a regular laboratory. On the other hand, the non-orthogonality of the time lines to the three-dimensional section can be implemented not only by a mechanical rotation of the laboratory space, but also using other physical factors. Thus, we are looking for how to do it using a strong electromagnetic field (the latter is not a problem for modern technologies). We introduce a space-time metric, which rotates due to its non-holonomy, and the gravitational field is neglected. Then, substituting the components of the obtained metric into Einstein's field equations with the electromagnetic energy-momentum tensor on the right hand side, we obtain the conditions under which the equations vanish and, therefore, the metric space is Riemannian and contains an electromagnetic field. As a result, we obtain how the electromagnetic field parameters can replace the rotation of space in the teleportation condition. The obtained result shows how to teleport physical bodies from an earth-bound laboratory to any remote point in the Universe using a super-strong electromagnetic field. Creating such devices is a very interesting task for engineers in the near future.

1 The background

In 1991, in the course of our extensive research on the application of the General Theory of Relativity to biophysics, we set ourselves the following primary task. We aimed to deduce such physical conditions, under which the four-dimensional pseudo-Riemannian space, which is the basic space-time of the General Theory of Relativity, is fully degenerate from the point of view of a regular observer. In such a fully degenerate region, the four-dimensional space-time interval is equal to zero, as well as the three-dimensional spatial interval and the interval of time, which are observed by a regular observer outside this region (in a regular non-degenerate region of the space-time), are also equal to zero.

In particular, the condition that the interval of physically observable time between two events is equal to zero enables instant displacement (non-quantum teleportation) of a physical body from the observer's laboratory to any remote point in the Universe.

The source and logical basis of this idea was the fact that a partial degeneration of the space-time was already known. In this case the four-dimensional (space-time) interval is equal to zero, and the observable three-dimensional interval and the interval of observable time are not equal to zero, but are equal to each other. Such a partially degenerate region of the space-

time is home to light-like trajectories and light-like (massless) particles moving along them, for example, photons (photons belong to the family of massless light-like particles).

In our mathematical search for physical conditions, under which the space-time fully degenerates, we used, as always in our theoretical work, the mathematical apparatus of chronometric invariants, which are physically observable quantities in the General Theory of Relativity. This mathematical apparatus was created in 1944 by our esteemed teacher A. L. Zelmanov (1913–1987), who published it first in 1944 in his PhD thesis [1] and then in two brief journal articles [2, 3]. It just so happened that after Zelmanov's death, we remain the only ones who professionally master this mathematical apparatus and apply it in scientific research. For this reason, before explaining our current study of the non-quantum teleportation condition, we give below a brief introduction to the theory of chronometric invariants.

2 A brief introduction to chronometric invariants

Briefly, *chronometric invariants* are the quantities that are invariant everywhere along a three-dimensional spatial section of the space-time and a line of time, which are linked to a real observer and his laboratory. Mathematically, chronometrically invariant quantities are projections of four-dimensional

(general covariant) quantities onto the three-dimensional spatial section and the line of time of the observer. In the general case, such a real three-dimensional spatial section (local three-dimensional space) can be curved, inhomogeneous, anisotropic, deformed, rotating, be filled with a gravitational field and also have some other properties such as viscosity etc. The lines of real time can have different density of time coordinates depending on the gravitational potential, as well as be non-orthogonal to the three-dimensional spatial section (the latter property is called the space-time non-holonomy, which is manifested as a three-dimensional rotation of the spatial section). As a result, the reference frame of a real observer, consisting of a coordinate grid paved on his real three-dimensional spatial section, as well as a system of real clocks located at each point of the section, has all the geometric and physical properties of his local space. Therefore, chronometrically invariant quantities as projections of four-dimensional (general covariant) quantities onto the real spatial section and real time line in his reference frame take into account the influence of all the geometric and physical factors present in his local space. So, the chronometrically invariant projections of any four-dimensional (general covariant) quantity calculated in the real reference frame of an observer are truly *physically observable quantities* registered by the observer.

The operator of projection onto the time line of an observer is the unit-length four-dimensional vector tangential to the observer's world line at each of its points

$$b^\alpha = \frac{dx^\alpha}{ds}, \quad b_\alpha b^\alpha = 1,$$

while the operator of projection onto his three-dimensional spatial section is the four-dimensional symmetric tensor

$$h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta.$$

These operators are orthogonal to each other, i.e., their common contraction is always equal to zero

$$h_{\alpha\beta} b^\alpha = 0, \quad h^{\alpha\beta} b_\alpha = 0, \quad h^\alpha_\beta b_\alpha = 0, \quad h^\beta_\alpha b^\alpha = 0.$$

A regular observer rests with respect to his reference body ($b^i = 0$) and, thus, accompanies to his reference space. Thus, the components of the projection operator b^α are

$$b^0 = \frac{1}{\sqrt{g_{00}}}, \quad b^i = 0, \quad b_0 = \sqrt{g_{00}}, \quad b_i = \frac{g_{i0}}{\sqrt{g_{00}}},$$

while the components of $h_{\alpha\beta}$ have the form

$$\begin{aligned} h_{00} &= 0, & h^{00} &= -g^{00} + \frac{1}{g_{00}}, & h^0_0 &= 0, \\ h_{0i} &= 0, & h^{0i} &= -g^{0i}, & h^i_0 &= 0, \\ h_{i0} &= 0, & h^{i0} &= -g^{i0}, & h^0_i &= \frac{g_{i0}}{g_{00}}, \\ h_{ik} &= -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}, & h^{ik} &= -g^{ik}, & h^i_k &= \delta^i_k. \end{aligned}$$

According to Zelmanov's theorem on the chronometrically invariant (physically observable) projections, the chr.inv.-projections of a four-dimensional vector Q^α are

$$b^\alpha Q_\alpha = \frac{Q_0}{\sqrt{g_{00}}}, \quad h^i_\alpha Q^\alpha = Q^i,$$

while for a symmetric 2nd rank tensor $Q^{\alpha\beta}$ these are

$$b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}, \quad h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}, \quad h^i_\alpha h^k_\beta Q^{\alpha\beta} = Q^{ik}.$$

Thus, the chr.inv.-projections of a four-dimensional interval dx^α are the physically observable time interval

$$d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i$$

and the observable three-dimensional interval dx^i which coincides with the spatial coordinate interval. The physically observable velocity is the three-dimensional chr.inv.-vector

$$v^i = \frac{dx^i}{d\tau}, \quad v_i v^i = h_{ik} v^i v^k = v^2,$$

which, on the trajectories of light, transforms to the three-dimensional chr.inv.-vector of the physically observable velocity of light c^i , the square of which is $c_i c^i = h_{ik} c^i c^k = c^2$.

Calculating the spatial chr.inv.-projections of the fundamental metric tensor $g_{\alpha\beta}$, we see that

$$h^\alpha_i h^\beta_k g_{\alpha\beta} = -h_{ik}, \quad h^i_\alpha h^k_\beta g^{\alpha\beta} = -h^{ik},$$

i.e., h_{ik} is the physically observable chr.inv.-metric tensor. It has all properties of the fundamental metric tensor $g_{\alpha\beta}$ in the observer's three-dimensional spatial section

$$h^\alpha_i h^\beta_k g_{\alpha\beta} = \delta^k_i - b_i b^k = \delta^k_i,$$

where δ^k_i is the unit three-dimensional tensor, which is part of the four-dimensional unit tensor δ^α_β . Therefore, the chr.inv.-metric tensor h_{ik} can lift and lower indices in chronometrically invariant quantities.

The chr.inv.-operators of derivation

$$\frac{*}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t}, \quad \frac{*}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{g_{0i}}{g_{00}} \frac{\partial}{\partial x^0}$$

are non-commutative

$$\frac{*}{\partial x^i \partial t} - \frac{*}{\partial t \partial x^i} = \frac{1}{c^2} F_i \frac{*}{\partial t}, \quad \frac{*}{\partial x^i \partial x^k} - \frac{*}{\partial x^k \partial x^i} = \frac{2}{c^2} A_{ik} \frac{*}{\partial t},$$

where

$$F_i = \frac{1}{1 - \frac{w}{c^2}} \left(\frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t} \right)$$

is the chr.inv.-vector of the gravitational inertial force,

$$A_{ik} = \frac{1}{2} \left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k} \right) + \frac{1}{2c^2} (F_i v_k - F_k v_i)$$

is the antisymmetric chr.inv.-tensor of the three-dimensional angular velocity of rotation of the observer's space, w is the gravitational potential, and v_i is the three-dimensional linear velocity of rotation of the observer's space due to the space-time non-holonomy (non-orthogonality of the time lines to the three-dimensional spatial section)

$$w = c^2 (1 - \sqrt{g_{00}}), \quad v_i = -c \frac{g_{0i}}{\sqrt{g_{00}}}, \quad v^i = -c g^{0i} \sqrt{g_{00}}.$$

In particular, v_i gives a detailed formula for the chr.inv.-metric tensor h_{ik} , which is

$$h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k.$$

It should be noted that the quantities w and v_i do not have chronometric invariance, despite the fact that $v_i = h_{ik} v^k$ and $v^2 = v_k v^k = h_{ik} v^i v^k$ as for a chr.inv.-quantity.

The reference space can deform, changing its coordinate grids with time that is expressed with the three-dimensional symmetric chr.inv.-tensor of the space deformation

$$D_{ik} = \frac{1}{2} \frac{\partial h_{ik}}{\partial t}, \quad D^{ik} = -\frac{1}{2} \frac{\partial h^{ik}}{\partial t}, \quad D = h^{ik} D_{ik} = \frac{\partial \ln \sqrt{h}}{\partial t},$$

where $h = \det \| h_{ik} \|$.

The regular 2nd rank Christoffel symbols $\Gamma_{\mu\nu}^\alpha$ and the 1st rank Christoffel symbols $\Gamma_{\mu\nu,\sigma}$ are replaced with the respective chr.inv.-Christoffel symbols

$$\Delta_{jk}^i = h^{im} \Delta_{jk,m} = \frac{1}{2} h^{im} \left(\frac{\partial h_{jm}}{\partial x^k} + \frac{\partial h_{km}}{\partial x^j} - \frac{\partial h_{jk}}{\partial x^m} \right),$$

where the chr.inv.-metric tensor h_{ik} is used instead of the fundamental metric tensor $g_{\alpha\beta}$.

The chr.inv.-curvature tensor is derived similarly to the Riemann-Christoffel tensor from the non-commutativity of the 2nd chr.inv.-derivatives of an arbitrary vector

$${}^* \nabla_i {}^* \nabla_k Q_l - {}^* \nabla_k {}^* \nabla_i Q_l = \frac{2A_{ik}}{c^2} \frac{\partial Q_l}{\partial t} + H_{lki} {}^* Q_j,$$

where the 4th rank chr.inv.-tensor

$$H_{lki} {}^* j = \frac{\partial \Delta_{il}^j}{\partial x^k} - \frac{\partial \Delta_{kl}^j}{\partial x^i} + \Delta_{il}^m \Delta_{km}^j - \Delta_{kl}^m \Delta_{im}^j,$$

is the basis for the chr.inv.-curvature tensor C_{lkij} ,

$$C_{lkij} = \frac{1}{4} (H_{lki} {}^* j - H_{jkl} {}^* i + H_{kji} {}^* l - H_{ilj} {}^* k),$$

$$C_{lk} = C_{lki} {}^* i, \quad C = h^{lk} C_{lk},$$

which has all properties of the Riemann-Christoffel tensor in the observer's three-dimensional spatial section, and its contraction gives the observable chr.inv.-curvature C . Also

$$H_{lkij} = C_{lkij} + \frac{1}{2} (2A_{ki} D_{jl} + A_{ij} D_{kl} + A_{jk} D_{il} + A_{kl} D_{ij} + A_{li} D_{jk}),$$

$$H_{lk} = C_{lk} + \frac{1}{2} (A_{kj} D_l^j + A_{lj} D_k^j + A_{kl} D),$$

$$H = h^{lk} H_{lk} = C.$$

Please note that, as was found by Zelmanov, the physically observable chr.inv.-curvature of a space is depended on not only the gravitational inertial force acting in the space, but also the space rotation and deformation, and, therefore, does not vanish in the absence of the gravitational field.

The general covariant Einstein equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\kappa T_{\alpha\beta} + \lambda g_{\alpha\beta}$$

with taking all possible factors into account have the chr.inv.-projections called the chr.inv.-Einstein equations

$$\left. \begin{aligned} \frac{{}^* \partial D}{\partial t} + D_{jl} D^{jl} + A_{jl} A^{lj} + {}^* \nabla_j F^j - \frac{1}{c^2} F_j F^j &= \\ &= -\frac{\kappa}{2} (\varrho c^2 + U) + \lambda c^2 \\ {}^* \nabla_j (h^{ij} D - D^{ij} - A^{ij}) + \frac{2}{c^2} F_j A^{ij} &= \kappa J^i \\ \frac{{}^* \partial D_{ik}}{\partial t} - (D_{ij} + A_{ij}) (D_k^j + A_k^j) + D D_{ik} + 3 A_{ij} A_k^j - \\ - \frac{1}{c^2} F_i F_k + \frac{1}{2} ({}^* \nabla_i F_k + {}^* \nabla_k F_i) - c^2 C_{ik} &= \\ &= \frac{\kappa}{2} (\varrho c^2 h_{ik} + 2 U_{ik} - U h_{ik}) + \lambda c^2 h_{ik} \end{aligned} \right\},$$

where the chr.inv.-derivative of the A^{ij} by x^j

$${}^* \nabla_j A^{ij} = \frac{{}^* \partial A^{ij}}{\partial x^j} + \Delta_{jl}^i A^{jl} + \Delta_{lj}^i A^{ij}, \quad \Delta_{lj}^i = \frac{{}^* \partial \ln \sqrt{h}}{\partial x^j}$$

is determined, as well as all other chr.inv.-derivatives

$$\begin{aligned} {}^* \nabla_i Q_k &= \frac{{}^* \partial Q_k}{\partial x^i} - \Delta_{ik}^l Q_l, \\ {}^* \nabla_i Q^k &= \frac{{}^* \partial Q^k}{\partial x^i} + \Delta_{il}^k Q^l, \\ {}^* \nabla_i Q_{jk} &= \frac{{}^* \partial Q_{jk}}{\partial x^i} - \Delta_{ij}^l Q_{lk} - \Delta_{ik}^l Q_{jl}, \\ {}^* \nabla_i Q_j^k &= \frac{{}^* \partial Q_j^k}{\partial x^i} - \Delta_{ij}^l Q_l^k + \Delta_{il}^k Q_j^l, \\ {}^* \nabla_i Q^{jk} &= \frac{{}^* \partial Q^{jk}}{\partial x^i} + \Delta_{il}^j Q^{lk} + \Delta_{il}^k Q^{jl}, \\ {}^* \nabla_i Q^j &= \frac{{}^* \partial Q^j}{\partial x^i} + \Delta_{ji}^l Q^l, & \Delta_{ji}^l &= \frac{{}^* \partial \ln \sqrt{h}}{\partial x^i}, \\ {}^* \nabla_i Q^{ji} &= \frac{{}^* \partial Q^{ji}}{\partial x^i} + \Delta_{il}^j Q^{il} + \Delta_{li}^j Q^{ji}, & \Delta_{li}^j &= \frac{{}^* \partial \ln \sqrt{h}}{\partial x^i}, \end{aligned}$$

by analogy with the respective absolute derivative, and

$$\varrho = \frac{T_{00}}{g_{00}}, \quad J^i = \frac{c T_0^i}{\sqrt{g_{00}}}, \quad U^{ik} = c^2 T^{ik}$$

are the chr.inv.-projections of the energy-momentum tensor $T_{\alpha\beta}$ of the distributed matter that fills the space, e.g., an electromagnetic field: ϱ is the physically observable density of the field energy, J^i is the physically observable density of the field momentum, and U^{ik} is the physically observable stress-tensor of the field (its trace is $U = h^{mn} U_{mn}$).

The electromagnetic field tensor is the curl of the four-dimensional electromagnetic field potential A^α , i.e.,

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu},$$

where

$$\nabla_\mu A_\nu = \frac{\partial A_\nu}{\partial x^\mu} - \Gamma_{\nu\mu}^\sigma A_\sigma$$

is the absolute derivative of the A_ν by x^μ . The electromagnetic field tensor has the physically observable projections

$$E^i = \frac{F_{0i}}{\sqrt{g_{00}}} = \frac{g^{i\alpha} F_{0\alpha}}{\sqrt{g_{00}}}, \quad H^{ik} = F^{ik} = g^{i\alpha} g^{k\beta} F_{\alpha\beta},$$

called the chr.inv.-electric strength E^i and chr.inv.-magnetic strength H^{ik} of the field. The respective chr.inv.-pseudovector H^{*i} and chr.inv.-pseudotensor E^{*ik}

$$H_{*i} = \frac{1}{2} \varepsilon_{ikm} H^{km}, \quad H^{*i} = \frac{1}{2} \varepsilon^{ikm} H_{km},$$

$$E^{*ik} = -\varepsilon^{ikm} E_m, \quad \varepsilon^{ipq} H_{*i} = \frac{1}{2} \varepsilon^{ipq} \varepsilon_{imn} H^{mn} = H^{pq}$$

are created in accordance with the transposition of indices in the antisymmetric “discriminant” chr.inv.-tensor

$$\varepsilon^{ikm} = \frac{e^{ikm}}{\sqrt{h}}, \quad \varepsilon_{ikm} = e_{ikm} \sqrt{h},$$

which was introduced by Zelmanov by analogy with the Levi-Civita antisymmetric unit tensor e^{ikm} . Using ε^{ikm} and ε_{ikm} , we can transform chr.inv.-tensors into chr.inv.-pseudotensors (see §2.3 in our monograph [5]).

Thus, the general covariant energy-momentum tensor of an electromagnetic field

$$T_{\alpha\beta} = \frac{1}{4\pi} \left(-F_{\alpha\sigma} F_{\beta}^{\cdot\sigma} + \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \right)$$

has the following chr.inv.-projections

$$\varrho = \frac{T_{00}}{g_{00}} = \frac{1}{8\pi} (E_i E^i + H_{*i} H^{*i}),$$

$$J^i = \frac{c T_0^i}{\sqrt{g_{00}}} = \frac{c}{4\pi} \varepsilon^{ikm} E_k H_{*m},$$

$$U^{ik} = c^2 T^{ik} = \varrho c^2 h^{ik} - \frac{c^2}{4\pi} (E^i E^k + H^{*i} H^{*k}).$$

Generally speaking, the mathematical apparatus of chronometric invariants is extensive. We have given above only

that part of it that is necessary for understanding this article. For a deeper study of this mathematics, we recommend the respective chapters of our monographs [4, 5], especially — the chapter Tensor Algebra and the Analysis in [5]. You can also study Zelmanov’s publications [1–3], of which his 1957 presentation [3] is the most useful and complete.

3 The physical conditions under which the space-time is fully degenerate

To deduce the physical conditions, under which the space-time is fully degenerate, we considered the square of the four-dimensional space-time interval $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ in the form, expressed in terms of chr.inv.-quantities, i.e.

$$ds^2 = c^2 d\tau^2 - d\sigma^2,$$

where $d\tau$ is the interval of physically observable time, $d\sigma$ is the physically observable three-dimensional interval

$$d\tau = \left(1 - \frac{w}{c^2}\right) dt - \frac{1}{c^2} v_i dx^i, \quad d\sigma^2 = h_{ik} dx^i dx^k,$$

while w is the gravitational potential, and v_i is the linear velocity of rotation of the observer’s space due to the space-time non-holonomy. Thus, considering the space-time interval the path travelled by a particle, we have

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{v^2}{c^2}\right), \quad v^i = \frac{dx^i}{d\tau},$$

where v^i is the physically observable chr.inv.-velocity of the particle registered by the observer (see above).

Prior to our study, two types of trajectories and, respectively, two types of particles were known in the General Theory of Relativity. First, these are the so-called non-isotropic trajectories, along which, in terms of chr.inv.-quantities,

$$ds^2 = c^2 d\tau^2 - d\sigma^2 \neq 0, \quad c^2 d\tau^2 \neq d\sigma^2 \neq 0.$$

They lie in the so-called non-isotropic region of the space-time, which is home to mass-bearing particles, and “mass-bearing” means that the rest-mass of such a particle is non-zero ($m_0 \neq 0$). The relativistic mass (mass of motion) of such a particle is non-zero too ($m \neq 0$). Such particles make up substances.

Trajectories of the second type are the so-called isotropic trajectories, along which, in terms of chr.inv.-quantities,

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = 0, \quad c^2 d\tau^2 = d\sigma^2 \neq 0.$$

They lie in the so-called isotropic region of the space-time, which is home to massless particles, for which “massless” means that the rest-mass of such a particle is equal to zero ($m_0 = 0$), while its relativistic mass (mass of motion) is non-zero ($m \neq 0$). Re-writting $ds^2 = 0$ in the form

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{v^2}{c^2}\right) = 0, \quad c^2 d\tau^2 \neq 0,$$

we see that massless particles travel at the velocity of light ($v^2 = h_{ik} v^i v^k = c^2$). The latter mean that massless particles are related to the light-like family of particles.

The fact that $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$ and $c^2 d\tau^2 = d\sigma^2 \neq 0$ along the isotropic trajectories means that this is a partially degenerate region of the space-time.

Taking the above into account, we logically supposed that the space-time becomes fully degenerate, if

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = 0, \quad c^2 d\tau^2 = 0, \quad d\sigma^2 = 0.$$

Our expectations found full justification. Below we will explain why.

As it is known from the geometry of metric spaces, a metric space is fully degenerate if the determinant of its metric tensor is equal to zero. In the four-dimensional pseudo-Riemannian space, which is the basic space-time of the General Theory of Relativity, the determinant of the fundamental metric tensor is $g < 0$. This means that the basic space-time of the General Theory of Relativity is non-degenerate.

The condition $d\tau = 0$ means that the physically observable time interval between any two events in this space-time region, when registered by an observer, whose home is the regular (non-degenerate) space-time region, is equal to zero. We re-write $d\tau = 0$ in the form

$$d\tau = \left[1 - \frac{1}{c^2} (w + v_i u^i) \right] dt = 0, \quad u^i = \frac{dx^i}{dt},$$

where u^i is the three-dimensional coordinate velocity of motion with respect to the observer, which is not a physically observable chr.inv.-quantity; the u^i is based on the time coordinate increment dt , which is not equal to zero between the events ($dt \neq 0$).

The condition $d\sigma^2 = 0$ in the extended form is

$$d\sigma^2 = h_{ik} dx^i dx^k = 0, \quad dx^i \neq 0$$

and means that in this space-time region the physically observable three-dimensional distance $d\sigma$ between any two different points ($dx^i \neq 0$) when registered by an observer, whose home is a regular non-degenerate space-time region, is equal to zero. This condition satisfies only if the determinant of the chr.inv.-metric tensor h_{ik} is equal to zero

$$h = \det \| h_{ik} \| = h_{11}h_{22}h_{33} + h_{31}h_{12}h_{23} + h_{21}h_{13}h_{32} - h_{31}h_{22}h_{13} - h_{21}h_{12}h_{33} - h_{11}h_{23}h_{32} = 0.$$

Zelmanov proved that the determinant of the fundamental metric tensor $g = \det \| g_{\alpha\beta} \|$ is connected with that of the chr.inv.-metric tensor $h = \det \| h_{ik} \|$ by the formula

$$h = -\frac{g}{g_{00}},$$

i.e., once the chr.inv.-metric tensor h_{ik} is degenerate, the fundamental metric tensor $g_{\alpha\beta}$ is degenerate too.

The above is an exact proof to why the entire space of the Universe or a local space region in it, wherein $c^2 d\tau^2 = 0$ and $d\sigma^2 = 0$, is fully degenerate. We therefore called such a space-time *zero-space*, while the trajectories that lie in it — *zero-trajectories*.

Using the formulae for $d\tau$ and h_{ik} , we obtained the *physical conditions for full degeneracy*, i.e., the physical conditions in a fully degenerate space-time (zero-space)

$$w + v_i u^i = c^2, \quad \left(1 - \frac{w}{c^2} \right)^2 c^2 dt^2 = g_{ik} dx^i dx^k.$$

From a geometric point of view, the conditions for full degeneracy mean the following.

Within the infinitesimal vicinity of any point in a Rimeanian space, we can introduce a flat space, which is tangential to the Riemannian space in this point. The latter means that the basis vectors $\vec{e}_{(a)}$ of the tangential flat space are tangential to the curved coordinate lines of the Riemannian space. But, since the coordinate lines in a Riemannian space are curved and non-orthogonal to each other (if the space is non-holonomic), the lengths of the basis vectors $\vec{e}_{(a)}$ in the tangential flat space are different from the unit length. The vector of an infinitesimal displacement in the Riemannian space is expressed through the tangential basis vectors as

$$d\vec{r} = \vec{e}_{(a)} dx^a,$$

and, since the scalar product of the vector $d\vec{r}$ with itself gives $d\vec{r}d\vec{r} = ds^2$ and also it is $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$, we obtain

$$g_{\alpha\beta} = \vec{e}_{(a)} \vec{e}_{(b)} = e_{(a)} e_{(b)} \cos(x^\alpha; x^\beta),$$

i.e., $g_{00} = e_{(0)}^2$, $g_{0i} = e_{(0)} e_{(i)} \cos(x^0; x^i)$, $g_{ik} = e_{(i)} e_{(k)} \cos(x^i; x^k)$. Thus, according to the definitions of v_i and h_{ik} , we have

$$v_i = -c e_{(i)} \cos(x^0; x^i),$$

$$h_{ik} = e_{(i)} e_{(k)} [\cos(x^0; x^i) \cos(x^0; x^k) - \cos(x^i; x^k)].$$

Taking into account that $d\tau = 0$ and $d\sigma^2 = 0$ in the zero-space (the latter, as was shown above, means $h = 0$), we obtain the *geometric conditions for full degeneracy*

$$e_{(0)} = -\frac{1}{c} e_{(i)} u^i \cos(x^0; x^i),$$

$$\cos(x^0; x^i) \cos(x^0; x^k) = \cos(x^i; x^k).$$

So, once the rotation of the observer's space reaches the light speed, $\cos(x^0; x^i) = 1$ and, thus, $\cos(x^i; x^k) = 1$: the lines of time become "fallen" into the three-dimensional spatial section (time becomes "fallen" into space), wherein all three spatial axes become coinciding with each other.

As for the particles located in the zero-space, their physical sense is derived based on Levi-Civita's rule, according to which, in a Riemannian space of n dimensions the length of

any n -dimensional vector Q^α transferred in parallel to itself remains unchanged ($Q_\alpha Q^\alpha = \text{const}$).

As it is known, any mass-bearing particle is characterized by the four-dimensional momentum vector P^α , and any massless (i.e., having the zero rest-mass) particle is characterized by the four-dimensional wave vector K^α ,

$$P^\alpha = m_0 \frac{dx^\alpha}{ds}, \quad K^\alpha = \frac{\omega_0}{c} \frac{dx^\alpha}{ds},$$

each of which is transferred in parallel to itself along the particle's trajectory in the space-time. The chr.inv.-projections of the P^α and K^α onto the time line and the three-dimensional spatial section of a regular observer are equal to

$$\begin{aligned} \frac{P_0}{\sqrt{g_{00}}} &= m, & P^i &= \frac{m}{c} v^i, \\ \frac{K_0}{\sqrt{g_{00}}} &= \frac{\omega}{c}, & K^i &= \frac{\omega}{c} c^i. \end{aligned}$$

To adapt the P^α and K^α to the zero-space condition, we are looking for the condition in their structure. Based on the interval of physically observable time $d\tau$ (page 32), we obtain how the physically observable velocity depends on the condition for full degeneracy $w + v_i u^i = c^2$, i.e.,

$$v^i = \frac{u^i}{1 - \frac{1}{c^2} (w + v_k u^k)},$$

and then express ds^2 in the form

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{v^2}{c^2} \right) = c^2 d\tau^2 \left\{ \left[1 - \frac{1}{c^2} (w + v_k u^k) \right]^2 - \frac{u^2}{c^2} \right\},$$

which gives

$$P^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{M}{c} \frac{dx^\alpha}{dt}, \quad K^\alpha = \frac{\omega_0}{c} \frac{dx^\alpha}{ds} = \frac{\omega}{c^2} \frac{dx^\alpha}{dt},$$

where $dt \neq 0$, and

$$M = \frac{m}{1 - \frac{1}{c^2} (w + v_k u^k)}, \quad \omega = \frac{\omega_0}{1 - \frac{1}{c^2} (w + v_k u^k)}$$

take the condition for full degeneracy $w + v_i u^i = c^2$ into account and are not equal to zero in the zero-space.

For zero-space particles, the chr.inv.-projections of their momentum vector P^α and wave vector K^α onto the time line and the three-dimensional space of a regular observer outside the zero-space are equal to

$$\begin{aligned} \frac{P_0}{\sqrt{g_{00}}} &= M \left[1 - \frac{1}{c^2} (w + v_i u^i) \right] = 0, & P^i &= \frac{1}{c} M u^i \neq 0, \\ \frac{K_0}{\sqrt{g_{00}}} &= \frac{\omega}{c} \left[1 - \frac{1}{c^2} (w + v_i u^i) \right] = 0, & K^i &= \frac{1}{c^2} \omega u^i \neq 0. \end{aligned}$$

The above result means that all zero-space particles have zero rest-masses $m_0 = 0$, zero relativistic masses $m = 0$ and zero relativistic frequencies $\omega = 0$. We therefore called the particles, whose home is the zero-space, *zero-particles*.

This is the third, new type of particles in addition to mass-bearing and massless (light-like) particles, already known in the General Theory of Relativity.

As it is known, for any regular mass-bearing and massless particle (their home is the regular non-degenerate space-time), the relation between its energy and momentum remains unchanged along its trajectory

$$E^2 - c^2 p^2 = \text{const}.$$

This follows from Levi-Civita's rule $P_\alpha P^\alpha = \text{const}$ and $K_\alpha K^\alpha \neq \text{const}$ having the form for mass-bearing particles and massless particles, respectively,

$$E^2 - c^2 p^2 = E_0^2, \quad E^2 - c^2 p^2 = 0,$$

where $E = mc^2$, $p^2 = m^2 v^2$, $E_0 = m_0 c^2$. For massless particles this relation, taking into account that $p^2 = m^2 v^2 = m^2 h_{ik} v^i v^k$, transforms into the banal formula $h_{ik} v^i v^k = c^2$ meaning that they travel at the velocity of light.

On the other hand, $P_\alpha P^\alpha \neq \text{const}$ and $K_\alpha K^\alpha \neq \text{const}$ for zero-particles: anyone can verify this fact by his own calculations based on the above. This fact means that Levi-Civita's rule is violated along the trajectories of zero-particles, and, hence, the observed geometry along their trajectories is not Riemannian.

The said does not necessarily mean that the zero-space geometry is non-Riemannian itself, but only that it looks like that from the point of view of a regular observer.

Of all the types of particles known in modern physics, only virtual particles have $E^2 - c^2 p^2 \neq \text{const}$. Feynman diagrams show that virtual particles are carriers of the interaction between elementary particles, i.e., between each two branching points on the diagrams. According to Quantum Electrodynamics, all physical processes in our world are based on the emission and absorption of virtual particles by real mass and massless (light-like) particles.

That is, the interaction between particles in our regular space-time is transmitted through an "exchange buffer" that is the zero-space, while zero-particles transmitting the interaction through this "buffer space" (zero-space) are *virtual particles* known in Quantum Electrodynamics.

The above is the solely interpretation of virtual particles and Feynman diagrams in the framework of the space-time geometry, and is a "bridge" connecting Quantum Electrodynamics with the General Theory of Relativity.

To understand how zero-particles could be registered in an experiment conducted by a regular observer, consider them as waves travelling along their space-time trajectories.

As it is known, any massless particle in the framework of the geometric optics approximation is characterized by the

four-dimensional wave vector determined in the lower-index form K_α through the wave phase ψ called eikonal. In analogy to it, we introduce the four-dimensional momentum vector characteristic of any mass-bearing particle, respectively,

$$K_\alpha = \frac{\partial \psi}{\partial x^\alpha}, \quad P_\alpha = \frac{\hbar}{c} \frac{\partial \psi}{\partial x^\alpha},$$

where \hbar is Planck's constant. Their physically observable chr.inv.-projections onto the observer's line of time are

$$\frac{K_0}{\sqrt{g_{00}}} = \frac{1}{c} \frac{\partial \psi}{\partial t}, \quad \frac{P_0}{\sqrt{g_{00}}} = \frac{\hbar}{c^2} \frac{\partial \psi}{\partial t},$$

and, since these chr.inv.-projections are also equal to ω/c and m (see above), we obtain that, in the framework of the geometric optics approximation,

$$\omega = \frac{\partial \psi}{\partial t}, \quad m = \frac{\hbar}{c^2} \frac{\partial \psi}{\partial t}.$$

Therefore, on the transition to the zero-space, i.e., under the condition for full degeneracy $w + v_i u^i = c^2$, since $\omega = 0$ and $m = 0$ (see above), we obtain

$$\frac{\partial \psi}{\partial t} = 0.$$

The eikonal equation $K_\alpha K^\alpha = \text{const}$ means that the length of the four-dimensional wave vector transferred in parallel to itself remains unchanged. The chr.inv.-eikonal equation for regular massless (light-like) particles and mass-bearing particles, taking the main property $g_{\alpha\sigma} g^{\beta\sigma} = \delta_\alpha^\beta$ of the fundamental metric tensor $g_{\alpha\beta}$ into account, has the form, respectively,

$$\frac{1}{c^2} \left(\frac{\partial \psi}{\partial t} \right)^2 - h^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = 0,$$

$$\frac{1}{c^2} \left(\frac{\partial \psi}{\partial t} \right)^2 - h^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = \frac{m_0^2 c^2}{\hbar^2},$$

and is a travelling wave equation. On the transition to the zero-space, the above eikonal equations take the same form

$$h^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = 0,$$

which is a standing wave equation.

To understand the result we have obtained, we should take into account the fact that a regular observer does not register zero-space objects themselves, but only what he sees on the transition to or from the zero-space (we assume that Levi-Civita's rule is satisfied on this boundary), and the zero-space itself is the fully degenerate case of the isotropic space (home to massless light-like particles).

Therefore, zero-particles, i.e., all particles, whose home is the zero-space, should appear to a regular observer outside the zero-space as *standing light waves*, while the zero-space should appear as a point containing a system of standing light waves (a *light-like hologram*) inside itself.

4 Non-quantum teleportation

Teleportation is the instant displacement of particles from one point in the three-dimensional space to another.

Initially, scientists considered only quantum teleportation. In fact, quantum teleportation is not a real instant displacement, but a "probabilistic trick" based on the laws of Quantum Mechanics [7]. This is despite the fact that, using quantum teleportation, photons were first "teleported" in 1998 [8], and atoms were "teleported" in 2004 [9, 10].

On the contrary, we considered instant displacement in accordance with the geometric structure of the space-time of the General Theory of Relativity, which is real teleportation without any "probabilistic tricks". This is why we called this regular non-quantum method of particle teleportation *non-quantum teleportation*.

In terms of physically observable chr.inv.-quantities, teleportation is a process of displacement in which the interval of physically observable time between its beginning and end is equal to zero ($d\tau = 0$). If a mass-bearing particle is teleported (mass-bearing particles make up substances), the teleportation condition $d\tau = 0$ is added with the physically observable three-dimensional interval between the point of departure and the point of arrival, which is not equal to zero, i.e. $d\sigma \neq 0$. Therefore, the space-time metric along the trajectories of non-quantum teleportation of mass-bearing particles is

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = -d\sigma^2, \quad c^2 d\tau^2 = 0, \quad d\sigma^2 \neq 0.$$

Since $d\sigma^2 = h_{ik} dx^i dx^k$ and taking into account the condition $w + v_i u^i = c^2$ under which $d\tau = 0$, the space-time metric takes the form, which we called the *non-quantum teleportation metric*

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{w}{c^2}\right)^2 c^2 dt^2 + g_{ik} dx^i dx^k =$$

$$= -\frac{1}{c^2} v_i u^i v_k u^k dt^2 + g_{ik} dx^i dx^k.$$

As you can see, in the non-quantum teleportation metric, the regular signature (+---) of space-time is replaced with the inverted signature (-+++). That is, from the point of view of a regular observer, "time" and "space" are replaced with each other on the teleportation trajectories: "time" of a teleporting particle is "space" of a regular observer, and "space" of the teleporting particle is "time" of the regular observer.

The same is true for the non-quantum teleportation metric, derived for massless (light-like) particles. If a massless (light-like) particle is teleported, the teleportation condition $d\tau = 0$ is added with $c^2 d\tau^2 = d\sigma^2$, since the latter is characteristic of the isotropic region of the space-time, which is home to such particles. Therefore, the space-time metric along the trajectories of non-quantum teleportation of massless (light-like) particles is fully degenerate

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = 0, \quad c^2 d\tau^2 = d\sigma^2 = 0,$$

which means that the trajectories along which massless particles are teleported lie in the fully degenerate space-time (zero-space). The equation of such trajectories is derived from the non-quantum teleportation metric (see above) equalized to zero, and is the fully degenerate light hypercone equation

$$\left(1 - \frac{w}{c^2}\right)^2 c^2 dt^2 = g_{ik} dx^i dx^k.$$

So, according to the General Theory of Relativity, as soon as we realize the physical condition

$$w + v_i u^i = c^2$$

in the local space inside a device in our laboratory (under this condition, $d\tau = 0$), a mass-bearing or massless (light-like) particle that is inside this device enters a teleportation trajectory and, thus, can be instantly teleported to any other place in our Universe. For this reason, we call $w + v_i u^i = c^2$ also the *physical condition for non-quantum teleportation*.

5 Finding the teleportation condition accessible in a real laboratory. Problem statement

The above results, which we obtained in the early 1990s, were presented in our two monographs in 2001 [4, 5], and then in the brief article [6]. The reason for such a long overview of these results in the present article is that without a detailed acquaintance with the above results, it would be impossible to understand everything that follows, including the engineering implementation of non-quantum teleportation at any distance in our Universe.

So, the physical condition $w + v_i u^i = c^2$ under which the interval of physically observable time is degenerate ($d\tau = 0$) is also the physical condition for non-quantum teleportation. To implement this physical condition, a super-strong gravitational potential and a near-light-speed rotation of the observer's space are required. Obviously, in a real laboratory, this is extremely difficult, if not impossible.

On the other hand, when deriving the teleportation condition $w + v_i u^i = c^2$ from $d\tau = 0$, we did not indicate the formulas for the individual components of the fundamental metric tensor $g_{\alpha\beta}$ and the chr.inv.-metric tensor h_{ik} . That is, we did not specify the specific local space of the real laboratory in which we are going to teleport particles.

It is obvious that, as soon as we specify the metric of the local space in a real laboratory, the teleportation condition derived from this metric will be different from its general form $w + v_i u^i = c^2$. Say, we have an electromagnetic field generator installed and running in our laboratory. If so, then the local space in our laboratory has an electromagnetic field. Accordingly, we expect that the characteristics of the electromagnetic field will appear in the teleportation condition derived from $d\tau = 0$. In particular, if the generated electromagnetic field is super-strong (this is not a big problem when using modern technologies), then the numerical values of the electromagnetic field terms in the teleportation condition can be

so significant that “replace” the gravitational potential and rotation of the laboratory space. In such a case, the teleportation condition, i.e., the condition under which particles enter teleportation trajectories and, thus, can be instantly teleported to any other place in our Universe, can be implemented in a real laboratory.

Which specific space metric is suitable for a real laboratory? Such a local space is connected either with the Earth, or with another planet, or with another star system, and, at first glance, is described by Schwarzschild's mass-point metric. On the other hand, the mass-point metric does not take into account the rotation of space, which is one of the two “core” factors in the teleportation condition $w + v_i u^i = c^2$. Another drawback is that the space described by the mass-point metric is filled only with a gravitational field, and does not have an electromagnetic field. The third drawback is that the gravitational field is so weak in a real earth-bound laboratory that this factor can be neglected in the teleportation condition.

We therefore have drafted the following research plan for the next Sections of this article.

At our first step we will derive and analyze the teleportation condition for each of the three most popular space metrics. These are Schwarzschild's mass-point metric, Schwarzschild's metric inside a sphere filled with an incompressible liquid and de Sitter's metric of a space filled with the physical vacuum. Then, based on the above metrics, we will introduce three similar metrics containing a three-dimensional rotation due to the space-time non-holonomy (expressed by $g_{0i} \neq 0$). After that we will derive and analyze the teleportation condition in each of these three types of rotating space.

At our second step, we will introduce the metric of a space that rotates due to the space-time non-holonomy, but free from the gravitational field. This metric will be our “working metric” in this research.

It is not a fact that the introduced metric containing rotation describes a Riemannian space. As it is known, a Riemannian space metric must not only have the Riemann square form $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$, determined by the Riemann fundamental metric tensor $g_{\alpha\beta}$, and be invariant $ds^2 = inv$ everywhere in the space. It must also satisfy Einstein's field equations — the relation between the Ricci curvature tensor, the fundamental metric tensor multiplied by the curvature scalar, and the energy-momentum tensor of the “space filler”, which is satisfied in any Riemannian space. The latter means that as soon as we substitute the components of the fundamental metric tensor $g_{\alpha\beta}$ (taken from the formula of a particular Riemannian space metric) and the components of the energy-momentum tensor of the medium filling the space into the component notation of the field equations, this must turn the field equations into the zero identity. This is why not many space metrics are proven to be Riemannian and, thus, are used in the General Theory of Relativity.

So, most likely, the introduced metric containing rotation will turn out to be non-Riemannian due to the term taking the

three-dimensional space rotation into account.

To correct this situation, at our second step, we will take the $g_{\alpha\beta}$ components from the introduced metric, then substitute them into the chr.inv.-Einstein equations, the right hand side of which is non-zero and contains the energy-momentum tensor of an electromagnetic field. The relations that vanish the resulting chr.inv.-Einstein equations (we call them the *Riemannian conditions*), are the conditions under which the metric is Riemannian and describes a non-holonomic (rotating) space-time filled with an electromagnetic field.

Please note that, as was found by Zelmanov (see page 33), the physically observable chr.inv.-curvature of a space is depended on not only the acting gravitational inertial force, but also the space rotation and deformation, and, therefore, does not vanish in the absence of the gravitational field.

At our third step, we will consider the teleportation condition, derived for the introduced metric containing rotation, and the Riemannian conditions for this metric in the presence of an electromagnetic field (the latter follow from the Einstein equations, see above).

The obtained system of equations will show how strong the electromagnetic field should be and what additional conditions are required to launch particles on teleportation trajectories in a slow rotating laboratory space. Super-strong electromagnetic fields are not a big problem when using modern technologies. For this reason, the obtained electromagnetic field parameters and additional conditions will show how, under the conditions of a real earth-bound laboratory, real physical bodies and photons can be instantly teleported to any other place in our Universe.

6 The teleportation condition in the space of a mass-point body

Schwarzschild's mass-point metric describes a spherically symmetric space filled with the gravitational field created in emptiness by a spherically symmetrical massive island, which is considered as a point-like mass. The metric has the form

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2),$$

where r is the distance from the centre of the island, while $r_g = 2GM/c^2$ is its gravitational radius.

Here and below, in terms of the spherical coordinates, r is the radial coordinate, θ is the polar angle, φ is the geographical longitude, dr is the elementary segment length along the r -axis, $r d\theta$ is the elementary arc length along the θ -axis, and $r \sin\theta d\varphi$ is the elementary arc length along the φ -axis.

Therefore, the non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of the mass-point metric expressed in terms of the spherical coordinates are equal to

$$g_{00} = 1 - \frac{r_g}{r}, \quad g_{11} = -\frac{1}{1 - \frac{r_g}{r}}, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2\theta.$$

With the above $g_{\alpha\beta}$ components, we obtain that the interval of physically observable time in the space of the mass-point metric has the form

$$d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i = \sqrt{1 - \frac{r_g}{r}} dt,$$

and the teleportation condition, which is $d\tau = 0$ with $dt \neq 0$, has the following form

$$1 - \frac{r_g}{r} = 0 \quad \Rightarrow \quad r = r_g.$$

In addition to the above, because g_{00} is expressed through the gravitational potential w in the form

$$g_{00} = \left(1 - \frac{w}{c^2}\right)^2,$$

the obtained teleportation condition can be re-written as

$$1 - \frac{w}{c^2} = 0 \quad \Rightarrow \quad w = c^2.$$

The obtained result means that, in the space of the mass-point metric, i.e., in the field of a spherically symmetric non-rotating mass, a particle enters a teleportation trajectory under the condition of gravitational collapse, i.e., on the surface of a gravitational collapsar.

In other words, if you are in the field of a spherically symmetric non-rotating mass, in order to launch a particle on a teleportation trajectory, you need to simulate a mini black hole in your laboratory.

7 The teleportation condition in the space of a rotating mass-point body

Introduce a mass-point metric, where a gravitational field is created in emptiness by a spherically symmetrical massive island, which rotates due to the space-time non-holonomy. We use Schwarzschild's mass-point metric as a basis. Assume that the space rotates along the φ -axis (along the geographical longitudes) with the linear velocity $v_3 = \omega r^2 \sin^2\theta$, where $\omega = \text{const}$ is the angular velocity of this rotation. Since, according to the definition of v_i ,

$$v_3 = \omega r^2 \sin^2\theta = -\frac{c g_{03}}{\sqrt{g_{00}}},$$

we obtain

$$g_{03} = -\frac{1}{c} v_3 \sqrt{g_{00}} = -\frac{\omega r^2 \sin^2\theta}{c} \sqrt{1 - \frac{r_g}{r}},$$

and, thus, we obtain a Schwarzschild-like mass-point metric containing the above rotation, i.e.,

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - 2\omega r^2 \sin^2\theta \sqrt{1 - \frac{r_g}{r}} dt d\varphi - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2).$$

Accordingly, the interval of physically observable time in the rotating space of the Schwarzschild-like metric we have introduced is

$$\begin{aligned} d\tau &= \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i = \\ &= \left(\sqrt{1 - \frac{r_g}{r}} - \frac{\omega r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt} \right) dt, \end{aligned}$$

and the teleportation condition, i.e., $d\tau = 0$ with $dt \neq 0$, written in the spherical coordinates has the form

$$\sqrt{1 - \frac{r_g}{r}} - \frac{\omega r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt} = 0,$$

or, that is the same

$$\omega + \omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = c^2,$$

where $\sin \theta = 1$ for the observer's laboratory located at the equator, and the last multiplier is the coordinate velocity of the teleporting particle along the φ -direction, which is the geographical longitude (we assume that the particle travels either in the same or in the opposite direction in which the space rotates).

This condition is different from that in the space of the Schwarzschild mass-point metric in only the second term depending on the rotation of space due to the space-time non-holonomy: the faster the rotation of space and the faster the teleporting particle, the farther the teleportation trajectory from the surface of gravitational collapse.

8 The teleportation condition in the space inside a liquid sphere

Consider the metric of the space inside a liquid sphere, which was introduced by Schwarzschild. It describes the space inside a sphere, which is not empty, but filled with an incompressible liquid. The gravitational field inside such a sphere is created by a spherically symmetrical incompressible liquid that fills it. As it is known, this metric has the form

$$\begin{aligned} ds^2 &= \frac{1}{4} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right)^2 c^2 dt^2 - \\ &\quad - \frac{dr^2}{1 - \frac{r^2 r_g}{a^3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \end{aligned}$$

where $r_g = 2GM/c^2$ is the gravitational radius calculated for the entire mass M of the liquid (source of the gravitational field) inside the sphere, and $a = \text{const}$ is the radius of the sphere. Respectively, the non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of this metric are

$$g_{00} = \frac{1}{4} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right)^2,$$

$$g_{11} = -\frac{1}{1 - \frac{r^2 r_g}{a^3}}, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2 \theta.$$

As a result, we obtain that the interval of physically observable time inside such a sphere has the form

$$\begin{aligned} d\tau &= \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i = \\ &= \frac{1}{2} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right) dt, \end{aligned}$$

and the teleportation condition, i.e., $d\tau = 0$ with $dt \neq 0$, has the following form

$$3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} = 0.$$

The obtained formula is a condition under which a particle enters a teleportation trajectory inside a sphere filled with an incompressible liquid.

It is obvious that the obtained teleportation condition is satisfied if

$$r = r_g = a,$$

which means that a particle enters a teleportation trajectory on only the surface of the liquid sphere (where the particle's radial coordinate is $r = a$), and the liquid sphere is a gravitational collapsar ($a = r_g$).

9 The teleportation condition in the space inside a rotating liquid sphere

Introduce the metric of the space inside a sphere filled with an incompressible liquid, which rotates due to the space-time non-holonomy.

We use the metric of a liquid sphere as a basis. Assume that the liquid sphere has a radius $a = \text{const}$, a mass M and rotates along the φ -axis (along the geographical longitudes) with the linear velocity $v_3 = \omega r^2 \sin^2 \theta$, where $\omega = \text{const}$ is the angular velocity of this rotation. With these characteristic parameters, according to the definition of v_i , we obtain

$$\begin{aligned} v_3 &= \omega r^2 \sin^2 \theta = -\frac{c g_{03}}{\sqrt{g_{00}}}, \\ g_{03} &= -\frac{1}{c} v_3 \sqrt{g_{00}} = -\frac{\omega r^2 \sin^2 \theta}{2c} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right). \end{aligned}$$

Thus, the metric inside such a rotating liquid sphere is

$$\begin{aligned} ds^2 &= \frac{1}{4} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right)^2 c^2 dt^2 - \\ &\quad - \omega r^2 \sin^2 \theta \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right) dt d\varphi - \\ &\quad - \frac{dr^2}{1 - \frac{r^2 r_g}{a^3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \end{aligned}$$

Therefore, the interval of physically observable time inside such a rotating sphere has the form

$$d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i = \left\{ \frac{1}{2} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right) - \frac{\omega r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt} \right\} dt.$$

where $\sin \theta = 1$ for the observer's laboratory located at the equator, and the last multiplier is the coordinate velocity of the teleporting particle along the φ -direction, which is the geographical longitude (we assume that the particle travels either in the same or in the opposite direction in which the space rotates).

As a result, the teleportation condition ($d\tau = 0$ with $dt \neq 0$) inside such a rotating liquid sphere has the form

$$\frac{1}{2} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right) - \frac{\omega r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt} = 0.$$

This is a condition under which a particle enters a teleportation trajectory inside the space of a rotating sphere filled with an incompressible liquid. It is different from that in the space inside a non-rotating liquid sphere in only the second term depending on the rotation of space due to the space-time non-holonomy. The smaller the gravitational radius r_g of the rotating liquid sphere and the smaller the distance r from the centre of the sphere, the faster the sphere should rotate and a particle should travel in order for this particle to enter a teleportation trajectory.

According to the obtained teleportation condition, two ultimate cases of particle teleportation are conceivable in the space inside a rotating liquid sphere.

1. In the first ultimate case of particle teleportation, the gravitational radius r_g of the liquid sphere is much smaller than the distance r of the teleporting particle from the centre of the sphere, and this distance r is much smaller than the radius a of the sphere

$$r_g \ll r, \quad r \ll a,$$

which is possible if the liquid sphere has a small mass, the liquid itself is very rarefied, and the teleporting particle is close to the centre of the sphere.

In this case, the obtained teleportation condition takes the simplified form

$$\omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = c^2,$$

i.e., the liquid sphere should rotate at the velocity of light and the particle should travel at the velocity of light in order for this particle to enter a teleportation trajectory.

2. In the second ultimate case of particle teleportation,

$$r r_g = a^2 \quad \Rightarrow \quad r = r_g = a,$$

and g_{00} of the metric is equal to zero

$$g_{00} = \frac{1}{4} \left(3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right)^2 = 0,$$

which means gravitational collapse. In this case, the teleporting particle is on the surface of the liquid sphere, which is a gravitational collapsar. In this case, the obtained teleportation condition takes the form

$$\omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = 0 \quad \Rightarrow \quad \frac{d\varphi}{dt} = 0,$$

which means that the teleporting particle rests with respect to the liquid sphere, since the sphere rotates ($v_3 \neq 0$) according to the initial formulation of the problem.

In other words, in order for a particle to enter a teleportation trajectory on the surface of a liquid sphere, which is a gravitational collapsar, the particle should be at rest with respect to the sphere.

10 The teleportation condition in the space filled with the physical vacuum

De Sitter's metric describes a space filled with the physical vacuum (λ -field) and does not include any island of mass or a distributed matter. The curvature is the same everywhere in such a space, so it is a constant curvature space. The physical vacuum (λ -field) produces a non-Newtonian gravitational force, which is proportional to the distance in the space, i.e., the force of non-Newtonian gravitation (λ -force) grows with distance. If $\lambda < 0$, it is an attraction force. If $\lambda > 0$, it is a repulsion force.

For details about the physical vacuum, its physically observable properties, and also the non-Newtonian gravitational force, see Chapter 5 in our monograph [5].

As it is known, de Sitter's metric has the form

$$ds^2 = \left(1 - \frac{\lambda r^2}{3} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{\lambda r^2}{3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2),$$

and, hence, the non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of this metric are

$$g_{00} = 1 - \frac{\lambda r^2}{3}, \quad g_{11} = -\frac{1}{1 - \frac{\lambda r^2}{3}}, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2 \theta.$$

Using these components, we obtain that the interval of physically observable time in a de Sitter space is

$$d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i = \sqrt{1 - \frac{\lambda r^2}{3}} dt,$$

and the teleportation condition, i.e., $d\tau = 0$ with $dt \neq 0$, has the following form

$$1 - \frac{\lambda r^2}{3} = 0 \quad \Rightarrow \quad r = \sqrt{\frac{3}{\lambda}},$$

where, since $\lambda = \text{const}$, the r means the maximum distance in the space. As it is known, $\lambda \leq 10^{-56} \text{ cm}^{-2}$ with today's measurement accuracy. So, if our Universe is a de Sitter space, the maximum distance in it is $r \geq 10^{28} \text{ cm}$.

In addition, the teleportation condition we have obtained above means that the space is in the state of collapse, i.e.,

$$g_{00} = 1 - \frac{\lambda r^2}{3} = 0.$$

The above means that a particle in a de Sitter space, i.e., in a space filled with the physical vacuum in the absence of any other matter, enters a teleportation trajectory at the maximum distance from the observer, which is conceivable in the space. Besides that, since the state of collapse occurs at the same distance from the observer, we conclude that the entire space should be in the state of collapse, i.e., the entire space filled with the physical vacuum should be a collapsar.

11 The teleportation condition in the rotating space filled with the physical vacuum

Introduce the metric of a space filled with the physical vacuum in the absence of other matter, which rotates due to the space-time non-holonomy.

We derive the metric based on de Sitter's metric. Assume that the space rotates along the φ -axis (along the geographical longitudes) with the linear velocity $v_3 = \omega r^2 \sin^2 \theta$, where $\omega = \text{const}$ is the angular velocity of this rotation. Thus, according to the definition of v_i , we obtain

$$v_3 = \omega r^2 \sin^2 \theta = -\frac{c g_{03}}{\sqrt{g_{00}}},$$

$$g_{03} = -\frac{1}{c} v_3 \sqrt{g_{00}} = -\frac{\omega r^2 \sin^2 \theta}{c} \sqrt{1 - \frac{\lambda r^2}{3}}.$$

As a result, we obtain the metric of a rotating space filled with the physical vacuum

$$ds^2 = \left(1 - \frac{\lambda r^2}{3}\right) c^2 dt^2 - 2 \omega r^2 \sin^2 \theta \sqrt{1 - \frac{\lambda r^2}{3}} dt d\varphi - \frac{dr^2}{1 - \frac{\lambda r^2}{3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2),$$

and, hence, the interval of physically observable time in such a space has the form

$$d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c \sqrt{g_{00}}} dx^i = \left(\sqrt{1 - \frac{\lambda r^2}{3}} - \frac{\omega r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt} \right) dt,$$

and the teleportation condition ($d\tau = 0$ with $dt \neq 0$) has the following form

$$\sqrt{1 - \frac{\lambda r^2}{3}} - \frac{\omega r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt} = 0,$$

where r is the distance between the teleporting particle and the observer, $\sin \theta = 1$ for the observer's laboratory located at the equator, and the last multiplier is the coordinate velocity of the teleporting particle along the φ -direction, which is the geographical longitude (assuming that the particle travels either in the same or in the opposite direction in which the space rotates).

The above formula we have obtained is the condition under which a particle enters a teleportation trajectory in a rotating de Sitter space, which is a rotating space filled with the physical vacuum in the absence of any other matter.

According to the obtained teleportation condition, two ultimate cases are conceivable for particle teleportation in a rotating de Sitter space.

1. In the first ultimate case of particle teleportation,

$$\lambda r^2 \ll 1,$$

and the obtained teleportation condition takes the following simplified form

$$\omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = c^2.$$

Since $\lambda \leq 10^{-56} \text{ cm}^{-2}$ (according to modern astronomy), in this ultimate case of particle teleportation in a rotating de Sitter space, the teleporting particle should be at the distance $r \ll 10^{28} \text{ cm}$ from the observer. In addition, the space should rotate at the velocity of light and the particle should travel at the velocity of light.

2. In the second ultimate case of particle teleportation, the teleporting particle should be very far from the observer

$$r = \sqrt{\frac{3}{\lambda}} \geq 10^{28} \text{ cm},$$

i.e., at the edge of the observable Universe or even beyond that observable edge.

In this case, g_{00} of the metric is equal to zero

$$g_{00} = 1 - \frac{\lambda r^2}{3} = 0,$$

which means that the space is in the state of collapse (i.e., the entire space is a huge collapsar), and the obtained teleportation condition takes the form

$$\omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = 0 \implies \frac{d\varphi}{dt} = 0,$$

which, since the space rotates ($v_3 \neq 0$), means that the teleporting particle is at rest.

In other words, in the second ultimate case of particle teleportation in a rotating de Sitter space, the teleporting particle should be resting with respect to the space, be at the maximum distance from the observer, which is conceivable in the space, while the entire space should be in the state of collapse (it should be a huge collapsar).

12 The metric of the space, which rotates, but is free from the gravitational field

Introduce the metric of a space, where the three-dimensional space rotates due to the space-time non-holonomy, but there is no field of gravitation. This space metric most accurately describes the local space of an observer, who is located in an earth-bound laboratory, since the gravitational potential on the Earth's surface is so weak that its factor under the teleportation condition can be neglected. Only the factors of rotation of space and the teleporting particle's speed affect teleportation in this case. An addition, in the space of this metric, the effect of rotation of space due to the space-time non-holonomy is most clearly manifested.

For the above reasons, we will deduce the characteristics of such a simplest rotating space in more detail.

Assuming that the space rotates along the φ -axis (along the geographical longitudes) with the velocity $v_3 = \omega r^2 \sin^2 \theta$, where $\omega = \text{const}$ is the angular velocity of this rotation, and, according to the definition of v_i ,

$$v_3 = \omega r^2 \sin^2 \theta = -\frac{c g_{03}}{\sqrt{g_{00}}}, \quad g_{03} = -\frac{\omega r^2 \sin^2 \theta}{c},$$

we obtain the metric of such a space. It has the form

$$ds^2 = c^2 dt^2 - 2\omega r^2 \sin^2 \theta dt d\varphi - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2),$$

where the rest non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ are

$$g_{00} = 1, \quad g_{11} = -1, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2 \theta.$$

So forth, using the general formula for the chr.inv.-metric tensor, which is

$$h_{ik} = -g_{ik} + \frac{g_{0i} g_{0k}}{g_{00}} = -g_{ik} + \frac{1}{c^2} v_i v_k,$$

we obtain that its non-zero components in the specific space we are considering are equal to

$$h_{11} = 1, \quad h_{22} = r^2, \quad h_{33} = r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right),$$

$$h^{11} = 1, \quad h^{22} = \frac{1}{r^2}, \quad h^{33} = \frac{1}{r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)},$$

where, since the matrix h_{ik} is diagonal, the upper-index components of h_{ik} are obtained as $h^{ik} = (h_{ik})^{-1}$ just like the invertible matrix components to any diagonal matrix.

To check the correctness of the above construction of the space metric, we calculate $v^2 = v_i v^i = h_{ik} v^i v^k$. Since $v^i = h^{ik} v_k$, we obtain the following

$$v^2 = v_i v^i = \frac{\omega^2 r^2 \sin^2 \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}, \quad v = \frac{\omega r \sin \theta}{\sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}},$$

hence, the dimension of v is [cm/sec]. If the space rotates slowly, the above formula transforms to $v = \omega r \sin \theta$ [cm/sec] that is completely "comme il faut".

Based on the above formula for v_3 and using the corresponding h^{ik} components, we obtain that the antisymmetric chr.inv.-tensor of the angular velocity of rotation of space, A_{ik} (see page 32), has the following non-zero components

$$A_{13} = \omega r \sin^2 \theta, \quad A_{31} = -A_{13},$$

$$A_{23} = \omega r^2 \sin \theta \cos \theta, \quad A_{32} = -A_{23},$$

$$A^{13} = \frac{\omega}{r \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}, \quad A^{31} = -A^{13},$$

$$A^{23} = \frac{\omega \cot \theta}{r^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}, \quad A^{32} = -A^{23}.$$

To check the correctness of the above, we calculate the square of the chr.inv.-pseudovector of the angular velocity of rotation of space, $\Omega^2 = \Omega_{*i} \Omega^{*i} = h_{ik} \Omega^{*i} \Omega^{*k}$. Since

$$\Omega^{*i} = \frac{1}{2} \varepsilon^{ikm} A_{km}, \quad \varepsilon^{ikm} = \frac{e^{ikm}}{\sqrt{h}}, \quad \Omega_{*i} = h_{ik} \Omega^{*k}$$

as for any pseudovector (see page 34 in this paper; for more details on pseudovectors and pseudotensors see §2.3 of our monograph [5]), after some algebra we obtain

$$\Omega^2 = \Omega_{*i} \Omega^{*i} = \frac{\omega^2}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}, \quad \Omega = \frac{\omega}{\sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}},$$

so, the dimension of Ω is [sec^{-1}]. If the space rotates slowly, the obtained formula transforms to $\Omega = \omega$ [sec^{-1}] that is completely "comme il faut".

Using the non-zero h_{ik} components, we obtain the determinant of the chr.inv.-metric tensor h_{ik} (see page 35)

$$h = \det \| h_{ik} \| = r^4 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right).$$

So forth, we obtain nonzero chr.inv.-derivatives of $\ln \sqrt{h}$. According to the mathematical apparatus of chronometric invariants, they are equal to the respective chr.inv.-Christoffel symbols, in which two indices have been contracted, i.e., Δ_{ik}^i . Such Christoffel symbols are used in our further calculation of the chr.inv.-divergence of A^{ik} , as well as the chr.inv.-Ricci curvature tensor C_{ik} , which are the left hand side terms of the chr.inv.-Einstein equations. After some algebra, we obtain

$$\Delta_{i1}^i = \frac{* \partial \ln \sqrt{h}}{\partial r} = \frac{2}{r \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \left(1 + \frac{3\omega^2 r^2 \sin^2 \theta}{2c^2}\right),$$

$$\Delta_{i2}^i = \frac{* \partial \ln \sqrt{h}}{\partial \theta} = \frac{\cot \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} \left(1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2}\right),$$

as well as their chr.inv.-derivatives

$$\begin{aligned}\frac{{}^*\partial\Delta_{i1}^i}{\partial r} &= -\frac{2}{r^2\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2} - \frac{3\omega^2 \sin^2\theta}{c^2\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2} - \\ &\quad - \frac{3\omega^4 r^2 \sin^4\theta}{c^4\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2}, \\ \frac{{}^*\partial\Delta_{i1}^i}{\partial\theta} &= \frac{2\omega^2 r \sin\theta \cos\theta}{c^2\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2}, \\ \frac{{}^*\partial\Delta_{i2}^i}{\partial r} &= \frac{2\omega^2 r \sin\theta \cos\theta}{c^2\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2} = \frac{{}^*\partial\Delta_{i1}^i}{\partial\theta}, \\ \frac{{}^*\partial\Delta_{i2}^i}{\partial\theta} &= -\frac{1}{\sin^2\theta\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)} - \frac{2\omega^2 r^2 \sin^2\theta}{c^2\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2} - \\ &\quad - \frac{2\omega^4 r^4 \sin^4\theta}{c^4\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^2}.\end{aligned}$$

The chr.inv.-Ricci curvature tensor C_{lk} is one of the terms contained in the tensorial equation of the chr.inv.-Einstein equations (see page 33). Its formula (page 33) is based on the chr.inv.-Christoffel symbols Δ_{jk}^i and their chr.inv.-derivatives. Therefore, to calculate the chr.inv.-Ricci curvature tensor C_{lk} in the specific space we are considering, we need to calculate the chr.inv.-Christoffel symbols. They are re-combinations of the chr.inv.-derivatives of the chr.inv.-metric tensor h_{ik} (see page 33). Thus, first, we obtain non-zero chr.inv.-derivatives of the chr.inv.-metric tensor h_{ik} for the space we are considering. They have the form

$$\begin{aligned}\frac{{}^*\partial h_{22}}{\partial r} &= 2r, \\ \frac{{}^*\partial h_{33}}{\partial r} &= 2r \sin^2\theta \left(1 + \frac{2\omega^2 r^2 \sin^2\theta}{c^2}\right), \\ \frac{{}^*\partial h_{33}}{\partial\theta} &= 2r^2 \sin\theta \cos\theta \left(1 + \frac{2\omega^2 r^2 \sin^2\theta}{c^2}\right).\end{aligned}$$

So forth, according to the general formula for the chr.inv.-Christoffel symbols (see page 33), we calculate all them one by one in the specific space we are considering. After some algebra, we obtain formulae for those of them that are different from zero, i.e.,

$$\begin{aligned}\Delta_{22}^1 &= -r, \\ \Delta_{33}^1 &= -r \sin^2\theta \left(1 + \frac{2\omega^2 r^2 \sin^2\theta}{c^2}\right), \\ \Delta_{12}^2 &= \Delta_{21}^2 = \frac{1}{r}, \\ \Delta_{33}^2 &= -\sin\theta \cos\theta \left(1 + \frac{2\omega^2 r^2 \sin^2\theta}{c^2}\right),\end{aligned}$$

$$\begin{aligned}\Delta_{13}^3 &= \Delta_{13}^3 = \frac{1}{r\left(1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)} \left(1 + \frac{2\omega^2 r^2 \sin^2\theta}{c^2}\right), \\ \Delta_{23}^3 &= \Delta_{32}^3 = \frac{\cot\theta}{1+\frac{\omega^2 r^2 \sin^2\theta}{c^2}} \left(1 + \frac{2\omega^2 r^2 \sin^2\theta}{c^2}\right).\end{aligned}$$

Then, we look for non-zero components of the contracted 4th rank chr.inv.-tensor $H_{lki}^{\dots i}$, which, since the space we are considering is free from deformations ($D_{ik}=0$), is equal to the chr.inv.-Ricci curvature tensor C_{lk} (for the full formulae of the chr.inv.-curvature tensors see page 33)

$$C_{lk} = C_{lki}^{\dots i} = H_{lki}^{\dots i} = \frac{{}^*\partial\Delta_{il}^i}{\partial x^k} - \frac{{}^*\partial\Delta_{kl}^i}{\partial x^i} + \Delta_{il}^m \Delta_{km}^i - \Delta_{kl}^m \Delta_{im}^i,$$

where, according to the mathematical apparatus of chronometric invariants (see page 33), we have

$$\Delta_{ik}^i = \frac{{}^*\partial \ln \sqrt{h}}{\partial x^k}.$$

As a result, we obtain that the chr.inv.-Ricci tensor in the specific space we are considering has the following non-zero components

$$\begin{aligned}C_{11} &= H_{11i}^{\dots i} = \frac{{}^*\partial\Delta_{i1}^i}{\partial r} + \Delta_{21}^2 \Delta_{12}^2 + \Delta_{31}^3 \Delta_{13}^3, \\ C_{12} &= H_{12i}^{\dots i} = \frac{{}^*\partial\Delta_{i1}^i}{\partial\theta} + \Delta_{31}^3 \Delta_{23}^3 - \Delta_{21}^2 \Delta_{12}^i, \\ C_{21} &= H_{21i}^{\dots i} = \frac{{}^*\partial\Delta_{i2}^i}{\partial r} + \Delta_{32}^3 \Delta_{13}^3 - \Delta_{12}^2 \Delta_{i2}^i, \\ C_{22} &= H_{22i}^{\dots i} = \frac{{}^*\partial\Delta_{i2}^i}{\partial\theta} - \frac{{}^*\partial\Delta_{22}^1}{\partial r} + \\ &\quad + 2\Delta_{12}^2 \Delta_{22}^1 + \Delta_{32}^3 \Delta_{23}^3 - \Delta_{22}^1 \Delta_{i1}^i, \\ C_{33} &= H_{33i}^{\dots i} = -\frac{{}^*\partial\Delta_{33}^1}{\partial r} - \frac{{}^*\partial\Delta_{33}^2}{\partial\theta} + \\ &\quad + 2\Delta_{13}^3 \Delta_{33}^1 + 2\Delta_{23}^3 \Delta_{33}^2 - \Delta_{33}^1 \Delta_{i1}^i - \Delta_{33}^2 \Delta_{i2}^i,\end{aligned}$$

where

$$\Delta_{i1}^i = \frac{{}^*\partial \ln \sqrt{h}}{\partial r}, \quad \Delta_{i2}^i = \frac{{}^*\partial \ln \sqrt{h}}{\partial\theta}.$$

To calculate the components of the chr.inv.-Ricci tensor, we already have the specific formulae for Δ_{i1}^i and Δ_{i2}^i in the metric we are considering (see page 43). In addition, we need formulae for the chr.inv.-derivatives of Δ_{33}^1 with respect to r and Δ_{33}^2 with respect to θ , which are contained in the chr.inv.-Ricci tensor. We obtain that they are equal to

$$\begin{aligned}\frac{{}^*\partial\Delta_{33}^1}{\partial r} &= -\sin^2\theta \left(1 + \frac{6\omega^2 r^2 \sin^2\theta}{c^2}\right), \\ \frac{{}^*\partial\Delta_{33}^2}{\partial\theta} &= \sin^2\theta + \frac{2\omega^2 r^2 \sin^4\theta}{c^2} - \cos^2\theta - \frac{6\omega^2 r^2 \sin^2\theta \cos^2\theta}{c^2}.\end{aligned}$$

So forth, after some algebra, we obtain formulae for the non-zero components of the chr.inv.-Ricci tensor

$$\begin{aligned} C_{11} &= \frac{3\omega^2 \sin^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} - \frac{\omega^4 r^2 \sin^4 \theta}{c^4 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ C_{12} &= \frac{3\omega^2 r \sin \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} - \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^4 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ C_{21} &= \frac{3\omega^2 r \sin \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} - \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^4 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ C_{22} &= \frac{3\omega^2 r^2 \cos^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} - \frac{\omega^4 r^4 \sin^2 \theta \cos^2 \theta}{c^4 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ C_{33} &= \frac{3\omega^2 r^2 \sin^2 \theta}{c^2} - \frac{\omega^4 r^4 \sin^4 \theta}{c^4 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}, \end{aligned}$$

where, in particular, we see that $C_{12} = C_{21}$ that means a certain curvature symmetry in the space we are considering.

As a result, the physically observable chr.inv.-scalar curvature $C = h^{lk} C_{lk}$ (see page 33) of the rotating space we are considering is equal to

$$C = \frac{6\omega^2}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} - \frac{2\omega^4 r^2 \sin^2 \theta}{c^4 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2},$$

i.e., the origin of the physically observable chr.inv.-curvature of such a space is only its three-dimensional rotation due to the space-time non-holonomy (non-orthogonality of the time lines to the three-dimensional spatial section).

As you can see, the obtained formula for the scalar curvature and also every component of the obtained chr.inv.-Ricci curvature tensor (used in the chr.inv.-Einstein equations, see below) consists of two terms: the first order term, the goal of which is very significant, and the second order (additional) term, the influence of which is tiny. When Larissa first saw the above formulae, she immediately said: “You just made a fundamental theoretical discovery: if a space rotates due its space-time non-holonomy, its curvature produces the first order effect.”

The above characteristics of the space we are considering will be used further to calculate the individual components of the chr.inv.-Einstein equations in this space.

So forth, we obtain that the interval of physically observable time in such a rotating space has the formula

$$d\tau = \sqrt{g_{00}} dt + \frac{g_{0i}}{c\sqrt{g_{00}}} dx^i = \left(1 - \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \frac{d\varphi}{dt}\right) dt,$$

where $\sin \theta = 1$ (the polar angle θ is equal to $\frac{\pi}{2}$) for the observer's laboratory located at the equator, and the last multiplier is the coordinate velocity of the teleporting particle

along the φ -direction, which is the geographical longitude (assuming that it travels either in the same or in the opposite direction in which the space rotates).

As a result, we obtain that the teleportation condition, i.e., $d\tau = 0$ with $dt \neq 0$, has the form

$$\omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = c^2.$$

The obtained formula is the teleportation condition in a rotating space that is free from the field of gravitation and a distributed matter. In this case, as you can see from the above formula, a particle enters a teleportation trajectory in such a space, if it travels at the velocity of light, and the space rotates at the velocity of light.

Next, we will look how this condition changes if the rotating space is not empty, but filled with an electromagnetic field. To do it we will consider Einstein's field equations for a space of the above metric, where the right hand side of the equations is non-zero, but contains the energy-momentum tensor of the electromagnetic field (such Einstein equations characterize a space filled with an electromagnetic field).

As it is known, Einstein's equations are one of the necessary conditions for a space metric to be Riemannian. Therefore, the considered rotating space filled with an electromagnetic field is Riemannian under some particular conditions by which the Einstein equations for this space metric vanish (for this reason we call them *Riemannian conditions*).

We hope, the derived Riemannian conditions will somehow replace the rotation of space (the main factor in the teleportation condition) with the electromagnetic field parameters, thereby giving us the opportunity to “strengthen” the space-time non-holonomy to the level necessary for particle teleportation without the need to mechanically rotate the observer's local space at the light speed.

13 Using Einstein's field equations to find conditions under which the introduced metric is Riemannian

In an empty rotating space of the metric we have introduced above, the gravitational inertial force, the space deformation and the λ -term are equal to zero, while the space curvature and rotation are non-zero

$$F_i = 0, \quad D_{ik} = 0, \quad \lambda = 0, \quad C_{ik} \neq 0, \quad A_{ik} \neq 0.$$

The chr.inv.-Einstein equations (for their full formulae see page 33) very simplify under the above conditions. If the rotating space is filled with a distributed matter, they have the non-zero right hand side and take the form

$$\left. \begin{aligned} A_{ik} A^{ki} &= -\frac{\varkappa}{2} (\varrho c^2 + U) \\ {}^* \nabla_k A^{ik} &= -\varkappa J^i \\ 2A_{ij} A_k^j - c^2 C_{ik} &= \frac{\varkappa}{2} (\varrho c^2 h_{ik} + 2U_{ik} - U h_{ik}) \end{aligned} \right\},$$

where the right hand side contains the physically observable projections of the energy-momentum tensor of the matter that fills the space: ϱ is the chr.inv.-density of the field energy, J^i is the chr.inv.-density of the field momentum, and U^{ik} is the chr.inv.-stress-tensor of the field.

Calculate $U = h_{mn}U^{mn}$, i.e., the trace of the electromagnetic field chr.inv.-stress-tensor U^{ik} (see page 34). Since the trace of the chr.inv.-metric tensor is $h_{mn}h^{mn} = 3$, we obtain $U = \varrho c^2$. Thus, the chr.inv.-Einstein equations in a rotating space filled with an electromagnetic field have the form

$$\left. \begin{aligned} A_{ik}A^{ki} &= -\kappa\varrho c^2 \\ {}^*\nabla_k A^{ik} &= -\kappa J^i \\ 2A_{ij}A_k{}^j - c^2 C_{ik} &= \kappa U_{ik} \end{aligned} \right\},$$

or, extending the electromagnetic field characteristics,

$$\left. \begin{aligned} A_{ik}A^{ki} &= -\frac{\kappa c^2}{8\pi} (E_i E^i + H_{*i} H^{*i}) \\ {}^*\nabla_k A^{ik} &= -\frac{\kappa c}{4\pi} \varepsilon^{ikm} E_k H_{*m} \\ 2A_{ij}A_k{}^j - c^2 C_{ik} &= \\ &= \frac{\kappa c^2}{8\pi} (E_j E^j + H_{*j} H^{*j}) h_{ik} - \frac{\kappa c^2}{4\pi} (E_i E_k + H_{*i} H_{*k}) \end{aligned} \right\}.$$

Taking into account the characteristics of the space metric we are considering (see above), after some algebra we obtain non-zero components of the left hand side terms

$$\begin{aligned} A_{ik}A^{ki} &= -\frac{2\omega^2}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}, \\ {}^*\nabla_k A^{3k} &= \frac{\omega}{r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \left\{ 1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \right\}, \\ 2A_{1j}A_1{}^j - c^2 C_{11} &= -\frac{\omega^2 \sin^2 \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} + \frac{\omega^4 r^2 \sin^4 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ 2A_{1j}A_2{}^j - c^2 C_{12} &= -\frac{\omega^2 r \sin \theta \cos \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} + \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ 2A_{2j}A_1{}^j - c^2 C_{21} &= -\frac{\omega^2 r \sin \theta \cos \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} + \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ 2A_{2j}A_2{}^j - c^2 C_{22} &= -\frac{\omega^2 r^2 \cos^2 \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} + \frac{\omega^4 r^4 \sin^2 \theta \cos^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2}, \\ 2A_{3j}A_3{}^j - c^2 C_{33} &= -\omega^2 r^2 \sin^2 \theta + \frac{\omega^4 r^4 \sin^4 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}. \end{aligned}$$

We see that the left hand side of the chr.inv.-Einstein equations does not vanish. This means that a rotating space characterized by the considered metric is not Riemannian, if it is empty. To be Riemannian, such a space must be filled with a distributed matter so that the right hand side of the Einstein equations equalized the non-zero left hand side.

Using the obtained left hand side of the chr.inv.-Einstein equations, as well as the formulae for the electromagnetic field characteristics ϱ , J^i , U_{ik} (see page 34), we get the above chr.inv.-Einstein equations in the final form

$$\left. \begin{aligned} \frac{\omega^2}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} &= \frac{\kappa c^2}{16\pi} (E_i E^i + H_{*i} H^{*i}) \\ \frac{\omega}{r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \left\{ 1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \right\} &= \\ &= -\frac{\kappa c}{4\pi} \varepsilon^{3km} E_k H_{*m} \\ \frac{\omega^2 (2 + \sin^2 \theta)}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^2 \sin^4 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} &= \\ &= \frac{\kappa c^2}{4\pi} (E_1 E_1 + H_{*1} H_{*1}) \\ \frac{\omega^2 r \sin \theta \cos \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} &= \\ &= \frac{\kappa c^2}{4\pi} (E_1 E_2 + H_{*1} H_{*2}) \\ \frac{\omega^2 r \sin \theta \cos \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} &= \\ &= \frac{\kappa c^2}{4\pi} (E_2 E_1 + H_{*2} H_{*1}) \\ \frac{\omega^2 r^2 (2 + \cos^2 \theta)}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^4 \sin^2 \theta \cos^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} &= \\ &= \frac{\kappa c^2}{4\pi} (E_2 E_2 + H_{*2} H_{*2}) \\ 3\omega^2 r^2 \sin^2 \theta - \frac{\omega^4 r^4 \sin^4 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} &= \\ &= \frac{\kappa c^2}{4\pi} (E_3 E_3 + H_{*3} H_{*3}) \end{aligned} \right\},$$

where the right hand side is expressed through the chr.inv.-electric strength vector E^i and the chr.inv.-magnetic strength pseudovector H^{*i} of the field (see page 34 for detail).

Note that the dimension of the electric and magnetic field strengths here is $[\text{gram}^{1/2} \text{cm}^{-3/2}]$ as well as everywhere in the

relativistic electrodynamics. To avoid confusion, we note that in our earlier works for these quantities we used the “electromagnetic” dimension $[\text{gram}^{1/2} \text{cm}^{-1/2} \text{sec}^{-1}]$, as is customary in Classical Electrodynamics and technology. It is different from the above by a unit coefficient, the dimension of which is the same as that of the velocity of light.

Mathematically, the obtained chr.inv.-Einstein equations mean that a rotating space filled with an electromagnetic field of the specific configuration, as indicated in the equations, is Riemannian. Therefore, the above Einstein equations are the *Riemannian conditions* for this space metric. That is, we can consider a rotating space in the General Theory of Relativity only if it is filled with an electromagnetic field of the specific structure determined by the Einstein equations.

14 The structure of the electromagnetic field

To obtain some information about the structure of the particular electromagnetic field determined by the obtained Einstein equations, we analyze the equations in detail.

The scalar and tensorial equations give trivial relations between E and H .

Since just one component ${}^*\nabla_k A^{3k}$ of the vectorial Einstein equation is non-zero, ${}^*\nabla_k A^{1k} = 0$ and ${}^*\nabla_k A^{2k} = 0$ give

$$\varepsilon^{1km} E_k H_{*m} = \varepsilon^{123} E_2 H_{*3} + \varepsilon^{132} E_3 H_{*2} = 0,$$

$$\varepsilon^{2km} E_k H_{*m} = \varepsilon^{213} E_1 H_{*3} + \varepsilon^{231} E_3 H_{*1} = 0,$$

from which, since $\varepsilon^{123} = -\varepsilon^{132} = \varepsilon^{312}$ and so on, we obtain

$$E_2 H_{*3} - E_3 H_{*2} = 0,$$

$$E_1 H_{*3} - E_3 H_{*1} = 0.$$

The non-zero vectorial Einstein equation means

$$\begin{aligned} \varepsilon^{3km} E_k H_{*m} &= \varepsilon^{312} E_1 H_{*2} + \varepsilon^{321} E_2 H_{*1} = \\ &= -\frac{4\pi\omega}{\kappa c r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \left\{1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}\right\}, \end{aligned}$$

which, taking into account that (see page 34)

$$\varepsilon^{ikm} = \frac{e^{ikm}}{\sqrt{h}}, \quad e^{123} = +1, \quad e^{312} = -e^{132} = e^{123} = +1,$$

gives the following

$$\begin{aligned} E_1 H_{*2} - E_2 H_{*1} &= \\ &= -\frac{4\pi\omega}{\kappa c \sin \theta \sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}} \left\{1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}\right\}. \end{aligned}$$

Taking the above into account, we conclude that the electromagnetic field determined by the obtained Einstein equa-

tions is characterized by the system of relations

$$\left. \begin{aligned} E^2 + H^2 &= \frac{16\pi\omega^2}{\kappa c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \\ E_2 H_{*3} - E_3 H_{*2} &= 0 \\ E_1 H_{*3} - E_3 H_{*1} &= 0 \\ E_1 H_{*2} - E_2 H_{*1} &= \\ &= -\frac{4\pi\omega}{\kappa c \sin \theta \sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}}} \left\{1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}\right\} \\ E_1 E_1 + H_{*1} H_{*1} &= \\ &= \frac{4\pi}{\kappa c^2} \left\{ \frac{\omega^2 (2 + \sin^2 \theta)}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^2 \sin^4 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} \right\} \\ E_1 E_2 + H_{*1} H_{*2} &= \\ &= \frac{4\pi}{\kappa c^2} \left\{ \frac{\omega^2 r \sin \theta \cos \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^3 \sin^3 \theta \cos \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} \right\} \\ E_2 E_2 + H_{*2} H_{*2} &= \\ &= \frac{4\pi}{\kappa c^2} \left\{ \frac{\omega^2 r^2 (2 + \cos^2 \theta)}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} - \frac{\omega^4 r^4 \sin^2 \theta \cos^2 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)^2} \right\} \\ E_3 E_3 + H_{*3} H_{*3} &= \\ &= \frac{4\pi}{\kappa c^2} \left\{ 3\omega^2 r^2 \sin^2 \theta - \frac{\omega^4 r^4 \sin^4 \theta}{c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)} \right\} \end{aligned} \right\},$$

which at small ω simplifies to

$$\left. \begin{aligned} E^2 + H^2 &= \frac{16\pi\omega^2}{\kappa c^2} \\ E_2 H_{*3} - E_3 H_{*2} &= 0 \\ E_1 H_{*3} - E_3 H_{*1} &= 0 \\ E_1 H_{*2} - E_2 H_{*1} &= -\frac{4\pi\omega}{\kappa c \sin \theta} \\ E_1 E_1 + H_{*1} H_{*1} &= \frac{4\pi\omega^2 (2 + \sin^2 \theta)}{\kappa c^2} \\ E_1 E_2 + H_{*1} H_{*2} &= \frac{4\pi\omega^2 r \sin \theta \cos \theta}{\kappa c^2} \\ E_2 E_2 + H_{*2} H_{*2} &= \frac{4\pi\omega^2 r^2 (2 + \cos^2 \theta)}{\kappa c^2} \\ E_3 E_3 + H_{*3} H_{*3} &= \frac{12\pi\omega^2 r^2 \sin^2 \theta}{\kappa c^2} \end{aligned} \right\}.$$

Please note that $E^2 = E_i E^i = h_{ik} E^i E^k$ and $H^2 = H_{*i} H^{*i} = h_{ik} H^{*i} H^{*k}$. The θ denotes the polar angle, i.e., $\sin \theta = 1$ and $\cos \theta = 0$ in the laboratory located at the equator.

What do the above relations between the electric and magnetic strength of the field mean and how to implement them in a real laboratory is rather better to ask engineers.

15 Non-quantum teleportation in the condition of a real laboratory using a strong electromagnetic field

Looking at the obtained Einstein equations (see page 46), you can see that the mechanical rotation of space, which appears due to the non-holonomy of the space-time, can be replaced by the magnetic or electric strength of the electromagnetic field that fills the space. Two natural questions arise in this regard: 1) Why is this even possible? 2) Is it possible to increase the space-time non-holonomy by an electromagnetic field in a real laboratory to such a level as to realize the non-quantum teleportation condition?

1. To understand why this is possible, you need to understand what is the three-dimensional rotation of space due to the non-holonomy of the space-time. An ordinary three-dimensional rotation is expressed in terms of $g_{ik} \neq 0$ in the space metric and, therefore, can be removed by a coordinate transformation (moving to another, non-rotating coordinate system on the observer's reference body). On the contrary, a rotation resulting from the non-orthogonality of the time lines to the three-dimensional spatial section originates from $g_{0i} \neq 0$ in the metric, and, therefore is not removable by a coordinate transformation; this is one of the fundamental properties of the observer's reference space.

For example, consider an observer in a laboratory located on the surface of the Earth. For him, the ordinary rotation expressed in terms of $g_{ik} \neq 0$, which can be removed by a coordinate transformation, is the rotation of an observed external body, say, the Moon. On the contrary, the non-removable rotation expressed in terms of $g_{0i} \neq 0$ is the rotation of his reference body, the Earth, around its own axis.

In terms of the basis vectors $\vec{e}_{(i)}$ tangential to the curved coordinate lines of the Riemannian space (see page 35),

$$g_{0i} = e_{(0)} e_{(i)} \cos(x^0; x^i), \quad v_i = -c e_{(i)} \cos(x^0; x^i),$$

which means that the linear velocity v_i of such rotation is merely a manifestation of the inclination of the time coordinate lines to the three-dimensional spatial section. Cosine takes numeric values from +1 to -1. The length of the tangential basis vectors is equal to 1 in the absence of perturbing factors and decreases with increasing curvature of the coordinate lines. Therefore, such rotation of space cannot be mechanically increased to superluminal speed.

On the other hand, according to the obtained Einstein equations, the stronger the electromagnetic field, the faster the rotation of space: the limit for increasing the rotation of

space is only the power of the electromagnetic field generator installed in your laboratory. This is because the angular velocity ω of rotation of space contained in them (and in the teleportation condition) has the same origin as the angular velocity ω in the definition of v_i .

As a result, we arrive at the conclusion that there are two types of rotation of space, which cannot be removed by a coordinate transformation. The source of the first type of rotation is a mechanical rotation of the observer's reference body, say, the planet Earth. Such rotation cannot exceed the speed of light. The second type is a "virtual rotation" that appears in a space filled with a distributed matter, due to the non-zero right hand side of the Einstein equations. Such "virtual rotation" is formally added to the first type of rotation of space, despite the fact that the observer's reference body still mechanically rotates at its own rotation speed, as before. This summation occurs because the angular velocity ω of both types of rotation has the same mathematical origin. For example, the Einstein equations showed that such "virtual rotation" can be as fast as the electromagnetic field strong.

This situation is similar to that with the equations of motion of particles. A free particle travels along a geodesic (i.e., shortest) trajectory. The equation of its motion is the equation of geodesic line: the right hand side of the equation is equal to zero. If an external factor perturbs the particle's motion, it deviates from the geodesic line. In this case, its motion is non-geodesic, and the equation of its motion contains the deviating force on the right hand side.

2. Now a second question arises: can we increase ω with an electromagnetic field to the level necessary to implement the teleportation condition in a real laboratory? To answer this question, let us consider the scalar chr.inv.-Einstein equation we have obtained (see page 46)

$$E^2 + H^2 = \frac{16\pi\omega^2}{\kappa c^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}\right)}.$$

If the electric component of the electromagnetic field is much weaker than its magnetic component ($E \ll H$), then in the first order approximation we obtain the relation

$$\omega \approx \sqrt{\frac{\kappa c^2}{16\pi}} H$$

connecting the angular velocity of the "virtual rotation" of space with the magnetic strength of the electromagnetic field (which is the source of this "virtual rotation").

On the other hand, the non-quantum teleportation condition is a rotating space filled with an electromagnetic field has the form (see page 45)

$$\omega r^2 \sin^2 \theta \frac{d\varphi}{dt} = c^2,$$

where we assume that the space rotates with the linear velocity $v_3 = \omega r^2 \sin^2 \theta$ along the φ -axis (geographical latitude),

$\omega = \text{const}$ is the angular velocity of this rotation, and the last multiplier is the coordinate velocity $\tilde{\omega}$ of the teleporting particle along the φ -direction (we assume that the particle travels either in the same or in the opposite direction in which the space rotates).

Assume that the observer's laboratory is located on the Earth's equator. In this case, $\sin \theta = 1$. Then the ω necessary to launch a particle onto a teleportation trajectory in the observer's laboratory has the form

$$\omega = \frac{c^2}{\tilde{\omega} r^2}.$$

Substituting here the formula for ω obtained above from the scalar chr.inv.-Einstein equation, we obtain the magnetic strength required for non-quantum teleportation in the condition of the earth-bound laboratory

$$H \approx \sqrt{\frac{16\pi}{\kappa}} \frac{c}{\tilde{\omega} r^2}.$$

That is, as soon as the magnetic strength inside the experimental setup reaches a numerical value according to this formula (and with the configuration of the electromagnetic field according to the obtained Einstein equations), a teleportation channel opens between this experimental setup and another remote experimental setup located anywhere else in the Universe. Synchronization of these two experimental setups is implemented using the same fine tuning of the magnetic field configuration and other characteristics, which allows physical bodies to be teleported only between these two setups, and not to some other place in the Universe.

Regarding the specific numerical value of H , necessary to implement the teleportation condition in the earth-bound laboratory, it depends on the understanding of the physical sense of the $\tilde{\omega}$ and r in the above formula, as well as on the system of dimensions of electromagnetic quantities. Meanwhile, even on the basis of draft calculations and other information (that cannot be made public), we are sure that such an experimental setup is quite possible using a super-powerful pulsed magnetic field generator. These specific calculations, as well as the creation of such an experimental setup, are a task for engineers rather than for a theoretical physicist who is far from technology.

A century ago, Nikola Tesla claimed that the use of super-strong electromagnetic fields will allow us to travel instantly to any point in the Universe. We have no idea where he got this information from. Nevertheless, we are very glad that the words he uttered a century ago have now received a solid mathematical foundation in Einstein's theory.

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Deflection of Light Rays and Mass-Bearing Particles in the Field of a Rotating Body

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As proved earlier, the space of a rotating body is Riemannian only if it is filled with a distributed matter (Progr. Phys., 2022, v.18, 31–49). In this paper we consider motion of massless (light-like) and mass-bearing particles in the space of a rotating body filled with an electromagnetic field, where the influence of gravitation is negligible. Solving the equations of motion of a particle that does not have an electric charge, we find that its motion deflects from a straight line due to the space curvature caused by the rotation of space. That is, the trajectories of light rays and mass-bearing particles are deflected near a rotating body due to the curvature of space caused by its rotation. This is one more fundamental effect of the General Theory of Relativity, in addition to the deflection of light rays in the field of a gravitating body.

In this small paper, which is a continuation of the previous one [1], we consider the equations of motion of a massless (light-like) particle and a mass-bearing particle in the space of a rotating body, which is filled with an electromagnetic field, and the influence of gravitation is negligible.

As proved in the previous paper, a rotating space has a significant curvature due to its space-time non-holonomy (non-orthogonality of the time lines to the three-dimensional spatial section). For this reason, we expect to find that the space curvature caused by the rotation of space deflects light rays and mass-bearing particles near a rotating body.

Please note that, as proved earlier using Einstein's equations [1], the space of a rotating body is Riemannian only if it is filled with a distributed matter, say, an electromagnetic field. Therefore, the above problem statement will not be valid and mathematically correct in an empty space or in a space filled only with a gravitational field. In the case we are considering, as in the previous article, the "space filler" is an electromagnetic field.

In this work, as well as in our other works, we use the mathematical apparatus of chronometric invariants, which are physically observable quantities in the General Theory of Relativity. This mathematical apparatus was created in 1944 by our esteemed teacher A. L. Zelmanov (1913–1987). Its basics can be learned from Zelmanov's publications [2–4], of which his 1957 presentation [4] is the most useful and complete, and also from our previous article [1]. For a deeper study of this mathematical apparatus, read the respective chapters of our monographs [5, 6], especially — the chapter Tensor Algebra and the Analysis in [6].

The equations of motion of both mass-bearing and massless (light-like) particles were studied in detail in our two monographs [5, 6]. The first one [5] focused onto free motion of particles, and the second one [6] focused onto non-geodesic motion of particles: the right hand side of the equations of non-geodesic motion is non-zero, and contains the

external force deflecting the particles from geodesic (shortest) trajectories.

The chronometrically invariant equations of motion are the physically observable projections of the general covariant four-dimensional equations of motion onto the time line and the three-dimensional spatial section of a particular observer. Such projections are invariant along the spatial section of the observer (his observed space) and are expressed through the properties of his local reference space. Those who are interested in how the equations of motion are derived can refer to the respective chapters of our monographs, where all these equations are explained in detail.

In this paper we consider mass-bearing particles that do not have an electric charge, and massless (light-like) particles are not electrically charged by definition. As a result, the right hand side of the equations of their motion, containing the force acting on electrically charged particles from the electromagnetic field, is equal to zero. Therefore, these are free particles, and the equations of their motion are the equations of motion along geodesic lines.

The chr.inv.-equations of motion of a free mass-bearing particle describe the motion along an ordinary geodesic line

$$\left. \begin{aligned} \frac{dm}{d\tau} - \frac{m}{c^2} F_i v^i + \frac{m}{c^2} D_{ik} v^i v^k &= 0 \\ \frac{d(m v^i)}{d\tau} + 2m (D_k^i + A_k^i) v^k - m F^i + m \Delta_{nk}^i v^n v^k &= 0 \end{aligned} \right\},$$

and the chr.inv.-equations of motion of a free massless (light-like) particle describe the motion along an isotropic geodesic line (a.k.a. a null geodesic line)

$$\left. \begin{aligned} \frac{d\omega}{d\tau} - \frac{\omega}{c^2} F_i c^i + \frac{\omega}{c^2} D_{ik} c^i c^k &= 0 \\ \frac{d(\omega c^i)}{d\tau} + 2\omega (D_k^i + A_k^i) c^k - \omega F^i + \omega \Delta_{nk}^i c^n c^k &= 0 \end{aligned} \right\}.$$

Here in the equations of motion and so forth, m is the relativistic mass of the travelling particle, ω is the relativistic frequency of the massless (light-like) particle, $d\tau$ is the physically observable time interval expressed through the linear velocity v_i of the rotation of space

$$d\tau = \sqrt{g_{00}} dt - \frac{1}{c^2} v_i dx^i,$$

$$v_i = -\frac{c g_{0i}}{\sqrt{g_{00}}}, \quad v^i = -c g^{0i} \sqrt{g_{00}},$$

and, respectively the chr.inv.-vector of the physically observable velocity of the travelling particle has the form

$$v^i = \frac{dx^i}{d\tau}, \quad v_i v^i = h_{ik} v^i v^k = v^2,$$

which in the ultimate case transforms into the chr.inv.-vector of the physically observable velocity of light, the square of which is $c_i c^i = h_{ik} c^i c^k = c^2$.

Please note that, according to the theory of chronometric invariants, the square of any chr.inv.-quantity, and also lifting and lowering indices in chr.inv.-quantities is determined through the chr.inv.-metric tensor

$$h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k, \quad h^{ik} = -g^{ik}, \quad h_k^i = -g_k^i = \delta_k^i,$$

which is obtained as the spatial chr.inv.-projection of the fundamental metric tensor $g_{\alpha\beta}$ and has all its properties everywhere in the observer's three-dimensional spatial section.

Concerning the physically observable characteristics of space, which are terms in the equations of motion, these are the chr.inv.-vector of the gravitational inertial force F^i (where $w = c^2(1 - \sqrt{g_{00}})$ is the gravitational potential), the antisymmetric chr.inv.-tensor of the angular velocity of rotation of space, A_{ik} , the symmetric chr.inv.-tensor of deformation of space, D_{ik} , and the chr.inv.-Christoffel symbols Δ_{jk}^i (coherence coefficients of space), i.e.

$$F_i = \frac{1}{1 - \frac{w}{c^2}} \left(\frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t} \right),$$

$$A_{ik} = \frac{1}{2} \left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k} \right) + \frac{1}{2c^2} (F_i v_k - F_k v_i),$$

$$D_{ik} = \frac{1}{2} \frac{\partial h_{ik}}{\partial t}, \quad D^{ik} = -\frac{1}{2} \frac{\partial h^{ik}}{\partial t},$$

$$\Delta_{jk}^i = h^{im} \Delta_{jk,m} = \frac{1}{2} h^{im} \left(\frac{\partial h_{jm}}{\partial x^k} + \frac{\partial h_{km}}{\partial x^j} - \frac{\partial h_{jk}}{\partial x^m} \right),$$

where the chr.inv.-operators of derivation have the form

$$\frac{\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{g_{0i}}{g_{00}} \frac{\partial}{\partial x^0}.$$

In our further calculation of the deflection of light rays and mass-bearing particles in the field of a rotating body we

will use the same space metric that we introduced in the previous paper [1]. This is the metric of a space, where the three-dimensional space rotates due to the non-holonomy of the space-time, but there is no field of gravitation (or, to be more exact, the influence of gravitation is negligible).

Assume that the space rotates along the equatorial axis φ , i.e., along the geographical longitudes, with the velocity $v_3 = \omega r^2 \sin^2 \theta$, where $\omega = \text{const}$ is the angular velocity of this rotation. Then, according to the definition of v_i ,

$$v_3 = \omega r^2 \sin^2 \theta = -\frac{c g_{03}}{\sqrt{g_{00}}},$$

we obtain the metric of such a space

$$ds^2 = c^2 dt^2 - 2\omega r^2 \sin^2 \theta dt d\varphi - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).$$

As you can see, the non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of this metric are equal to

$$g_{00} = 1, \quad g_{03} = -\frac{\omega r^2 \sin^2 \theta}{c},$$

$$g_{11} = -1, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2 \theta,$$

and, according to the definition of the chr.inv.-metric tensor h_{ik} , its non-zero components in the metric are equal to

$$h_{11} = 1, \quad h_{22} = r^2, \quad h_{33} = r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \right),$$

$$h^{11} = 1, \quad h^{22} = \frac{1}{r^2}, \quad h^{33} = \frac{1}{r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \right)},$$

where, since the matrix h_{ik} is diagonal, the upper-index components of h_{ik} are obtained as $h^{ik} = (h_{ik})^{-1}$ just like the invertible matrix components to any diagonal matrix.

Using the definition of the antisymmetric chr.inv.-tensor of the angular velocity of rotation of space, A_{ik} , we obtain that its non-zero components in the rotating space we are considering are equal to

$$A_{13} = \omega r \sin^2 \theta, \quad A_{31} = -A_{13},$$

$$A_{23} = \omega r^2 \sin \theta \cos \theta, \quad A_{32} = -A_{23},$$

$$A^{13} = \frac{\omega}{r \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \right)}, \quad A^{31} = -A^{13},$$

$$A^{23} = \frac{\omega \cot \theta}{r^2 \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \right)}, \quad A^{32} = -A^{23}.$$

Using the definition of the chr.inv.-Christoffel symbols Δ_{jk}^i (coherence coefficients of space), after some algebra, we obtain formulae for their non-zero components in the rotating

space we are considering. They have the form

$$\begin{aligned}\Delta_{22}^1 &= -r, \\ \Delta_{33}^1 &= -r \sin^2 \theta \left(1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2} \right), \\ \Delta_{12}^2 &= \Delta_{21}^2 = \frac{1}{r}, \\ \Delta_{33}^2 &= -\sin \theta \cos \theta \left(1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2} \right), \\ \Delta_{13}^3 &= \Delta_{13}^3 = \frac{1}{r \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \right)} \left(1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2} \right), \\ \Delta_{23}^3 &= \Delta_{32}^3 = \frac{\cot \theta}{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} \left(1 + \frac{2\omega^2 r^2 \sin^2 \theta}{c^2} \right).\end{aligned}$$

Now, using the obtained physically observable characteristics of the rotating space we are considering, we will modify the general formulae of the chr.inv.-equations of free motion (see above) in accordance with the space metric. As a result, we will obtain the chr.inv.-equations of motion of a free mass-bearing particle and a free massless (light-like) particle in the rotating space. The solution of these equations will show the effect of deflection of light rays and mass-bearing particles in the field of a rotating body.

Since $g_{00} = 1$ in the metric, and the rotation of space is stationary ($v_3 = \omega r^2 \sin^2 \theta$ is not time-dependent), then the gravitational potential $w = c^2 (1 - \sqrt{g_{00}})$ is equal to zero and, hence, the gravitational inertial force vanish, $F_i = 0$.

In addition, you can see that neither the fundamental metric tensor $g_{\alpha\beta}$ nor the chr.inv.-metric tensor h_{ik} of the metric are not time-dependent, the rotating space we are considering does not deform and, hence, the tensor of deformation of space vanish, $D_{ik} = 0$.

As a result, since $F_i = 0$ and $D_{ik} = 0$, the chr.inv.-equations of motion of a free mass-bearing particle in the rotating space we are considering take the simplified form

$$\left. \begin{aligned}\frac{dm}{d\tau} &= 0 \\ \frac{d(mv^i)}{d\tau} + 2mA_k^i v^k + m\Delta_{nk}^i v^n v^k &= 0\end{aligned} \right\},$$

and the chr.inv.-equations of motion of a free massless (light-like) particle are simplified to the form

$$\left. \begin{aligned}\frac{d\omega}{d\tau} &= 0 \\ \frac{d(\omega c^i)}{d\tau} + 2\omega A_k^i c^k + \omega \Delta_{nk}^i c^n c^k &= 0\end{aligned} \right\}.$$

The above equations are identical. Therefore they are solved in the same way and have the same solution.

Consider the above equations of motion of a free mass-bearing particle as a sample (the solution for a free massless particle will be the same).

The scalar equation of motion solves as $m = \text{const}$. With this solution taken into account, we substitute here the obtained formulae for the tensor of the angular velocity of rotation of space, A_{ik} , and the Christoffel symbols Δ_{jk}^i . As a result, neglecting higher order terms (otherwise the equations are unsolvable), we obtain the vectorial equations of motion in the component form suitable for their further analysis

$$\left. \begin{aligned}\frac{dv^1}{d\tau} - 2\omega r \sin^2 \theta v^3 - r v^2 v^2 - r \sin^2 \theta v^3 v^3 &= 0 \\ \frac{dv^2}{d\tau} - 2\omega \sin \theta \cos \theta v^3 + \frac{2}{r} v^1 v^2 - \sin \theta \cos \theta v^3 v^3 &= 0 \\ \frac{dv^3}{d\tau} + \frac{2\omega}{r} v^1 + 2\omega \cot \theta v^2 + \frac{2}{r} v^1 v^3 + 2 \cot \theta v^2 v^3 &= 0\end{aligned} \right\}.$$

Even a brief look at the obtained equations of motion shows that the three possible effects are conceivable:

1. The deflection of a travelling free particle along the geographic longitudes (the third equation in the above system);
2. The deflection of a travelling free particle along the geographic latitudes (the second equation);
3. The acceleration or braking of a travelling free particle in the radial direction (the first equation).

The problem is that the above system of differential equations is unsolvable in the general form. Therefore, we will consider a simplified particular case of the equations, and calculate all three of the above effects just for this case.

Consider a particle travelling at a very high radial velocity v^1 in the equatorial plane exactly along the radial axis to the origin of the coordinates. Say, a particle from the near-Earth space travels freely in the equatorial plane directly to the Earth's surface. In this case, the velocities of its deflection along the geographical latitudes and longitudes, v^2 and v^3 , are negligible compared to v^1 , and the above equations take the simplified form

$$\left. \begin{aligned}\frac{dv^1}{d\tau} - 2\omega r v^3 - r v^2 v^2 - r v^3 v^3 &= 0 \\ \frac{dv^2}{d\tau} + \frac{2}{r} v^1 v^2 &= 0 \\ \frac{dv^3}{d\tau} + \frac{2\omega}{r} v^1 + \frac{2}{r} v^1 v^3 &= 0\end{aligned} \right\}.$$

In addition, we assume that the particle's velocity in the radial direction gains only a very small increment or decrement α' compared to its numerical value v^1 , which, according to our initial assumption, is very large. As a result, we set $v^1 = \text{const}$ in the equations of motion along the equatorially

longitudinal axis φ (third equation) and the latitudinal axis θ (second equation), but solve the equation of motion along the radial axis r (first equation) with respect to $v^1 + \alpha'$, i.e., with respect to the small parameter α . Otherwise, the above system of differential equations is unsolvable.

1. Consider the third equation of motion (along the equatorial axis φ). With the above assumptions, this equation takes the form, respectively,

$$y' + ay + b = 0, \quad \varphi'' + a\varphi' + b = 0,$$

where we used the following notations

$$y = v^3 = \frac{d\varphi}{d\tau}, \quad a = \frac{2}{r} v^1 = \text{const}, \quad b = \frac{2\omega}{r} v^1 = \text{const}.$$

The above differential equations for the velocity $y = v^3$ and the coordinate φ with respect to the physically observable time $\tau = x$ are solved as

$$y = \frac{C}{e^{ax}} - \frac{b}{a}, \quad \varphi = \frac{C_1}{e^{ax}} - \frac{bx}{a} + C_2,$$

where the constants of integration found using the initial conditions $x = x_0 = 0$ and $y = y_0 = 0$, are equal to

$$C = \frac{b}{a} = \omega, \quad C_1 = -\frac{b}{a^2} = -\frac{\omega r}{2v^1}, \quad C_2 = -C_1 = \frac{\omega r}{2v^1}.$$

As a result, we obtain solutions for the particle's velocity $y = v^3$ along the equatorial axis (along the geographic longitudes), as well as for the equatorial coordinate φ (geographical longitude) of the arrival point of this particle.

The obtained solution for the particle's velocity along the equatorial axis φ has the form

$$v^3 = -\omega + \omega e^{-\frac{2}{r} v^1 \tau}.$$

Here the first term $-\omega$ is the particle's basics equatorial velocity, the origin of which is the banally shift of the equatorial coordinate φ to its negative numerical values due to the Earth's turn over the particle's travel to the Earth.

The second, additional term means that a particle freely travelling to the surface of a rotating body gains an additional velocity directed along the equator (geographical longitudes) opposite to the rotation of the body.

The obtained solution for the equatorial coordinate φ of the arrival point of this particle has the form

$$\varphi = \varphi_0 - \omega \tau + \frac{\omega r}{2v^1} \left(1 - e^{-\frac{2}{r} v^1 \tau} \right).$$

The third, additional term of this solution means that a particle freely travelling to the surface of a rotating body is deflected along the equator (geographical longitudes) opposite to the rotation of the body.

All this is because the rotation of any body gets space curved near it, thereby creating a "slope of the hill" slowing

"down" along the equator towards the rotation of this body. In other words, space is curved by a rotating body in the direction of its rotation. As a result, a particle freely travelling to a rotating body "rolls down the curvature hill" of space along the equator in the direction in which the body rotates.

The same effect is expected for light rays, since the equations of motion for a massless (light-like) particle and a mass-bearing particle are identical, and, hence, their solutions coincide (see above). Only the mass-bearing particle's velocity is replaced with the physically observable velocity of light.

Please note that, as Zelmanov showed in 1944 using the mathematical apparatus of chronometric invariants, the vectorial components of the physically observable velocity of light depend on the geometric properties of space, as well as on the physical properties of distributed matter, despite the fact that the square of the velocity remains invariant.

As a result, the solution for the equatorial coordinate φ of the arrival point of a light ray falling down from space onto the Earth's surface in the equatorial plane has the form

$$\varphi = \varphi_0 - \omega \tau + \frac{\omega r}{2c^1} \left(1 - e^{-\frac{2}{r} c^1 \tau} \right),$$

where c^1 is the physically observable velocity of light in the radial direction.

Since the Earth, as well as any other planet or star, has its own gravitational field, a mass-bearing particle freely travelling to its surface gains a substantial acceleration. In this case, the particle's radial velocity cannot be assumed to be constant even in the first order approximation. For this reason, we will calculate the numerical value of the above effect, which we theoretically discovered, for a light ray.

Consider a light ray travelling, say, from the Moon to the Earth's surface along the radial axis r in the equatorial plane of the Earth. In this case, the physically observable velocity of light is equal to $c^1 = -3 \times 10^{10}$ cm/sec, since the vector of the velocity of light is directed opposite to the reading of the radial coordinates, the origin of which is the centre of the Earth. The Earth rotates around its axis with the angular velocity $\omega = 1$ rev/day $= 1.16 \times 10^{-5}$ rev/sec, and the Earth's radius is equal to $r = 6.4 \times 10^8$ cm. As a result, we obtain that the curvature of space caused by the Earth's rotation around its axis deflects a light ray coming to the Earth's surface from the Moon ($\tau = 1$ sec) in the longitudinal direction in which the Earth rotates by the angle equal to

$$\Delta\varphi = \frac{\omega r}{2c^1} \left(1 - e^{-\frac{2}{r} c^1 \tau} \right) = 1.2 \times 10^{-7} \text{ rev} = 0.16'',$$

where the main goal into the effect is made due to the first term, and the second term is equal to 1.5×10^{-41} and, therefore, can be neglected.

The effect calculated for the Earth is small. Meanwhile, this effect increases with the radius and rotation velocity of the cosmic body. For example, the Sun has the radius equal to $r = 7.0 \times 10^{10}$ cm, and rotates around its axis with the angular

velocity $\omega = 4.5 \times 10^{-7}$ rev/sec. Therefore, the curvature of space caused by the Sun's rotation around its axis deflects a light ray coming to the Sun's surface in the longitudinal direction in which the Sun rotates by the angle equal to

$$\Delta\varphi = 5.3 \times 10^{-7} \text{ rev} = 0.68''.$$

Obviously, this effect has a much larger numerical value near a rapidly rotating star, such as Wolf-Rayet stars or neutron stars.

2. Now consider the second equation of motion (along the geographical latitudes, where the polar angle θ is read from the North pole). With the same assumptions as those we used in the third equation above, neglecting higher order terms and taking the obtained solution $v^3 = -\omega$ into account, this equation takes the form, respectively,

$$y' + ay = 0, \quad \theta'' + a\theta' = 0,$$

where

$$y = v^2 = \frac{d\theta}{d\tau}, \quad a = \frac{2}{r} v^1 = \text{const.}$$

The above differential equations are solved as

$$y = \frac{C}{e^{ax}}, \quad \theta = \frac{C_1}{e^{ax}} + C_2,$$

where the constants of integration found using the initial conditions $x = x_0 = 0$ and $y = y_0 = 0$, are equal to $C = 0$, $C_1 = 0$ and $C_2 = \theta_0$. As a result, the solutions take the final form

$$v^2 = 0, \quad \theta = \theta_0,$$

i.e., a particle freely travelling to the surface of a rotating body is not deflected up or down the geographical latitudes.

3. Finally, consider the first equation of motion (along the radial coordinates). As is explained in the beginning, we assume that the particle's velocity in the radial direction gains a very small increment or decrement α' compared to its numerical value v^1 , which, according to our initial assumption, is very large. Thus, we assume $v^1 = \text{const}$ and solve the first equation of motion with respect to $v^1 + \alpha'$, i.e., with respect to the small parameter α . Neglecting higher order terms and taking the obtained solutions $v^3 = -\omega$ and $v^2 = 0$ into account, the first equation of motion takes the form, respectively,

$$y' + b = 0, \quad \alpha'' + b = 0,$$

where $y = \alpha'$ and $b = \omega^2 r = \text{const}$ (here r is the radius of the rotating body). These simplest equations are solved as

$$y = C - bx, \quad \alpha = -\frac{bx^2}{2} + C_2 x + C_1,$$

where, using the initial conditions $x = x_0 = 0$, $\alpha = \alpha_0 = 0$ and $y = y_0 = 0$, we find that the constants of integration are equal to zero. As a result, we obtain

$$\alpha' = -\omega^2 r \tau, \quad \alpha = -\frac{\omega^2 r \tau^2}{2}.$$

This solution means that a particle freely travelling to a rotating body gains an additional speed, and the length of its path is physically "stretched" due to the curvature of space caused by the body's rotation. As a result, the particle reaches the body later (with a delay in time) compared if the body did not rotate.

Thus, according to the obtained solution, the increment of the path length of a light ray that travelled, say, from the Moon to the Earth, and also the delay in time of its arrival are equal to

$$\alpha = -1.7 \text{ cm}, \quad \Delta\tau = \frac{\alpha}{c^1} = 5.7 \times 10^{-11} \text{ sec},$$

while such corrections for a light ray that travelled from the Earth to the Sun are equal to

$$\alpha = -6.6 \times 10^4 \text{ cm}, \quad \Delta\tau = \frac{\alpha}{c^1} = 2.2 \times 10^{-6} \text{ sec}.$$

So, we theoretically found that a particle travelling freely to a rotating body is deflected slightly from its radial trajectory in the equatorial direction, in which the body rotates, i.e., along the geographical longitudes. In addition, during the travel, the particle gains a small increase of its velocity, and its path is physically "stretched" for a little, as a result of which the particle reaches the body with a delay in time compared to if the body did not rotate.

These two effects take place both for mass-bearing particles and for light rays (massless light-like particles).

The origin of these effects is the space curvature caused by the rotation of space. When any body rotates, the space around it curves towards the direction of its rotation and the centre of the body (the centre of rotation), thereby creating a "slope of the hill" descending "down" along the equator in the direction, in which the body rotates, and also to the centre of the body. When a particle travels freely to a rotating body, it "rolls down" the slope of the space curvature along the equator in the direction, in which the body rotates, as well as to the centre of the body.

These are two new fundamental effects of the General Theory of Relativity, we have discovered "au bout d'un stylo" in addition to the Einstein effect of the deflection of light rays in the field of a gravitating body.

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Physics of Transcendental Numbers as Forming Factor of the Solar System

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Transcendental ratios of physical quantities can inhibit the occurrence of destabilizing parametric resonance and in this way, provide stability in systems of coupled periodic processes. In this paper we apply this approach to the solar system and show that it can explain the current set of rotational and orbital periods and distances including observed tendencies of their evolution.

Introduction

One of the unsolved fundamental problems in physics [1] is the stability of systems of a large number of coupled periodic processes, for instance, the stability of planetary systems. If numerous bodies are gravitationally bound to one another, perturbation models predict long-term highly unstable states [2] that contradict the physical reality of the solar system and thousands of exoplanetary systems.

Another issue is that in theory, there are infinitely many pairs of orbital periods and distances that fulfill Kepler's laws. Regrettably, Einstein's field equations do not reduce the theoretical variety of possible orbits, but increases it even more. As a consequence, the current orbital system of the Sun seems to be accidental, and its stability a miracle.

Furthermore, there is no known law concerning the rotation of celestial bodies besides conservation of the angular momentum [3] that they retain from the protoplanetary disk, so that also the current distribution of the rotational periods appears as to be accidental.

However, many planets in extrasolar systems like Trappist 1 or Kepler 20 have almost the same orbital periods as the large moons of Jupiter, Saturn, Uranus and Neptune [4]. Trappist 1 is 40 light years away from our solar system [5] and Kepler 20 nearly 1000 light years [6].

The question is, why they prefer similar orbital periods if there are infinite possibilities? Obviously, there are orbital periods preferred anywhere in the galaxy. Why these orbital periods are preferred? What makes them attractive?

In this paper, we introduce an approach to the problem of stability based on the physical interpretation of certain statements of number theory. This approach leads us to the conclusion that in real systems, bound periodic processes approximate transcendental frequency ratios that allow them to avoid destabilizing parametric resonance. We illustrate this conclusion on some well-known features of the solar system which are still unexplained.

Theoretical Approach

The starting point of our approach is the measurement as it is the source of data that allow us developing and proofing theoretical models of the reality. The result of a measure-

ment is the ratio of physical quantities where one of them is the reference quantity called unit of measurement. Whether measuring a wavelength or phase, a frequency, the speed or duration of some process, the mass of a body or its temperature, initially this ratio is a real number, regardless of its subsequent interpretation as component of a vector or tensor, for example. As real value, this ratio can approximate an integer, rational, irrational algebraic or transcendental number. In [7] we have shown that the difference between rational, irrational algebraic and transcendental numbers is not only a mathematical task, but it is also an essential aspect of stability in systems of bound periodic processes. For instance, integer frequency ratios, in particular fractions of small integers, make possible parametric resonance that can destabilize such a system [8, 9]. For instance, asteroids cannot maintain orbits that are unstable because of their resonance with Jupiter [10]. These orbits form the Kirkwood Gaps, which are areas in the asteroid belt where asteroids are absent.

According to this idea, irrational ratios should not cause destabilizing resonance interactions, because irrational numbers cannot be represented as a ratio of integers. However, algebraic irrational numbers, being real roots of algebraic equations, can be converted to rational numbers by multiplication. For example, the algebraic irrational number $\sqrt{2} = 1.41421 \dots$ cannot become a frequency scaling factor in real systems of coupled periodic processes, because $\sqrt{2} \cdot \sqrt{2} = 2$ creates the conditions for the occurrence of parametric resonance. Thus, only transcendental ratios can prevent parametric resonance, because they cannot be converted to rational or integer numbers by multiplication.

Actually, it is transcendental numbers, that define the preferred frequency ratios which allow to avoid destabilizing resonance [11]. In this way, transcendental frequency ratios sustain the lasting stability of coupled periodic processes. With reference to the evolution of a planetary system and its stability, we may therefore expect that the ratio of any two orbital periods should finally approximate a transcendental number.

Among all transcendental numbers, Euler's number $e = 2.71828 \dots$ is unique, because its real power function e^x coincides with its own derivatives. In the consequence, Euler's number allows inhibiting parametric resonance between any coupled periodic processes including their derivatives.

Because of this unique property of Euler's number, we expect that periodic processes in real systems prefer frequency ratios close to Euler's number and its roots. The natural logarithms of those frequency ratios are therefore close to integer $0, \pm 1, \pm 2, \dots$ or rational $\pm 1/2, \pm 1/3, \pm 1/4, \dots$ values. For rational exponents, the natural exponential function is always transcendental [12]. As shown by A. Khinchine [13], any rational number has a biunique presentation as a finite continued fraction. Consequently, we can present the natural logarithms of the frequency ratios we are looking for as finite continued fractions:

$$\ln(\omega_A/\omega_B) = \mathcal{F} = \langle n_0; n_1, n_2, \dots, n_k \rangle \quad (1)$$

ω_A and ω_B are the angular frequencies of two bound periodic processes A and B avoiding parametric resonance. We use angle brackets for continued fractions. All denominators n_1, n_2, \dots, n_k of a continued fraction including the free link n_0 are integer numbers. All numerators equal 1. The length of a continued fraction is given by the number k of layers.

Finite continued fractions represent all rational numbers in the sense that there is no rational number that cannot be represented by a finite continued fraction. This universality of continued fractions evidences that the distribution of rational logarithms (1) in the number continuum is fractal.

The first layer of this fractal is given by the truncated after n_1 continued fractions:

$$\langle n_0; n_1 \rangle = n_0 + \frac{1}{n_1}$$

The denominators n_1 follow the sequence of integer numbers $\pm 1, \pm 2, \pm 3$ etc. The second layer is given by the truncated after n_2 continued fractions:

$$\langle n_0; n_1, n_2 \rangle = n_0 + \frac{1}{n_1 + \frac{1}{n_2}}$$

Figure 1 shows the first and the second layer in comparison. As we can see, reciprocal integers $\pm 1/2, \pm 1/3, \pm 1/4, \dots$ are the attractor points of the fractal. In these points, the distribution density of rational logarithms (1) reaches a local maximum. Integers $0, \pm 1, \pm 2, \dots$ define the main attractors. Consequently, integer arguments of the natural exponential function define attractor points of transcendental numbers and ranges of stability that allow bound periodic processes to avoid parametric resonance.

Figure 1 shows that integer logarithms $0, \pm 1, \pm 2, \dots$ form the widest ranges of stability. Half logarithms $\pm 1/2$ form smaller ranges, third logarithms $\pm 1/3$ form the next smaller ranges and fourth logarithms $\pm 1/4$ form even smaller ranges of stability etc. Increasing the length of the continued fraction (1), the distribution density of the transcendental frequency

ratios ω_A/ω_B is increasing as well. Nevertheless, their distribution is not homogeneous, but fractal. Applying continued fractions and truncating them, we can represent the logarithms $\ln(\omega_A/\omega_B)$ as rational numbers $\langle n_0; n_1, n_2, \dots, n_k \rangle$ and make visible their fractal distribution.



Fig. 1: The distribution of rational logarithms for $k = 1$ (above) and for $k = 2$ (below) in the range $-1 \leq \mathcal{F} \leq 1$.

Here I would like to underline that the application of continued fractions doesn't limit the universality of our conclusions, because continued fractions deliver biunique representations of all real numbers including transcendental. Therefore, the fractal distribution of transcendental ratios (1) is an inherent feature of the number continuum that we call the *Fundamental Fractal* [11].

The natural exponential function $\exp(\mathcal{F})$ of the rational argument $\mathcal{F} = \langle n_0; n_1, n_2, \dots, n_k \rangle$ generates a fractal set of transcendental frequency ratios $\omega_A/\omega_B = \exp(\mathcal{F})$ which allow to avoid destabilizing parametric resonance and in this way, provide the lasting stability of periodic processes bound in systems regardless of their complexity. This conclusion we have exemplified [14] in particle physics, astrophysics, geophysics, biophysics and engineering.

For bound harmonic quantum oscillators, the continued fractions \mathcal{F} define not only ratios of frequencies ω , oscillation periods $\tau = 1/\omega$ and wavelengths $\lambda = c/\omega$, but also ratios of accelerations $a = c \cdot \omega$, energies $E = \hbar \cdot \omega$ and masses $m = \omega \cdot \hbar/c^2$, which allow to avoid parametric resonance.

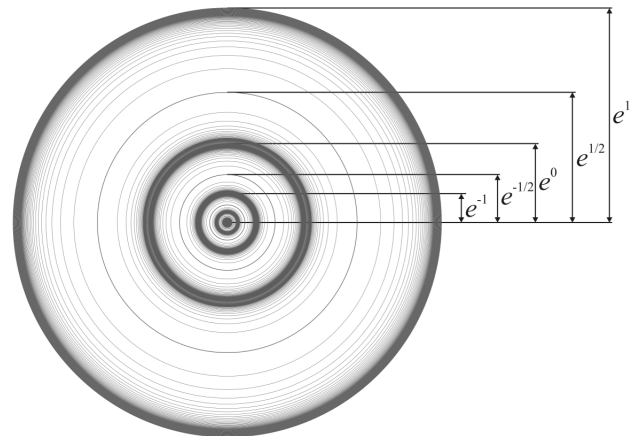


Fig. 2: The first layer ($k = 1$) of equipotential surfaces of the Fundamental Field in the 2D-projection in the range $-1 \leq \mathcal{F} \leq 1$.

The spatio-temporal projection of the Fundamental Fractal \mathcal{F} is a fractal scalar field of transcendental attractors, the *Fundamental Field* [15]. The connection between the spatial and temporal projections is given by the speed of light $c = 299792458$ m/s. The constancy of c makes both projections isomorphic, so that there is no arithmetic or geometric

difference. Only the units of measurement are different. Figure 2 shows the 2D-projection $\exp(\mathcal{F})$ of its first layer. The Fundamental Field is topologically 3-dimensional, a fractal set of embedded spheric equipotential surfaces. The logarithmic potential difference defines a gradient directed to the center of the field that causes a central force of attraction creating the effect of a field source. Because of the fractal logarithmic hyperbolic metric of the field, also every equipotential surface is an attractor. The logarithmic scalar potential difference $\Delta\mathcal{F}$ of sequent equipotential surfaces equals the difference of sequent continued fractions (1) on a given layer:

$$\Delta\mathcal{F} = \langle n_0; n_1, \dots, n_k \rangle - \langle n_0; n_1, \dots, n_k + 1 \rangle$$

Main equipotential surfaces at $k = 0$ correspond with integer logarithms (1); equipotential surfaces at deeper layers $k > 0$ correspond with rational logarithms.

The Fundamental Field is of pure arithmetic origin, and there is no particular physical mechanism required as field source. It is all about transcendental ratios of frequencies [11] that allow coupled periodic processes to avoid destabilizing parametric resonance. Hence, the Fundamental Field concerns all repetitive processes which share at least one characteristic – the frequency. Therefore, we postulate the universality of the Fundamental Field that affects any type of physical interaction, regardless of its complexity.

| PROPERTY | ELECTRON | PROTON |
|----------------------|----------------------------|----------------------------|
| $E = mc^2$ | 0.5109989461(31) MeV | 938.2720813(58) MeV |
| $\omega = E/\hbar$ | $7.76344 \cdot 10^{20}$ Hz | $1.42549 \cdot 10^{24}$ Hz |
| $\tau = 1/\omega$ | $1.28809 \cdot 10^{-21}$ s | $7.01515 \cdot 10^{-25}$ s |
| $\lambda = c/\omega$ | $3.86159 \cdot 10^{-13}$ m | $2.10309 \cdot 10^{-16}$ m |

Table 1: The basic set of physical properties of the electron and proton. Data from Particle Data Group [21]. Frequencies, oscillation periods and wavelengths are calculated.

In fact, scale relations in particle physics [16, 17], nuclear physics [18, 19] and astrophysics [15, 20] obey the same Fundamental Fractal (1), without any additional or particular settings. The proton-to-electron rest energy ratio approximates the first layer of the Fundamental Fractal that could explain their exceptional stability [14]. Normal matter is formed by nucleons and electrons because they are exceptionally stable quantum oscillators. In the concept of isospin, proton and neutron are viewed as two states of the same quantum oscillator. Furthermore, they have similar rest masses. However, a free neutron decays into a proton and an electron within 15 minutes while the life-spans of the proton and electron top everything that is measurable, exceeding 10^{29} years [21]. The proton-to-electron ratio (tab. 1) approximates the seventh

power of Euler's number and its square root:

$$\ln\left(\frac{E_p}{E_e}\right) = \ln\left(\frac{938.2720813 \text{ MeV}}{0.5109989 \text{ MeV}}\right) \simeq 7 + \frac{1}{2} = \langle 7; 2 \rangle$$

In the consequence of this potential difference of the proton relative to the electron, the scaling factor \sqrt{e} connects attractors of proton stability with similar attractors of electron stability in alternating sequence.

Applying Khinchine's [13] continued fraction method, we get the best approximation of the proton-to-electron ratio:

$$\ln\left(\frac{E_p}{E_e}\right) = 7 + \frac{1}{2} + \frac{1}{\frac{9}{64 + \frac{1}{11}}} = 7.515427769 \dots$$

Recent data [22] of the proton-to-electron ratio define the upper limit as 7.515427773 and the lower limit 7.515427702. The same method delivers for the neutron-to-proton ratio:

$$\ln\left(\frac{E_n}{E_p}\right) = \frac{1}{726}$$

By the way, $726 = 11 \cdot 11 \cdot 6$. The denominator 11 appears also in the W/Z-to-electron ratio [11], for example:

$$\ln\left(\frac{E_Z}{E_e}\right) = 12 + \frac{1}{11}$$

The unique properties of the electron and proton predestinate their physical characteristics as fundamental units. Table 1 shows the basic set of electron and proton units that can be considered as a *Fundamental Metrology* (c is the speed of light in a vacuum, \hbar is the Planck constant). In [23] was shown that the fundamental metrology (tab. 1) is completely compatible with Planck units [24]. Originally proposed in 1899 by Max Planck, these units are also known as natural units, because the origin of their definition comes only from properties of nature and not from any human construct. Max Planck wrote [25] that these units, “regardless of any particular bodies or substances, retain their importance for all times and for all cultures, including alien and non-human, and can therefore be called natural units of measurement”. Planck units reflect the characteristics of space-time.

We hypothesize that scale invariance according the Fundamental Fractal (1) calibrated on the physical properties of the proton and electron is a universal characteristic of organized matter and criterion of stability. This hypothesis we have called *Global Scaling* [14].

On this background, atoms and molecules emerge as stable eigenstates in fractal chain systems of harmonically oscillating protons and electrons. Andreas Ries [18] demonstrated that this model allows for the prediction of the most abundant isotope of a given chemical element.

In the following, we use the symbol \mathcal{F}_e for the Fundamental Fractal (1) calibrated on the properties of the electron,

and the symbol \mathcal{F}_p for the Fundamental Fractal calibrated on the properties of the proton. For example, $\mathcal{F}_e\langle 66 \rangle$ means the main attractor 66 of electron stability. In the solar system, this attractor stabilizes the orbital period of Jupiter [7].

In [15] we applied the Fundamental Fractal (1) to planetary systems interpreting gravity as macroscopic attractor effect of transcendental frequency ratios in chain systems of harmonic quantum oscillators – protons and electrons. In [26] we demonstrated that the Fundamental Field (fig. 2) in the interval of the main attractors $\langle 49 \rangle \leq \mathcal{F}_p \leq \langle 52 \rangle$ of proton stability reproduces the 3D profile of the Earth's interior confirmed by seismic exploration. As well, the stratification layers in planetary atmospheres follow the Fundamental Field [27]. In [28] we have shown that the Fundamental Fractal determines the Earth axial precession cycle, the obliquity variation cycle as well as the apsidal precession cycle and the orbital eccentricity cycle. There we have also shown that recently discovered geological cycles, like the 27 million years' cycle [29], as well as the periodic variations in the movement of the Solar system through the Galaxy, substantiate their determination by the Fundamental Fractal.

The orbital and rotational periods of planets, planetoids and large moons of the solar system correspond with attractors of electron and proton stability [23]. This is valid also for exoplanets [4] of the systems Trappist 1 and Kepler 20. In [15] we have shown that the maxima in the frequency distribution of the orbital periods of 1430 exoplanets listed in [30] correspond with attractors of the Fundamental Fractal. As well, the maxima in the frequency distribution of the number of stars in the solar neighborhood as function of the distance between them correspond with attractors of the Fundamental Fractal [20].

Exemplary applications

Jupiter's orbital period $T_O(Jupiter) = 4332.59$ days [31] approximates the main attractor $\mathcal{F}_e\langle 66 \rangle$ of electron stability that equals the 66th power of Euler's number multiplied by the oscillation period of the electron (see tab. 1):

$$\ln\left(\frac{T_O(Jupiter)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{4332.59 \cdot 86400 \text{ s}}{2\pi \cdot 1.28809 \cdot 10^{-21} \text{ s}}\right) = 66.00$$

Jupiter's distance from Sun approximates the main equipotential surface $\mathcal{F}_e\langle 56 \rangle$ of electron stability that equals the 56th power of Euler's number multiplied by the Compton wavelength of the electron. The aphelion $5.45492 \text{ AU} = 8.160444 \cdot 10^{11} \text{ m}$ delivers the upper approximation:

$$\ln\left(\frac{A_O(Jupiter)}{\lambda_e}\right) = \ln\left(\frac{8.160444 \cdot 10^{11} \text{ m}}{3.86159 \cdot 10^{-13} \text{ m}}\right) = 56.01$$

The perihelion $4.95029 \text{ AU} = 7.405528 \cdot 10^{11} \text{ m}$ delivers the lower approximation:

$$\ln\left(\frac{P_O(Jupiter)}{\lambda_e}\right) = \ln\left(\frac{7.405528 \cdot 10^{11} \text{ m}}{3.86159 \cdot 10^{-13} \text{ m}}\right) = 55.91$$

Now we can apply Kepler's 3rd law of planetary motion and express the gravitational parameter μ_{Sun} of the Sun through Euler's number, the speed of light c in a vacuum and the oscillation period τ_e of the electron:

$$\mu_{Sun} = \tau_e \cdot c^3 \cdot e^{36}$$

In logarithms, the cube of the mean orbit radius divided by the square of the orbital period $56 \cdot 3 - 66 \cdot 2 = 36$ results in the 36th power of Euler's number. In this way, within our numeric physical approach, the gravitational parameter of the Sun does not appear to be accidental, but is stabilized by Euler's number and origins from the quantum physical properties of the electron.

In a similar way, we can derive the attractor that the gravitational parameter of Jupiter is approximating. Thanks to the negligible eccentricities of the orbits of Jupiter's large moons, we can use the mean orbit radius for calculations. Callisto's orbit radius $R_O(Callisto) = 1.8827 \cdot 10^9 \text{ m}$ approaches the equipotential surface $\mathcal{F}_e\langle 50 \rangle$ of electron stability:

$$\ln\left(\frac{R_O(Callisto)}{\lambda_e}\right) = \ln\left(\frac{1.8827 \cdot 10^9 \text{ m}}{3.86159 \cdot 10^{-13} \text{ m}}\right) = 49.95$$

Callisto's orbital period $T_O(Callisto) = 16.689$ days is approaching the attractor $\mathcal{F}_e\langle 60; 2 \rangle$ of electron stability:

$$\ln\left(\frac{T_O(Callisto)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{16.689 \cdot 86400 \text{ s}}{2\pi \cdot 1.28809 \cdot 10^{-21} \text{ s}}\right) = 60.45$$

For reaching both attractors, Callisto must still increase its orbital period by 10 hours and of course, its mean orbit radius as well. Now we can apply Kepler's 3rd law of planetary motion and express the gravitational parameter $\mu_{Jupiter}$ of Jupiter through Euler's number:

$$\mu_{Jupiter} = \tau_e \cdot c^3 \cdot e^{29}$$

In logarithms, the cube of the mean orbit radius divided by the square of the orbital period $50 \cdot 3 - (60 + 1/2) \cdot 2 = 29$ results in the 29th power of Euler's number. In this way, Jupiter's gravitational parameter approximates the attractor $\mathcal{F}_e\langle 29 \rangle$ of electron stability.

Now we can derive the attractor that the gravitational parameter of the Earth is approximating. The orbital distance of the Moon from Earth approximates the equipotential surface $\mathcal{F}_e\langle 48; 3 \rangle$ of electron stability that equals the 48th power of Euler's number and its cubic root multiplied by the electron wavelength. The apoapsis of the Moon $A_O = 4.067 \cdot 10^8 \text{ m}$ delivers the upper approximation:

$$\ln\left(\frac{A_O(Moon)}{\lambda_e}\right) = \ln\left(\frac{4.067 \cdot 10^8 \text{ m}}{3.86159 \cdot 10^{-13} \text{ m}}\right) = 48.41$$

Periapsis $3.626 \cdot 10^8 \text{ m}$ delivers the lower approximation:

$$\ln\left(\frac{P_O(Moon)}{\lambda_e}\right) = \ln\left(\frac{3.626 \cdot 10^8 \text{ m}}{3.86159 \cdot 10^{-13} \text{ m}}\right) = 48.29$$

The orbital period $T_O(Moon) = 27.32166$ days approaches the main attractor $\mathcal{F}_e\langle 61 \rangle$ of electron stability:

$$\ln\left(\frac{T_O(Moon)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{27.32166 \cdot 86400 \text{ s}}{2\pi \cdot 1.28809 \cdot 10^{-21} \text{ s}}\right) = 60.95$$

For reaching this attractor, the Moon must increase its distance from Earth, and that's exactly what the Moon does [32]. However, our approach predicts an increase only until Moon's orbital period reaches the main attractor $\mathcal{F}_e\langle 61 \rangle = 29.08$ days. Now we can apply Kepler's 3rd law of planetary motion and express the gravitational parameter μ_{Earth} of the Earth through Euler's number:

$$\mu_{Earth} = \tau_e \cdot c^3 \cdot e^{23}$$

In logarithms, the cube of the mean orbit radius divided by the square of the orbital period $(48 + 1/3) \cdot 3 - 61 \cdot 2 = 23$ results in the 23th power of Euler's number. Consequently, also the gravitational parameter of the Earth does not appear to be accidental, but origins from the quantum physical properties of the electron and is approaching a main attractor of the Fundamental Fractal.

In a similar way, we can derive the attractors that the gravitational parameters of other planets are approximating. Phobos' mean orbit radius approximates the equipotential surface $\mathcal{F}_e\langle 45; -3 \rangle$ while its orbital period is stabilized by the attractor $\mathcal{F}_e\langle 56; 2 \rangle$. Consequently, the gravitational parameter of Mars approximates the attractor $\mathcal{F}_e\langle 21 \rangle$, because $(45 - 1/3) \cdot 3 - (56 + 1/2) \cdot 2 = 21$:

$$\mu_{Mars} = \tau_e \cdot c^3 \cdot e^{21}$$

The gravitational parameter of Uranus approximates the center of scale symmetry $(23 + 29)/2 = 26$ between the gravitational parameters of the Earth $\mathcal{F}_e\langle 23 \rangle$ and Jupiter $\mathcal{F}_e\langle 29 \rangle$:

$$\mu_{Uranus} = \tau_e \cdot c^3 \cdot e^{26}$$

Neptune's gravitational parameter approaches the same attractor $\mathcal{F}_e\langle 26 \rangle$, but for reaching it, Neptune's moon system must become larger. Saturn's gravitational parameter approximates the center of scale symmetry $(26 + 29)/2 = 27 + 1/2$ between the parameters of Uranus $\mathcal{F}_e\langle 26 \rangle$ and Jupiter $\mathcal{F}_e\langle 29 \rangle$:

$$\mu_{Saturn} = \tau_e \cdot c^3 \cdot e^{27+1/2}$$

Because the scaling factor \sqrt{e} links attractors of electron stability to corresponding attractors of proton stability, the mean orbit radius of Saturn's largest moon Titan approximates also the main equipotential surface $\mathcal{F}_p\langle 57 \rangle$. Titan's apoapsis $A_O = 1.25706 \cdot 10^9$ m delivers the upper approximation:

$$\ln\left(\frac{A_O(Titan)}{\lambda_p}\right) = \ln\left(\frac{1.25706 \cdot 10^9 \text{ m}}{2.10309 \cdot 10^{-16} \text{ m}}\right) = 57.05$$

Periapsis $1.18668 \cdot 10^9$ m delivers the lower approximation:

$$\ln\left(\frac{P_O(Titan)}{\lambda_p}\right) = \ln\left(\frac{1.18668 \cdot 10^9 \text{ m}}{2.10309 \cdot 10^{-16} \text{ m}}\right) = 56.99$$

And Titan's orbital period $T_O = 15.945$ days is approaching the main attractor $\mathcal{F}_p\langle 68 \rangle$ of proton stability:

$$\ln\left(\frac{T_O(Titan)}{2\pi \cdot \tau_p}\right) = \ln\left(\frac{15.945 \cdot 86400 \text{ s}}{2\pi \cdot 7.01515 \cdot 10^{-25} \text{ s}}\right) = 67.92$$

In this way, Saturn's gravitational parameter approximates also the attractor $\mathcal{F}_p\langle 35 \rangle$, because $57 \cdot 3 - 68 \cdot 2 = 35$ results in the 35th power of Euler's number, multiplied by the oscillation period of the proton:

$$\mu_{Saturn} = \tau_p \cdot c^3 \cdot e^{35}$$

Besides conservation of angular momentum [33], there is no known law concerning the rotation of celestial bodies. The more remarkable is the correspondence of the rotation periods of planets, planetoids and large moons with attractors of the Fundamental Fractal (1) as shown in [15]. Here we give some of the most expressive examples.

In the solar system, the 66th power of Euler's number stabilizes not only the orbital period 4332.59 days of Jupiter, but also the orbital period 686.971 days of Mars and the rotational period 9.074 hours of the planetoid Ceres, the largest body of the main asteroid belt that orbits the Sun between Mars and Jupiter. The difference lays in the reference units. While in the case of Jupiter's orbital period, the reference unit is the oscillation period of the electron $2\pi\tau_e$, in the case of Mars, it is the angular oscillation period of the electron τ_e :

$$\ln\left(\frac{T_O(Mars)}{\tau_e}\right) = \ln\left(\frac{686.971 \cdot 86400 \text{ s}}{1.28809 \cdot 10^{-21} \text{ s}}\right) = 66.00$$

And in the case of the rotational period of Ceres, the reference unit is the angular oscillation period of the proton τ_p :

$$\ln\left(\frac{T_R(Ceres)}{\tau_p}\right) = \ln\left(\frac{9.07417 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}}\right) = 66.01$$

The rotational periods of Mars and Earth approximate the next main attractor $\mathcal{F}_p\langle 67 \rangle$ of proton stability:

$$\ln\left(\frac{T_R(Mars)}{\tau_p}\right) = \ln\left(\frac{24.62278 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}}\right) = 67.01$$

$$\ln\left(\frac{T_R(Earth)}{\tau_p}\right) = \ln\left(\frac{23.93447 \cdot 3600 \text{ s}}{7.01515 \cdot 10^{-25} \text{ s}}\right) = 66.98$$

Mercury's period 58.64615 days of rotation is approaching the main attractor $\mathcal{F}_p\langle 71 \rangle$. Although Venus rotation is retrograde, its period 243.025 days approximates the attractor $\mathcal{F}_p\langle 72; 2 \rangle$ that coincides with $\mathcal{F}_e\langle 65 \rangle$. The rotation of further planets, planetoids and moons of the solar system we have analyzed in [15].

Conclusion

The application of our numeric-physical approach to the analysis of the orbital and rotational periods of the planets, planetoids and moons of the solar system and thousands of exoplanets [15] leads us to the conclusion that the avoidance of orbital, rotational, proton and electron parametric resonances by approximation of transcendental ratios can be viewed as a basic forming factor of planetary systems.

Studies of circumstellar disks around young stars conclude [34] that the planet formation process is observationally required to be both fast and common. Solid planets in the solar system should have then formed within less than a few million years, which is a major challenge for terrestrial planet formation theories [35].

Perhaps, our approach can explain the fast consolidation of the solar system. In fact, the scale-invariant fractal distribution of transcendental Euler attractors of stability is an inherent feature of the number continuum and therefore given a priori and does not require a long history of random collisions to find them.

The circumstance that the gravitational parameters of the Sun and the planets approximate main numeric attractors of electron and proton stability could be an approach to achieve a deeper understanding of gravitation.

In modern theoretical physics, numerical ratios usually remain outside the realm of theoretical interest. In this work we have tried to elucidate the physical meaning of numerical ratios and to show their theoretical and practical importance.

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Length Stretching and Time Dilation in the Field of a Rotating Body

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As was found in the first paper of this series of papers, the rotation of space produces a significant curvature (Progr. Phys., 2022, v.18, 31–49). In the second paper, we showed that light rays and mass-bearing particles are deflected near a rotating body due to the curvature of space caused by its rotation (ibid., 50–55). In this article we show that, since the rotation of the Earth around its axis curves the Earth's space making it "stretched" along the geographical longitudes, the measured length of a standard rod is greater when the rod is installed in the longitudinal direction. Due to the same reason, there is a time loss on board an airplane flying to the East (the direction in which the Earth's space rotates), and also a time gain when flying in the opposite direction, to the West. Both of the above effects are maximum at the equator (where the curvature of the Earth's space caused by its rotation is maximum and, therefore, space is maximally "stretched") and decrease towards the North and South Poles.

This paper is dedicated to the memory of Joseph C. Hafele, the outstanding American experimental physicist known due to his famous around-the-world-clocks experiment.

This is the third paper in the series of our papers on the effects of the space curvature caused by the rotation of space.

Recall that in the first paper [1], besides many other scientific results, it was found that the rotation of space produces a significant curvature due to its space-time non-holonomy (non-orthogonality of the time lines to the three-dimensional spatial section). In the second paper [2] that followed the first one, it was shown that light rays and mass-bearing particles are deflected near a rotating body due to the space curvature caused by the rotation of its space.

In particular, according to the formulae we have obtained, the curvature of the Earth's space, caused by its rotation, decreases from the equator, where it is maximum, to the geographical poles, and its effect depends on the direction of the measurement path with respect to the direction in which the Earth rotates.

This small paper is based on the previous two. We will calculate here the effects of length stretching and time loss/gain, which are due to the curvature of the Earth's space, caused by its rotation.

As always, we use the mathematical apparatus of chronometric invariants, which are physically observable quantities in the General Theory of Relativity. This mathematical apparatus was created in 1944 by our esteemed teacher A. L. Zelmanov (1913–1987) and published in his presentations [3–5], among which [5] is most complete. For a deeper study of this subject, read either our first article in this series [1] or the respective chapters in our monographs [6, 7].

Chronometrically invariant quantities are projections of four-dimensional (general covariant) quantities onto the line of time and the three-dimensional spatial section, which are

linked to the physical space of a real observer, and are invariant everywhere along the spatial section (his observed space). They are calculated using operators of projection, which take the structure of space into account. Since a real space can be curved, inhomogeneous, anisotropic, deforming, rotating, be filled with distributed matter etc., the lines of real time can have different density of time coordinates, and the three-dimensional coordinate grids can have different density of real three-dimensional coordinates. Therefore, chronometrically invariant quantities are truly physically observables registered by the observer.

In particular, the physically observable chr.inv.-projection of the four-dimensional interval dx^α onto the time line of an observer is the interval of physically observable time

$$d\tau = \sqrt{g_{00}} dt - \frac{1}{c^2} v_i dx^i,$$

and the physically observable chr.inv.-projections of dx^α onto his spatial section are the regular three-dimensional coordinate intervals dx^i . Here v_i is the linear velocity of the three-dimensional rotation of space, which arises due to the non-holonomy of the space-time (non-orthogonality of the time lines to the three-dimensional spatial section). It is determined as

$$v_i = -\frac{c g_{0i}}{\sqrt{g_{00}}}, \quad v^i = -c g^{0i} \sqrt{g_{00}},$$

where g_{00} is expressed through the gravitational potential w as usually, i.e., $w = c^2 (1 - \sqrt{g_{00}})$.

The fundamental metric tensor $g_{\alpha\beta}$, projected onto the three-dimensional spatial section of an observer, gives the chr.inv.-metric tensor h_{ik} of his space

$$h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k, \quad h^{ik} = -g^{ik}, \quad h_k^i = -g_k^i = \delta_k^i,$$

which has all properties of $g_{\alpha\beta}$ in the three-dimensional spatial section. Using the chr.inv.-metric tensor, we can lift and

lower indices in chr.inv.-quantities, and also get their squares. Thus, the square of the three-dimensional physically observable interval on the spatial section is calculated as

$$d\sigma^2 = h_{ik} dx^i dx^k = \left(-g_{ik} + \frac{1}{c^2} v_i v_k \right) dx^i dx^k.$$

In our further calculations, we will use the same space metric that we used in two previous papers. This is the metric of a space, where the three-dimensional space rotates due to the non-holonomy of the space-time, but there is no field of gravitation. More precisely, we neglect the influence of the Earth's gravitation, since in our further examples we do not change the altitude above the Earth's surface, so the influence of the gravitational potential remains constant.

Assuming that the space rotates along the equatorial axis φ , i.e., along the geographical longitudes, with the linear velocity $v_3 = \omega r^2 \sin^2 \theta$ (here $\omega = \text{const}$ is the angular velocity of this rotation), we obtain g_{03} from the definition of v_i ,

$$v_3 = \omega r^2 \sin^2 \theta = -\frac{c g_{03}}{\sqrt{g_{00}}},$$

and then we obtain the metric of such a space

$$ds^2 = c^2 dt^2 - 2\omega r^2 \sin^2 \theta dt d\varphi - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2).$$

The non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of this metric are obvious from the above

$$g_{00} = 1, \quad g_{03} = -\frac{\omega r^2 \sin^2 \theta}{c}, \\ g_{11} = -1, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2 \theta,$$

and the non-zero components of the chr.inv.-metric tensor h_{ik} , calculated from the above, are equal to

$$h_{11} = 1, \quad h_{22} = r^2, \quad h_{33} = r^2 \sin^2 \theta \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2} \right).$$

Now we have everything that is required for our further calculations.

So forth, we will calculate two effects due to the curvature of the Earth's space caused by its rotation.

First, we will calculate the effect of length stretching of a rod depending on its direction (in the equatorial, latitudinal and radial directions), as well as on the geographical latitude of the measurement site. According to the formulae for the Ricci curvature tensor and the scalar curvature, which we have obtained in the first paper [1, p. 45], the Earth's space is curved due to its rotation in the equatorial (longitudinal) direction, and its curvature decreases with the latitude from the equator, where the Earth's space is maximally "stretched", to the geographical poles of the Earth. Therefore, the measured

length of a standard rod is expected to be greater when the rod is installed in the direction along the geographical longitudes, and this effect of length stretching is maximum at the equator and decreases with the geographical longitudes towards the North and South Poles.

Second, we will calculate the difference in time on board an aircraft flying Westward and Eastward. It is expected that the rotation of the Earth's space causes a time loss when flying Eastward, the direction in which the Earth's space rotates, and a time gain when flying in the opposite direction, to the West. We also expect that the mentioned effects of time loss and time gain are greater when the airplane travels along the equator (where the curvature of the Earth's space caused by its rotation is maximum and, therefore, space is maximally "stretched") and decrease from the equator towards the North and South Poles.

1. Consider a standard rigid rod of an elementary length dl_0 , which is installed in a laboratory located somewhere on the surface of the Earth. Assume that the rod is installed in stages in three different positions: in the equatorial direction φ (along the geographical longitudes), in the polar direction θ (along the geographical latitudes), and in the radial direction r read from the centre of the Earth.

Using the formula for the square of the three-dimensional physically observable interval $d\sigma^2 = h_{ik} dx^i dx^k$ and the components of the physically observable chr.inv.-metric tensor h_{ik} we have obtained for an Earth-like rotating space (see above), we calculate the rod's length measured in each of the three indicated positions. It is respectively equal to

$$dl_r = \sqrt{h_{11} dr^2} = dr = dl_0,$$

$$dl_\theta = \sqrt{h_{22} d\theta^2} = r d\theta = dl_0,$$

$$dl_\varphi = \sqrt{h_{33} d\varphi^2} = \sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} r \sin \theta d\varphi = \sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} dl_0,$$

where $dr = dl_0$ is the length of an elementary segment along the radial r -axis, $r d\theta = dl_0$ is the length of an elementary arc along the latitudinal θ -axis (where θ is the polar angle read from the North Pole), and $r \sin \theta d\varphi = dl_0$ is the length of an elementary arc along the equatorial φ -axis.

As you can see from the above formulae, the rod retains its original physically observable length dl_0 , when installed in the positions along the radial direction ($dl_r = dl_0$) and along the geographical latitudes ($dl_\theta = dl_0$).

However, when the rod is installed in the position along the geographical longitudes, i.e., along the equatorial direction in which the Earth's space rotates, its physically observable length dl_φ becomes greater by a small amount δl depending on the factor specific of the curvature of space caused by

its rotation [1, p. 45], i.e.,

$$dl_\varphi = \sqrt{1 + \frac{\omega^2 r^2 \sin^2 \theta}{c^2}} dl_0 \simeq \left(1 + \frac{\omega^2 r^2 \sin^2 \theta}{2c^2}\right) dl_0,$$

$$\delta l \simeq \frac{\omega^2 r^2 \sin^2 \theta}{2c^2} dl_0.$$

Let us calculate the numerical value of this length stretching δl . The angular velocity of the Earth's rotation is equal to $\omega = 1 \text{ rev/day} = 7.27 \times 10^{-5} \text{ sec}^{-1}$. The Earth's radius is equal to $r = 6.4 \times 10^8 \text{ cm}$. Then the length stretching of a rod installed at the equator of the Earth in the direction along the longitudinal axis φ is equal to

$$\delta l \simeq 1.2 \times 10^{-12} dl_0$$

of the original length dl_0 of the rod. At the latitude of the Greenwich Observatory (51° North Lat., $\theta = 90^\circ - 51^\circ = 39^\circ$) the length stretching of a rod installed along the longitudinal axis φ is less than at the equator and is equal to

$$\delta l \simeq 5.0 \times 10^{-13} dl_0,$$

and this effect of length stretching vanishes at the geographical poles of the Earth, since there $\sin \theta = 0$ and, hence,

$$\delta l = 0, \quad dl_\varphi = dl_0.$$

So, we clearly see that the curvature of the Earth's space along the equatorial (longitudinal) axis, caused by the rotation of the Earth, and, as a result, the "stretching" of physical coordinates along the geographical longitudes, lead to the stretching of the physically observable length of a rod, installed in the position along the geographical longitudes.

The mentioned effect of length stretching is maximum at the equator, where the curvature of the Earth's space and the longitudinal stretching of physical coordinates caused by the Earth's rotation is maximum, and decreases towards the geographical poles, where the length stretching vanishes.

2. Consider an atomic clock installed on board an airplane flying, in stages, Westward and Eastward around the Earth. In this case, according to the definition of physically observable time, and taking the characteristics of an Earth-like rotating space into account (see above), the flight time τ registered on board the airplane is equal to

$$\tau = \left(1 - \frac{1}{c^2} v_3 u^3\right) t = \left(1 - \frac{\omega r^2 \sin^2 \theta}{c^2} u^3\right) t,$$

where t is the reference (coordinate) time counted using a reference clock installed at the point of departure (which is the same as at the point of arrival in an around-the-world flight), and u^3 is the linear coordinate velocity of the airplane, which is measured along the third, equatorial (longitudinal) axis φ

as the difference in the geographical longitudes traveled by the airplane per second.

If the airplane stays at the airport, its coordinate velocity is equal to zero $u^3 = 0$ and, therefore, the second term in the above formula vanishes. In this case, the clock installed on board the airplane count the same time as the reference clock at the airport ($\tau = t$).

Since the Earth rotates from West to East, an airplane, when flying Eastward, travels in the same direction in which the Earth's space rotates (the airplane's velocity is co-directed with the rotation velocity of the Earth's space). As a result, the clock installed on board the airplane should register a time loss, the amount of which is calculated as

$$\delta\tau_{\text{East}} = -\frac{\omega r^2 \sin^2 \theta}{c^2} u^3 t.$$

When an airplane flies Westward, its velocity is directed opposite the rotation velocity of the Earth's space. Accordingly, in this case, the clock on board the airplane should register a time gain, the amount of which is

$$\delta\tau_{\text{West}} = +\frac{\omega r^2 \sin^2 \theta}{c^2} u^3 t.$$

Assume that the airplane flies along the equator around the Earth at a constant cruising speed of 800 km/hour, which means that $u^3 = +3.5 \times 10^{-5} \text{ sec}^{-1}$ when flying Eastward and $u^3 = -3.5 \times 10^{-5} \text{ sec}^{-1}$ when flying Westward. Thus, the airplane returns to its point of departure in a time interval $t = 1.8 \times 10^5 \text{ sec}$. The angular velocity of the Earth's rotation is equal to $\omega = 1 \text{ rev/day} = 7.27 \times 10^{-5} \text{ sec}^{-1}$ and the Earth's radius is equal to $r = 6.4 \times 10^8 \text{ cm}$. Thus, we obtain that the clock on board this airplane should register a time loss when flying Eastward and a time gain when flying Westward, which are respectively equal to

$$\delta\tau_{\text{East}} = -210 \text{ nanosec}, \quad \delta\tau_{\text{West}} = +210 \text{ nanosec}.$$

That is, the rotation of the Earth's space results in a 210 nanosecond loss in time on board an Eastward-flying airplane travelled around the world along the equator, i.e., in the direction in which the Earth's space rotates, and a 210 nanosecond gain of time when travelled around the world in the opposite direction, to the West.

The above effect of time loss and time gain caused by the rotation of the Earth's space decreases with the geographical latitude due to the sine of the polar angle, which is a multiplier in the above formulae. For example, when flying Eastward and Westward around the Earth along the Greenwich parallel (51° North Lat., $\theta = 39^\circ$), the effect of time loss and time gain is respectively equal to

$$\delta\tau_{\text{East}} = -84 \text{ nanosec}, \quad \delta\tau_{\text{West}} = +84 \text{ nanosec}.$$

This effect obviously vanishes at the geographical poles of the Earth, since there $\sin \theta = 0$.

Yes, the expected loss/gain in the flight time is only 210 nanoseconds at the equator, and it decreases to the geographical poles. Compare, in the Hafele-Keating around-the-world-clocks experiment [8–10], the common effect of the relativistic addition of the Earth's rotation velocity to the airplane's velocity and also the decrease of the gravitational potential of the Earth with the flight altitude resulted a time loss of -59 ± 10 nanoseconds Eastward and a time gain of $+273 \pm 7$ nanoseconds Westward. Note that their results were different when flying to the West and to the East, because the relativistic velocity effect of Special Relativity was also taken into account (the airplane's velocity was added to or subtracted from the velocity of the Earth's rotation). The UK's National Measurement Laboratory commonly with the BBC repeated the Hafele-Keating experiment on its 25th anniversary in 2005, on board a London-Washington-London flight and with a better precision of ± 2 nanoseconds [11].

The loss/gain in the flight time, caused by the rotation of the Earth's space, is a "cumulative effect": it depends linearly on the flight time (see the formula above). That is, when an airplane will "wind circles" around the Earth, the effect of time loss/gain on its board, caused by the rotation of the Earth's space, will increase with each revolution. And, after three-four-five revolutions around the Earth, the expected effect caused by the rotation of the Earth's space will be many times (or even dozens of times) higher than the measurement precision.

This is the real way to register the effect of time loss/gain, caused by the rotation of the Earth's space. "Winding circles" around the Earth is easier not using an airplane, but on board a spacecraft orbiting the Earth because it travels around the Earth two dozen times a day anyway and without the need of aviation kerosene. Thus, having an atomic clock installed on board an orbital spacecraft, the effect of time loss/gain, caused by the rotation of the Earth's space, can be accumulated to a surely measurable numerical value in just a few days, without doing anything for this.

The aforementioned effects of length stretching and time loss/gain, occurring due to the curvature of the Earth's space caused by its rotation, are new fundamental effects of the General Theory of Relativity. They are in addition to the effect of deflection of light rays and mass-bearing particles in the field of a rotating body, which we theoretically discovered earlier, and the well-known Einstein effect of deflection of light rays in the field of a gravitating body.

We dedicate this paper to the memory of Joseph C. Hafele (1933–2014), the outstanding American experimental physicist who initiated and later performed (together with Richard E. Keating) the famous around-the-world-clocks experiment, now known as the Hafele-Keating experiment [8–10].

We had an extensive correspondence with Joseph Hafele

in the 2010s, in which we discussed various problems. Unfortunately, his sudden death had interrupted our acquaintance. He was a truly gentleman, good Catholic and a honest scientist who never compromised [12].

Surely he would be happy, if he read this article and saw it published.

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Space-Time Quantification

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The quantification of Length and Time in Kepler's laws implies an angular momentum quantum, identified with the reduced Planck's constant, showing a mass-symmetry with the Newtonian constant G . This leads to the Diophantine Coherence Theorem which generalizes the synthetic resolution of the Hydrogen spectrum by Arthur Haas, three years before Bohr. The Length quantum breaks the Planck wall by a factor 10^{61} , and the associated Holographic Cosmos is identified as the source of the Background Radiation in the Steady-State Cosmology. An Electricity-Gravitation symmetry, connected with the Combinatorial Hierarchy, defines the steady-state Universe with an invariant Hubble radius 13.812 milliard light-year, corresponding to 70.796 (km/s)/Mpc, a value deposited (1998) in a Closed Draft at the Paris Academy, confirmed by the WMAP value and the recent Carnegie-Chicago Hubble Program, and associated with the Eddington number and the Kotov-Lyuty non-local oscillation. This confirms definitely the Anthropic Principle and the Diophantine Holographic Topological Axis rehabilitating the tachyonic bosonic string theory. This specifies G , compatible with the BIPM measurements, but at 6σ from the official value, defined by merging discordant measurements.

1 The Diophantine Coherence Theorem (DCT)

For connecting different physical measurements, Physics uses multiplication while addition is forbidden. But multiplication is a generalization of addition [1]. This paradox may be suppressed by considering only numerical ratios of the same physical quantity, as in the third Kepler law, *introducing Space and Time quanta* L_1 and T_1 [15]. Considered as a Diophantine Equation, which uses only natural numbers n , it resolves directly:

$$\begin{aligned} (T_n/T_1)^2 &= (L_n/L_1)^3 \equiv n^6 \\ \Rightarrow T_n &= n^3 T_1 ; L_n = n^2 L_1 . \end{aligned} \quad (1)$$

This proceeds from the Holic Principle [27], a Diophantine form of the Holographic Principle, which states that Physics is described through the simplest degenerate Diophantine Equations, where the exponents identify with the dimensions 3 for Space, 2 for a 2D Time [39], 5 for Mass, and 7 for Field. The n -invariant L_n^3/T_n^2 is homogeneous to Gm_G , where G is Newton's gravitational constant, and m_G is a mass (here the usual central mass is divided by the factor $4\pi^2$). The other Kepler's law states that the orbital angular momentum per unit mass is an orbital invariant. Since the corresponding term L_n^2/T_n is proportional to n , this implies an orbital momentum quantum, identified to the reduced Planck constant, or *action quantum* \hbar , privileged by the particle physics in the spin concept. While the ratio of the kinematic parts of G and \hbar are homogeneous to a speed, these two universal constants presents a symmetry by respect to the mass concept, implying the as-

sociation of \hbar with a mass m_{\hbar} :

$$L_n^3/T_n^2 = Gm_G \quad ; \quad L_n^2/T_n = n\hbar/m_{\hbar} . \quad (2)$$

Any mass pair (m_G, m_{\hbar}) is associated to a series of Keplerian orbits (L_n, T_n) :

$$L_n = \frac{(n\hbar)^2}{Gm_G m_{\hbar}^2} \quad ; \quad T_n = \frac{(n\hbar)^3}{G^2 m_G^2 m_{\hbar}^3} . \quad (3)$$

For $n = 1$ and $m_G = m_{\hbar} = m$, the Special Non-Local Length and Time are:

$$L_{NL}(m) = \frac{\hbar^2}{Gm^3} \quad ; \quad T_{NL}(m) = \frac{\hbar^3}{G^2 m^5} . \quad (4)$$

Introducing the *formal* velocity $V_n = L_n/T_n$, this connects the reduced Planck energy $n\hbar/T_n$ with the gravitational potential energy pertaining to masses m_G and m_{\hbar} and the energy $m_{\hbar}V_n^2$:

$$\begin{aligned} V_n &= L_n/T_n = Gm_G m_{\hbar}/n\hbar \\ \Rightarrow n\hbar/T_n &= Gm_G m_{\hbar}/L_n = m_{\hbar}V_n^2 . \end{aligned} \quad (5)$$

With the Planck mass $m_P = \sqrt{\hbar c/G}$, where the light speed c is the third universal constant, this reads

$$\frac{n\hbar}{T_n} = \frac{Gm_G m_{\hbar}}{L_n} = m_{\hbar}V_n^2 \equiv m_{\hbar} \left(\frac{c}{nA} \right)^2 \quad ; \quad A = \frac{m_P^2}{m_G m_{\hbar}} . \quad (6)$$

This is called the *Diophantine Coherence Theorem* (DCT).

2 The atom H and the Holographic Cosmos

Three years before Bohr, Arthur Haas [3] considered the *electron orbital period* in the Rutherford model, and the corresponding Planck energy $nh\nu = nh/T_n = n\hbar v_n/L_n$ where $v_n = 2\pi V_n$ is the orbital velocity. The correct Hydrogen spectrum is obtained by equalizing it with the electric potential energy $\hbar c/aL_n$, where $a \approx 137.0359991$ is the electric constant, and the double (virial) kinetic electron energy $m_e v_n^2$ (the useful physical constants are listed in Table 1):

$$n\hbar \frac{v_n}{L_n} = \frac{\hbar c}{aL_n} = m_e v_n^2 \equiv m_e \left(\frac{c}{na} \right)^2. \quad (7)$$

Note that the so-called “properties of vacuum” ϵ_0 and μ_0 are unnecessary: they are only introduced for historical reasons, leading to the cumbersome, but official, choice of electrical units, hiding the true “electrical constant” a , whose inverse α , called “the fine structure constant” is of minor importance. For $n = 1$, this gives the bare Haas-Bohr radius: $r_{HB} = a\lambda_e$, where $\lambda_e \equiv \hbar/(m_e c)$ is the Reduced Electron wavelength (the effective electron mass effect defines the Bohr radius $r_B = r_{HB}/(1 + 1/p)$). This double equation shows up the same form that the above DCT (6), where additional 2π factors are integrated in the definitions of m_G and m_{\hbar} . The identification of potential energy terms implies $m_G m_{\hbar} = m_p^2/a$, thus in this case $A = a$. The simplest choice $m_{\hbar} = m_e$ implies the following m_G , where $m_N = am_e$ is the Nambu mass, a quasi-quantum in Particle Physics [17]:

$$m_{\hbar} = m_e \quad ; \quad m_G = \frac{m_p^2}{m_N} \quad ; \quad A = a. \quad (8)$$

This last mass is $m_G \approx 3.7939 \times 10^{12}$ kg, whose corresponding Special Length (4) is:

$$d_0 = L_{NL}(m_p^2/m_N) \approx 3.051 \times 10^{-96} \text{ meter}. \quad (9)$$

This is the Cosmic Space Quantum d_0 breaking the “Planck Wall” by a factor 10^{61} which has been associated to the Cosmos holographic radius R_{hol} [14]:

$$\pi \left(\frac{R_{hol}}{l_p} \right)^2 = 2\pi \frac{R_{hol}}{d_0}. \quad (10)$$

This is the Bekenstein-Hawking Entropy formula of the Holographic Principle [6] where the *Planck Length*

$$l_p \equiv (G\hbar/c^3)^{1/2} \equiv L_{NL}(m_P)$$

is a basic holographic length. The Cosmos radius R_C has been defined by the natural *mono-chromatic* holographic extension:

$$\pi \left(\frac{R_{hol}}{l_p} \right)^2 = 2\pi \frac{R_{hol}}{d_0} = 2\pi \frac{R_C}{l_p}, \quad (11)$$

leading to:

$$\begin{aligned} R_{hol} &= 2L_{NL}(m_N) \approx 18.105 \text{ Giga light-year (Glyr)} \\ R_C &= 2L_{NL}(m_N^2/m_P) \approx 9.075 \times 10^{86} \text{ meter}. \end{aligned} \quad (12)$$

Table 2 shows this symmetry between the Nambu mass m_N and the Planck mass m_P , whose large value is the source of the “Hierarchical Problem” [41]. From $P/\sqrt{a} \approx a_w n_i^3$, where $P = m_P/m_e$, these formula leads to a confirmation of the optimal G value in the ppb domain (Table 1), where $\beta = (H - p)^{-1}$

$$\left(\frac{P}{a_w} \right)^3 \approx \left(\frac{4\pi}{\sqrt{a}} \right)^8 \frac{(pH\beta^2)^5}{2} \approx \frac{aW}{137Z} (pH)^5 (16 \text{ ppm}), \quad (13)$$

showing the role of the geometrical factor 4π .

Now $L_{NL}(\sqrt{m_P m_N}) \approx \lambda_{CMB}/2a_s^2 (2a_s^2 \sim a)$, tying to 0.3% the strong coupling a_s and the nominal wavelength $\hbar c/kT_{CMB}$ of the Cosmic Microwave Background (CMB), whose source is lacking in the steady-state cosmology [7]. The simplest hypothesis is that the above Cosmos is this source. Indeed, the Wien CMB wavelength λ_{Wn} enters (0.1%):

$$4\pi \left(\frac{R_{hol}}{\lambda_{Wn}} \right)^2 \approx e^a. \quad (14)$$

This perfect holographic formula suggests that *the background would be coherent, meaning it brings information*. This could be the real significance of the CMB Anisotropy Statistics [29].

3 The gravitational hydrogen molecule

The Haas method was already applied to the *special three-body* dihydrogen molecule [13, p.391]:

$$n\hbar \frac{v_n}{L_n} = \frac{Gm_p m_H}{L_n} = m_e v_n^2, \quad (15)$$

The comparison with the above Haas equation implies the substitution: $a \rightarrow a_G = m_p^2/m_P m_H$, corresponding to the following m_G value:

$$m_{\hbar} = m_e \quad ; \quad m_G = m_{bc} \quad ; \quad A = a_G \quad (16)$$

where $m_{bc} = m_p m_H/m_e$ is close to the DNA bi-codon mass, which shows a central position in the Topological Axis [13], corresponding to the dimension 16. Indeed the topological term $f(16) = e^{16}$ is close to pH , and, more precisely, to $2n_i^4/a^3$ (0.04%).

For $n = 1$, this Haas-Sanchez radius R_{H_2} shows a direct Electricity-Gravitation symmetry, by respect to the Reduced Electron wavelength $\lambda_e = \hbar/m_e c$:

$$\begin{aligned} r_{HB} &= a\lambda_e = a \frac{\hbar}{m_e c} \\ R_{H_2} &= a_G \lambda_e = \frac{\hbar^2}{Gm_e m_P m_H} \equiv L_{NL}(m_0), \end{aligned} \quad (17)$$

where $m_0 = (m_e m_P m_H)^{1/3}$. Note that a and a_G are very close to the last two terms of the Combinatorial Hierarchy 137 and $N_L + 137$, with $N_L = 2^{127} - 1$, the Lucas Number [12].

Table 1: Physical constants

| Quantity | Value | Unit | 10^{-9} |
|--|---|--|-------------------|
| Electrical Constant a | 137.035999084(21) | - | 0.15 |
| Electron Excess Magnetic moment d_e | 1.00115965218096 | - | 0.26 |
| Official Strong Coupling constant Optimal Strong Coupling Constant a_s [15] | 8.45(5) 8.434502914 | - - | |
| Proton/Electron mass ratio p | 1836.152 673 43 | - | 0.06 |
| Proton/Electron Wyler mass ratio p_W [33] | $6\pi^5$ | - | exact |
| Neutron/Electron mass ratio n_t | 1838.683 661 7 | - | 0.5 |
| Hydrogen/Electron mass ratio H | 1837.152 660 14 | - | 0.06 |
| Hydrogen Relativist correction factor $\beta = 1/(H - p)$ | 1.0000266 | - | |
| Optimal Muon/Electron mass ratio μ [14] | 206.768 286 9 | - | |
| Optimal Higgs Boson mass m_{H_g} [15] | $495^2 m_e$ | - | |
| Action quantum \hbar | $1.054 571 81 \cdot 10^{-34}$ | J s | exact |
| Official Gravitation Constant G_{off} Optimal Gravitation Constant G | $6.674 30 \times 10^{-11}$ $6.67545272 \times 10^{-11}$ [14] | $\text{kg}^{-1}\text{m}^3\text{s}^{-2}$ $\text{kg}^{-1}\text{m}^3\text{s}^{-2}$ | |
| Speed of light in vacuum c | 299 792 458 | m s^{-1} | exact |
| Optimal Fermi Constant $G_F = \hbar^3/cm_F^2$ | $1.435 851 \cdot 10^{-62}$ | J m^3 | |
| Optimal Fermi mass ratio $m_F/m_e = F = a_w^{1/2}$ | 573007.3652 | - | |
| W boson mass ratio $W = m_W/m_e$ | 157298 ± 23 | - | 1.5×10^5 |
| Z boson mass ratio $Z = m_Z/m_e$ | 178450 ± 4 | - | 2.3×10^4 |
| Electron mass m_e | $9.109 383 701 5 \cdot 10^{-31}$ | kg | 0.3 |
| Boltzmann Constant k | $1.380649 \cdot 10^{-23}$ | J K^{-1} | exact |
| Reduced Electron Wavelength λ_e | $3.861 592 675 \cdot 10^{-13}$ | m | 0.3 |
| Measured CMB temperature T_{CMB} Optimal CMB Temperature T_{CMB} | 2.725 5(6) 2.725 820 138 [14] | Kelvin K | |
| Optimal CMB Wien wavelength λ_{Wn} | $1.063 082 472 \cdot 10^{-3}$ [14] | m | |
| Optimal CMB reduced wavelength $\hbar\lambda_{CMB} = \hbar c/kT_{CMB}$ | $8.400 716 617 \cdot 10^{-4}$ [14] | m | |
| Optimal CNB Temperature $T_{CNB} \equiv T_{CMB}(11/4)^{-1/3}$ | 1.945 597 [14] | Kelvin | |
| Optimal CNB reduced wavelength $\lambda_{CNB} = \hbar c/kT_{CNB}$ | $1 176 956 918 \cdot 10^{-3}$ [14] | m | |
| Optimal critical density $\rho_{cr} = 3c^2/8\pi GR^2$ | $9.411 979 89 \cdot 10^{-27}$ | $\text{kg m}^{-1/3}$ | |
| Kotov P_0 period t_K | 9600.606(12) [19] | s | 1200 |

In R_{H_2} the speed c is eliminated: for this reason, a precise approximation was immediately guessed by the c -free “dimensional analysis”, the so-called *Three Minutes Formula*, from the ternary symmetry Electron-Proton-Neutron (Closed Letter to the Paris Science Academy, March 1998) [22] (see Table 2). The associated Special time $T_{NL}(m_0)$ is very close (0.9%) to the time associated to the triplet: \hbar , the Fermi constant G_F and the associated critical steady-state density $\rho_{cr} =$

$3c^2/8\pi GR^2$ where $R = 2R_{H_2}$ and it is

$$\hbar^4/G_F^{5/2}\rho_{cr}^{3/2} \approx 3m_p^2 R_{hol}/cm_e m_Z$$

(0.01%), comforting the following steady-state Universe.

4 The Steady-State Universe revisited

A salient feature of the Universe is its critical character, relating its horizon radius R with its mass by $R = 2GM/c^2$. How-

Table 2: Values of the DCT Fundamental ($n = 1$) Radius $\hbar^2/Gm_Gm_{\tilde{h}}^2$ for specific values of m_G and $m_{\tilde{h}}$. Planck mass: m_P . Nambu mass: $m_N = am_e$. Holographic ratio $u = R_{hol}/R$. Proton mass: m_P . Hydrogen mass: m_H . Mean Atomic mass: $m_0 = (m_e m_P m_H)^{1/3}$. Bicodon mass $m_{bc} = m_P m_H / m_e$. Photon mass $m_{ph} = \hbar/c^2 t_K \approx 1.2222 \times 10^{-55}$ kg. Graviton mass: $m_{gr} = m_{ph}/a_w \approx 3.7223 \times 10^{-67}$ kg [14]. Optimal Higgs boson mass: $m_{Hg} = 495^2 m_e$.

| m_G | $m_{\tilde{h}}$ | Length | Symbol | Precision/offset |
|-------------------|------------------------|--|----------------------|----------------------|
| m_P^2/m_N | m_P^2/m_N | Space Quantum | d_0 | exact |
| m_P^2/m_0 | m_P^2/m_0 | Topon | λ_M | exact |
| m_{bc}/a_w | $m_e \sqrt{a_w a_G}$ | Reduced Electron Wavelength | λ_e | exact |
| m_P^2/m_N | m_e | Hass-Bohr radius $r_{HB} = a\lambda_e = r_B/(1 + 1/p)$ | r_{HB} | exact |
| $a^3 m_P$ | $\sqrt{m_P m_H}$ | Background Wien Wavelength | λ_W | 3.2×10^{-4} |
| m_{bc} | m_{bc} | Twice Kotov Length | $2l_K$ | 6.3×10^{-3} |
| m_{Hg} | m_{Hg} | $R\lambda_e/4\lambda_{CMB}$ $Ra_w^{1/2}/WZ^2$ | | - 0.23% + 0.25% |
| m_{bc} | m_e | Half Universe Radius | $R_{H_2} \equiv R/2$ | exact |
| m_N | m_N | Half Holographic Cosmos radius | $R_{hol}/2$ | exact |
| m_N^2/m_P | m_N^2/m_P | Half Cosmos Radius | $R_C/2$ | exact |
| $u \times m_{bc}$ | $\sqrt{m_{ph} m_{gr}}$ | Cosmos radius | R_C | 1.7×10^{-3} |

ever, in the initial “flat universe” model [32], the total mass M is only matter, while in the present Λ CDM standard model, it is separated between a material part with relative density Ω_m and a so-called “dark energy” part with relative density $1 - \Omega_m$ [29]. We have noted that Ω_m is compatible with $3/10$, which is both the density of the classical gravitational energy of a critical homogeneous ball and the density of the steady-state *non-relativist* recession kinetic energy [14]. While the standard cosmology uses an ad-hoc inflation to justify this observed critical condition, we consider rather the Universe as a particle (Topon) in the above Cosmos, with the Topon wavelength $\lambda_M \equiv \hbar/Mc = 2\hbar G/Rc^3 \equiv 2l_p^2/R$. Then, the critical condition results from the Bekeinstein-Hawking entropy holographic relation, as above (10), where the Topon appears as a secondary Length-Quantum, since the wavelength λ_m associated for any particle of mass m is a whole multiple n_m of the Topon, in conformity with the Field Quantum Theory. The geometrical interpretation is clear: it is a sphere area described by a whole number of sweeping circles, illustrating the fact that multiplication is a series of additions:

$$4\pi \left(\frac{R_{HB}}{l_p} \right)^2 = \pi \left(\frac{R}{l_p} \right)^2 = 2\pi \frac{R}{\lambda_M} \equiv 2\pi n_m \frac{R}{\lambda_m} \quad (18)$$

$$\Rightarrow M = \frac{Rc^2}{2G} \equiv \frac{R_{H_2} c^2}{G},$$

identifying twice the above Haas-Sanchez’s gravitational radius R_{H_2} with R , the steady-state Universe horizon radius, which is also the limit of a theoretical star radius when its

number of atoms shrinks to one [21], a central length in astrophysics, leading to the Machian formula:

$$R = 2 \frac{\hbar^2}{Gm_e m_P m_H} \Rightarrow M = \frac{m_P^4}{m_e m_P m_H}. \quad (19)$$

The effective electron mass $m'_e = m_e m_P / (m_P + m_e) \equiv M/n_e$, appears in the relation with Eddington number (Table 3) and introduces n_e , the *Universe Electron Quantum Number, canonical in Quantum Field Theory*. The Eddington Electron-Proton symmetry shows up in the following expression of the Large Number Correlation, where λ_{pH} is the geometrical mean of the reduced wavelengths of the proton and Hydrogen:

$$\frac{m_P^2}{m_P m_e} = n_e^{1/2} = \frac{R}{2\lambda_{pH}}, \quad (20)$$

which is extended by very precise dramatic expressions involving the symmetry between the weak bosons of masses $m_W = Wm_e$ and $m_Z = Zm_e$:

$$n_e^{1/2} \approx \frac{(WZ)^4}{2} \approx \left(\frac{m_F^2}{m_P m_H} \right)^7 \left(\frac{aZ}{W} \right)^3, \quad (21)$$

where appears as well a Planck-Fermi symmetry. It relates $a_G = m_P^2/m_P m_H$ to W and Z , specifying the known relation $a_G \approx W^8$ [5].

In the Topological Axis, the above Topon corresponds to the orbital number $k = 7$, while the gauge bosons correspond to $k = 3$ (weak bosons W, Z) and $k = 5$ (strong GUT boson

X), letting a single place $k = 1$ for a *non-standard massive Gluon* [14].

The particular values of the topological function $f(k) = \exp(2^{k+1/2})$ for $k = 7$ and 6 show up in (0.06%):

$$\begin{aligned} n_e &\approx f(7) \times 153^2 \\ R/\lambda_e &\approx f(6)/6, \end{aligned} \quad (22)$$

where $(f(6))^2 \equiv f(7)$ implies that $m_p/m_e \approx 1836 \equiv 6 \times 2 \times 153$, the Diophantine approximation of the Wyler formula $p_W = 6\pi^5$ [33]. The spectroscopic number associated to k is $2(2k+1)$, where 2 is the spin degeneracy and $2k+1$ the number of magnetic states [15]. For $k = 6$, this is 26 , the canonical dimension in the bosonic string theory [41].

This *invariable* Universe radius $R \approx 13.812$ Giga light-year (Glyr) of (19) is close to c times the variable standard Universe age. So the standard theoretical approach is correct, but not its Big Bang interpretation: it seems that a confusion is made somewhere between Time and Length, which readily occurs by putting $c = 1$. Moreover, the corresponding Hubble constant c/R is 70.793 (km/s)/Mpc, which is compatible with both the WMAP and the Carnegie-Chicago Hubble Program recent direct measurements (Table 3).

The above Universe gravitational potential energy $(3/10)Mc^2$ shows a Neutron Quantum Number (the number of neutron masses) very close (0.05%) to the large Eddington Number [14]. So it has nearly anticipated the correct Hubble Constant value (Table 3).

The Cosmos radius connects with the above radius R_{hol} and R by (27 ppm and 0.04%):

$$R_C(m_e/m_p)^2 \approx R_{hol} \left(\frac{WH}{3} \right)^2 \approx R(2FZ^2/3), \quad (23)$$

confirming very precisely, since $1/(H-p) \approx 27$ ppm, the optimal weak W boson mass [14] (Table 1).

5 The Cosmic Microwave Background (CMB)

This Universe radius $R = 2R_{H_2}$ enters a 1D-2D holographic relation: $2\pi R/\lambda_e = 4\pi\lambda_p\lambda_H/l_p^2$. The extension to the 3D holographic relation using λ_{H_2} , the reduced wavelength of the dihydrogen molecule H_2 , involves the reduced wavelength of the Cosmic Microwave Background (CMB) $\lambda_{CMB} = \hbar c/kT_{CMB}$:

$$2\pi \frac{R}{\lambda_e} = 4\pi \frac{\lambda_p\lambda_H}{l_p^2} \approx \frac{4\pi}{3} \left(\frac{\lambda_{CMB}}{\lambda_{H_2}} \right)^3, \quad (24)$$

leading to $T_{CMB} \approx (8G\hbar^4/3\lambda_p^5)^{1/3}/k \approx 2.729$ Kelvin, which is once more, apart the holographic factor $8/3$, a c -free three-fold (Mass, Length, Time) dimensional analysis, giving the energy kT_{CMB} from the constants G , \hbar , λ_p . Moreover, by substituting $a_G = R/2\lambda_e$ with the above Lucas Number N_L , this leads to a new holographic expression (analog to the area

of a 4D sphere), which gives T_{CMB} , compatible with the measured value $2.7255(6)$ Kelvin [14]:

$$\begin{aligned} N_L &\approx 2\pi^2 \frac{\lambda_{CMB}^3}{\lambda_e\lambda_H^2} \\ \Rightarrow T_{CMB} &= \frac{\hbar c}{k\lambda_{CMB}} \approx 2.7258205 \text{ Kelvin}. \end{aligned} \quad (25)$$

The standard cosmology predicts a Neutrino background with temperature $T_{CNB} = T_{CMB} \times (4/11)^{1/3} \approx 1.946$ Kelvin. The total CMB photon number is $n_{ph} = (3\xi(3)/8\pi)(R/\lambda_{CMB})^3$, exceeding the total Hydrogen number $n_H = M/m_H = R\lambda_H/2l_p^2$. But in terms of energy, the matter dominates. So one must consider also the ratio between the critical energy density $u_{cr} = 3c^4/8\pi GR^2$ and the total background energy density $u_{cmb+cnb} = yu_{cmb}$, with $y = 1 + (21/8)(4/11)^{4/3} \approx 1.681322$ [24] and $u_{cmb} = (\pi^2/15)\hbar c/\lambda_{CMB}^4$. We observed that these ratios are tied by an Eddington type relation:

$$\left(2 \frac{n_{ph}}{n_H} \right)^{1/2} \approx \frac{u_{cr}}{u_{cmb+cnb}} \Rightarrow T_{CMB} \approx 2.724 \text{ Kelvin}. \quad (26)$$

This confirms the existence of the Neutrino background. Now assuming that the total background Photon + Neutrino is the result of an ongoing Hydrogen-Helium transformation, producing $e_{He} = 6.40 \times 10^{14}$ Joule by kilogram of Helium, i.e. an efficiency $\epsilon_{He} = e_{He}/c^2 \approx 1/140$. The Helium mass density is $Y \times \rho_{bar}$; with the standard evaluation of baryonic density $\epsilon_{bar} = \rho_{bar}/\rho_{cr} \approx 0.045$ and $Y \approx 0.25$ [29], this leads to:

$$\begin{aligned} \left(\frac{\lambda_{CMB}^2}{l_p R} \right)^2 &\approx \frac{8\pi^3 y}{45Y\epsilon_{bar}\epsilon_{He}} \approx 1.15 \times 10^5 \\ \Rightarrow T_{CMB} &\approx 2.70 \text{ Kelvin}. \end{aligned} \quad (27)$$

In the standard model, the Universe age is far too small to explain a large Helium large density resulting from stellar activities [23]. Thus, it is not a real problem in the steady-state model.

6 The electron and the Kotov non-local period

This study confirms the central role of λ_e , the unit length in the Topological Axis [13]. So we look for a Diophantine series giving it for $n = 1$. This means:

$$\lambda_e \equiv \frac{\hbar}{m_e c} = \frac{\hbar^2}{Gm_G m_{\hbar}^2} \Rightarrow A \equiv \frac{m_p^2}{m_G m_{\hbar}} = \frac{m_{\hbar}}{m_e} \quad (28)$$

so that the fundamental ($n = 1$) energy is: $E \equiv m_{\hbar} c^2/A^2 = m_e c^2/A$. There is an *elimination of c* by considering the term A^2 as the product of the above gravitational constant $a_G = \hbar c/Gm_p m_H$ and the electro-weak one $a_w = \hbar^3/cG_F m_e^2$ [5], where G_F is the Fermi constant:

$$A^2 = a_G a_w \Rightarrow E = \frac{m_e c^2}{\sqrt{a_G a_w}} \quad (29)$$

Table 3: Prediction of Eddington Number ($N_E = 136 \times 2^{256}$) and Holo-physics formula for the *invariant* Hubble radius $R \approx 13.812$ Giga light-year (Gly) and the corresponding Hubble constant $H_0 = c/R$, which uses the length unit Megaparsec, compared to the main measurements. Lucas Number $N_L = 2^{127} - 1$. Topological Function $f(k) \equiv e^{2^{k+1/2}}$. Holographic ratio $u = R_{hol}/R$. For comparison, the so-called standard “Universe Age” is also presented, with unit in the c ratio.

| Date | Source $R = 2GM/c^2$ | Hubble radius Gly | Hubble Cst. km s ⁻¹ /Mpc | Univ. “Age” Gyr |
|------|--|-----------------------|--|--------------------|
| 1945 | Eddington Number [36] ; $N_E \approx (3/10) Mm_p/M_H m_n$ | 13.812 | 70.793 | |
| 1927 | Lemaître [34] | 1.6 | 620 | |
| 1929 | Hubble [35] | 1.8 | 540 | |
| 1956 | Humason, Maydal and Sandage [37] | 5.4 | 180 | |
| 1958 | Sandage [38] | 13 | 75 | |
| 1998 | $2\hbar^2/Gm_e m_p m_n$ Twice (3 mn Form.= Clsd Draft) | 13.800 | 70.852 | |
| 2006 | $2\hbar^2/Gm_e m_p m_n$ [22] | 13.800 | 70.852 | |
| 2006 | $2 N_L \lambda_e$ [22] | 13.889 | 70.397 | |
| 2017 | $(WZ)^4(\lambda_p \lambda_H)^{1/2}$ [5] [13] | 13.796 ± 0.002 | 70.87 ± 0.01 | |
| 2017 | $\lambda_e f(6)/6$ [13] | 13.821 | 70.744 | |
| 2017 | $\lambda_e (3^3)^{3^3}/u$ [13] | 13.812 | 70.793 | |
| 2017 | $2\hbar^2/Gm_e m_p m_H$ [13] Machian Formula | 13.812 | 70.793 | |
| 2017 | $2(ct_K)^2/a_w \lambda_e$ [13] | 13.812 | 70.793 | |
| 2017 | $(2/u)^{2 \times 3 \times 5 \times 7} \lambda_e$ [14] Complete Holic Principle | 13.856 | 70.565 | |
| 2021 | $(6/\pi)^{r_B/\lambda_e} \lambda_e$ [15] | 13.776 | 70.975 | |
| 2022 | $2N_L \lambda_e (1 - (137^2 + \pi^2 + e^2)/pH)$ | 13.812 (Machian prob) | 70.793 | |
| 1998 | PDG (Particle Data Group) | 14 ± 2 | 70 ± 10 | 11.5 ± 1.5 |
| 2002 | PDG | 13.7 ± 0.3 | 71 ± 3 | 15 ± 3 |
| 2005 | Hubble Space Telescope | 13.6 ± 1.5 | 72 ± 8 | 13.7 ± 0.2 |
| 2012 | WMAP [28] | 14.1 ± 0.2 | 69.3 ± 0.8 | 13.77 ± 0.06 |
| 2019 | Riess group [30] | 13.2 ± 0.3 | 74.2 ± 1.4 | |
| 2020 | Planck mission [29] | 14.5 ± 0.1 | 67.4 ± 0.5 | 13.82 ± 0.04 |
| 2020 | HOLICOW [31] | 13.4 ± 0.3 | 73.3 ± 1.8 | |
| 2021 | Carnegie-Chicago Hubble Program [18] | 14.0 ± 0.3 | 69.8 ± 1.6 | |

with $t_e \equiv \hbar/m_e c^2$ the electron period, this corresponds to the time:

$$t_e \sqrt{a_G a_w} \approx 9600.60 \text{ s}. \quad (30)$$

The identification with the Kotov P_0 period $t_K \approx 9600.606$ (12)s [16, 19] corresponds to $G \approx 6.6754527$ SI, specified to 10^{-8} by the Single-Electron Radius $R_1 \approx (4\pi p/p_w)^2 a_w c t_K$ [14] and consistent with the BIPM measurements [25], but at 6σ from the official value, a mean between *discordant* measurements. With the Fermi mass $m_F = m_e \sqrt{a_w}$, close to the mean nucleotide mass [13], the Lepton Mu mass m_μ , $u = R_{hol}/R$, the critical density $\rho_{cr} = 3c^2/8\pi G R^2$, $m_{GF} =$

$(m_p m_F)^{1/2}$, this defines our optimal strong coupling a_s :

$$\begin{aligned}
 m_G &= \frac{m_e m_p m_H}{m_F^2} \\
 m_{\hbar}/m_P &= \frac{m_F}{(m_p m_H)^{1/2}} \equiv \frac{m_\mu^2}{m_e m_N} \equiv 2\pi \frac{a_s m_p m_H}{m_e m_F} \\
 (GG_F)^{1/2} &\equiv \left(\frac{\hbar}{m_{GF}} \right)^2 = \frac{\hbar}{(m_p m_H)^{1/2}} \frac{\lambda_e^2}{t_K} \\
 \frac{G_F}{G m_p^2 l_p^2} &\approx \frac{a^4 m_p m_\mu}{m_e^2} (0.2\%) \\
 \frac{\hbar}{(G_F \rho_{cr})^{1/2}} &\approx \frac{\lambda_e^2}{u^{1/16} l_p} (0.01\%)
 \end{aligned} \quad (31)$$

exhibiting a symmetry between canonical area speeds. Note that $2ct_K \approx L_{NL}(m_{bc})$, confirming once more the bi-codon

mass, which enters also a relation involving the Cosmos, the Photon and Graviton masses [14] (Table 3). Moreover, with $P = m_P/m_e$, $F = m_P/m_e$, $H = m_H/m_e$, $p = m_p/m_H$, and the precise variant (0.14 ppm) of the Golden Number: $\Phi_0 = P/(a_w H)^3 \approx ((4\pi/3)(H/p)^2)^{1/3}$, one observes:

$$\begin{aligned} \frac{L_{NL}(m_{GF})}{r_{HB}} &\equiv \left(\frac{P}{F^3}\right)^{1/2} \frac{1}{a} \approx \Phi_0^2 \text{ (15 ppm)} \\ cT_{NL}(m_{GF}) &\equiv l_P \left(\frac{P}{F}\right)^{5/2} \\ &\approx \left(\frac{R_{hol} \lambda_e}{2}\right)^{1/2} \frac{1}{d_e^2} \text{ (74 ppm)} \end{aligned} \quad (32)$$

where d_e is the canonical Excess Electron Magnetic Moment (Table 1). This specifies the holographic relations $a^2 \approx (4\pi/3)p^{3/2}$ and $F^5/Pa^3 \approx \eta$, with $\eta = 1 + 2/(3 \times 139)$ (ppb precision) [15], where 139 is the complete Atiyah form [26], adding the dimensions of the four algebra (octonion, quaternion, complex, real): $139 = 137 + 2 = 2^7 + 2^3 + 2^1 + 2^0 \approx i^{-i\pi}$, and $3 \times 139 + 2 = 419$, the positive crystallographic number [40] in the superstring dimensions 10D and 11D [41], see Table 7 in [15]. Moreover, $T_{NL}(m_{GF}) \approx 19.14$ ms, typical of the Human nervous system, and the third octave down the flat La tone (Lab) for $La_3(A_4) = 442.9$ Hz, an anthropic argument far more pertinent and precise than the rough standard ones, principally based on a *cosmic Big Bang scenario* [5].

7 Conclusions

The quantification of Length and Time implies, through the Diophantine treatment of the Kepler laws, an angular momentum quantum identified with the reduced Planck constant \hbar . This leads to the Diophantine Coherence Theorem (DCT) which has the same structure than the Hass formulation in the Hydrogen atom spectrum. The DCT shows that the real invariant quantity is the Frequency, so that the Energy conservation would mean a Frequency Accordance, or “Coherence Principle”, mandatory in Practical Holography; the DCT conforms with the Harmony Principle of Pythagoras, the father of Natural Philosophy, the very root of Science. This confirms the pertinence of the Quantum Field Theory, where any Particle Field is defined by a whole number, entering the Holographic principle in the revisited critical steady-state Universe. In particular, both the Electron Quantum Number and the Neutron Quantum Number play a central role. The Universe Length Quantum (Topon) is associated to a Universe Time quantum (“Chronon” $t_M = \lambda_M/c$), which may be looked as the period of the *Permanent Bang oscillation matter-antimatter* [42].

The DCT shows that the Haas-Bohr radius is a *pseudo* length quantum, while the Universe itself appears as a pseudo quantum in a Cosmos, defined by the Holographic Principle where the Planck length is an intermediate holographic length, instead of the standard quantum. The Cosmic Length

Quantum breaks the “Planck wall” by the factor 10^{-61} . The main pseudo length quantum is the reduced Electron Wavelength which shows, through the DCT and the Kotov non-local period, a symmetry between gravitation and electroweak interaction. The Kotov-Lyuty Non-Doppler oscillation was overlooked: it is however a sign of the non-local character of Quantum Cosmology. It is mandatory to check the Lyuty Non-Doppler Quasar measurements [16].

The Planck mass enters naturally in the DCT, but plays no role in Particle Physics. However, the standard spin formulation rejoins our conclusion that the reduced Planck constant \hbar plays a more fundamental role than h . This is confirmed by the spiraling trajectory interpretation of the Single-Electron cosmic model [14].

The standard speed limit c excludes any explanation of the wave packet reduction phenomena, which requires a non-local or tachyonic Physics. So, it is logical that the bosonic string theory, which introduces tachyon, is confirmed by the Diophantine Topological Axis. Indeed, the central bosonic dimension $d = 26$ corresponds to the non-local universe radius (Machian Formula). The Holographic Principle and the DNA bi-codon mass are both decisive. So the DNA could be an helix-hologram, opening the way towards bio-computing [20]. The c -free Elementary Non-Local Three Minutes Formula giving the Universe half-radius is now fully established: this means a tight harmony between the Universe and Human Consciousness, a special and decisive manifestation of the Anthropic Principle.

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Iso-Representation of the Deuteron Spin and Magnetic Moment via Bohm's Hidden Variables

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In this paper, we review and upgrade the iso-representation of the spin $1/2$ of nucleons according to the isotopic branch of hadronic mechanics, known as *hadronic spin*, which is characterized by an isotopy of Pauli's matrices with an explicit and concrete realization of Bohm's *hidden variable* λ and show, apparently for the first time, that it allows a consistent and time invariant representation of the spin $J_D = 1$ of the Deuteron in its true ground state, that with null angular contributions $L_D = 0$. We then show, also apparently for the first time, that the indicated hadronic spin allows a numerically exact and time invariant representation of the magnetic moment of the Deuteron with the numeric value $\lambda = 2.65557$.

1 The Einstein-Podolsky-Rosen argument

In the preceding paper [1], we have outlined the axiom-preserving completion of 20th century applied mathematics into *iso-mathematics*, (see [2] for an extended presentation and [5–7] for independent studies), and the related *iso-mechanical branch of hadronic mechanics* (see [3] for a detailed treatment, [8–10] for independent studies and [11–13] for recent reviews) which isotopic methods have been used for the verification in [14, 15] of the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that *Quantum mechanics is not a complete theory* [16] (see [17] for the proceedings of the 2020 Teleconference in the EPR Argument, and its overviews [18, 19]).

Via the use of said isotopic methods, [1] achieved, apparently for the first time, a non-relativistic and relativistic representation of *all* characteristics of the muons (including their recently measured anomalous magnetic moment) as an extended and naturally unstable hadronic bound state of electrons and positrons produced free in the spontaneous decay with the lowest mode.

In the subsequent paper [20], we showed that said isotopic methods confirm the 1983 experimentally unresolved deviations [21] from the conventional formulation of time dilation for composite particles such as the muons, in favor of its axiom-preserving isotopic completion. We indicated in [20] that said deviations are due to incompatibility of the conventional time dilation with the time-irreversible character of the muon decay voiced since 1967 by R. M. Santilli [22] (see the 1995 full treatment [3]) and independently voiced in 1968 by D. I. Blokhintsev [23] for the incompatibility of the conventional time dilation with internal non-local effects of composite particles.

In this paper, we review and upgrade the notion of *hadronic spin* first introduced in [3, Section 6.8, page 250] and then used for verification [14] of the EPR argument [16] as well as in other applications [4]. The new notion of hadronic spin is

then used for the characterization of the spin $1/2$ of the nucleons, and realized via an isotopy of Pauli's matrices with an explicit and concrete realization of Bohm's *hidden variable* λ [43]. We then show, apparently for the first time, that said hadronic spin allows the first known exact and time-invariant representation of the spin $S_D = 1$ of the deuteron in the true ground state, that with null contributions from angular momenta $L_D = 0$.

We then show, also apparently for the first time, that said hadronic spin allows a numerically exact and time-invariant representation of the magnetic moment of the Deuteron with $\lambda = 2.65557$.

A technical understanding of this paper requires a technical knowledge at least of [2, 3]. A preliminary understanding of this paper requires a knowledge of reviews [11–13].

2 Iso-representation of the Deuteron spin

As it is well known, the quantum mechanical spin $1/2$ of nucleons is characterized by the fundamental irreducible representation of the special unitary Lie algebra $SU(2)$ which is notoriously given by the celebrated *Pauli matrices*

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (1)$$

(where σ_3 is set hereon along the spin direction) with commutation rules

$$[\sigma_i, \sigma_j] = \sigma_i \sigma_j - \sigma_j \sigma_i = i2\epsilon_{ijk}\sigma_k. \quad (2)$$

The value $S = 1/2$ of the nucleon spin is characterized by the eigenvalue equations on a Hilbert space \mathcal{H} over the field of complex numbers \mathcal{C} with basis $|b\rangle$

$$\begin{aligned} S_k &= \frac{1}{2} \sigma_k, \\ \sigma_3 |b\rangle &= \pm |b\rangle, \\ \sigma^2 |b\rangle &= (\sigma_1 \sigma_1 + \sigma_2 \sigma_2 + \sigma_3 \sigma_3) |b\rangle = 3 |b\rangle. \end{aligned} \quad (3)$$

A serious insufficiency of quantum mechanics in nuclear physics, which is fully supportive of the EPR argument [16], is that the representation of the spin 1/2 of nucleons via Pauli matrices does not allow a representation of the Deuteron spin $S_D = 1$ under the conditions of its experimental detection, that is, in its ground state with null orbital contributions $L_D = 0$. In fact, the sole possible *stable* bound state between a proton and a neutron permitted by quantum mechanics (qm) is the singlet

$$D = (n_{\uparrow}, n_{\downarrow})_{qm}, \quad (4)$$

for which the total spin is null, $J_D = 0$. In an attempt of resolving this insufficiency while preserving quantum mechanics, nuclear physicists have assumed for about one century that the Deuteron is a bound state of a proton and a neutron in *excited orbits* such that $L_D = 1$ (see e.g. [25]).

When at Harvard University with DOE support, R.M. Santilli noted that the most effective way of resolving the above and other insufficiencies of quantum mechanics (see next section) is to *exit from its class of unitary equivalence*. Therefore, Santilli proposed in two 1978 memoirs [26, 27] and in two Springer Verlag monographs [28, 29], the EPR generalization / completion of quantum mechanics into a new discipline which he called *hadronic mechanics* (see the Abstract and [27, pages 684,749,777] and [29, page 112]).

Hadronic mechanics was conceived to be an *axiom-preserving, thus isotopic non-unitary image of quantum mechanics for the representation of the dimension, shape and density of hadrons in interior conditions with ensuing potential as well as non-potential interactions due to mutual penetration*.

The proposal voiced in [26]–[29] suggested the construction of the *time irreversible completion of quantum mechanics into hadronic mechanics with the basic time evolution* (see [27, (4.15.24), page 742], [29, (19), page 153] and [3, (4.3.1), page 154])

$$\begin{aligned} i \frac{dA}{dt} &= (A, H) = ARH - HSA = \\ &= (ATH - HTA) + (AJH + HJA), \\ A(t) &= e^{HSit} A(0) e^{-iRH}, \\ R &= T + J, \quad s = -T + J, \end{aligned} \quad (5)$$

which is called Lie-admissible / Jordan-admissible since the bracket (A, H) clearly contains a Lie algebra $(ATH - HTA)$ and a Jordan algebra $(AJH + HJA)$ content.

By recalling that quantum mechanics can only represent systems whose time reversal images verify causality laws (because Heisenberg's equation is invariant under anti-Hermiticity), the aim of Santilli's proposal (stemming from his DOE support) was to achieve a consistent treatment of systems whose time reversal image violate causality, which is the case for all energy-releasing processes, with particular reference to nuclear fusions and fossil fuel combustion.

In this paper, we study *stable nuclei* that, as such, are time-reversal invariant. Consequently, our study requires the *Lie-isotopic branch of hadronic mechanics*, called for brevity *iso-mechanics*, which is based on the completion of the quantum mechanical enveloping associative algebra of Hermitean operators A, B, \dots on \mathcal{H} over \mathbb{C} with product $A \times B = AB$ and multiplicative unit I into the new product (first introduced in [26, (3.710), page 352] and [29, (5), page 71])

$$A \star B = A\hat{T}B, \quad \hat{T} > 0, \quad (6)$$

called *iso-product* because associativity-preserving, the positive-definite quantity \hat{T} being called the *isotopic element* and new compatible multiplicative unit

$$\hat{I} = 1/\hat{T} > 0, \quad \hat{I} \star A = A \star \hat{I} \equiv A \quad \forall A \in \mathcal{H}, \quad (7)$$

called *iso-unit* with ensuing basis time evolution first introduced in [27, (4.15.59), page 752] (see also [29, (18), page 163, Vol. II] and [3, (3.1.6), page 81])

$$i \frac{dA}{dt} = [A, H]^{\dagger} = A\hat{T}H - H\hat{T}A, \quad (8)$$

$$A(t) = e^{H\hat{T}t} A(0) e^{-i\hat{T}H} = W(t) A(0) W(t)^{\dagger}, \quad (9)$$

$$WW^{\dagger} \neq I,$$

which is called Lie-isotopic because of the clear verification of the Lie algebra axioms by the new brackets $[A, H]^{\star}$, although in a generalized form.

Following the identification of the basic structure (6) to (8), Santilli constructed in the 1983 monograph [29] the systematic isotopies in the 1983 volume [29] of the various branches of Lie's theory (universal enveloping associative algebra, Lie's theorems, Lie's transformation groups, etc.), resulting in a theory nowadays known as the *Lie-Santilli iso-theory* [5] (see also [30, 31]).

Santilli then constructed the isotopies of all known space-time symmetries [32]–[42]. In particular, systematic studies were conducted on the construction, classification and verification isotopies of the SU(2)-spin symmetry which can be found in [3, Chapter 6, page 209 on], in papers [33]–[37] with a summary in Section 3 of [12].

The *hadronic spin* (first introduced in [3, Section 6.8]) is the characterization of the spin of hadrons under strong interactions via the iso-irreducible, iso-unitary, iso-representations of the Lie-Santilli iso-symmetry $\widehat{\text{SU}}(2)$.

The simplest possible case of spin 1/2 of the nucleons can be outlined as following: all mathematical and physical aspects of the (regular [31]) isotopic branch of hadronic mechanics can be uniquely and unambiguously constructed via a simple, positive-definite *non-unitary transformation* set equal to the iso-unit of the new theory

$$UU^{\dagger} = \hat{I} > 0 \neq I, \quad \hat{T} = 1/\hat{I} = (UU^{\dagger})^{-1} > 0, \quad (10)$$

provided said non-unitary transformation is applied to the *totality* of the quantum mechanical, mathematical and physical quantities and their operations *with no exception known to the author*, to prevent insidious inconsistencies in mixing mathematics and iso-mathematics that generally remain undetected by non-experts in the field.

The indicated correct use of the above procedure permits the map of all quantum mechanical quantities, including unit, product, Lie algebras, etc., into their hadronic formulations that are generally denoted with a “hat”

$$\begin{aligned} I &\rightarrow UIU^\dagger = \hat{I}, \\ AB &\rightarrow U(AB)U^\dagger = \\ &= (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A} \star \hat{B}, \\ AB - BA &= [A, B] \rightarrow U(AB - BA)U^\dagger = \\ &= \hat{A} \star \hat{B} - \hat{B} \star \hat{A} = [\hat{A}, \hat{B}]^*, \text{ etc.} \end{aligned} \quad (11)$$

The hadronic spin 1/2 for nuclear constituents is given by the iso-fundamental, iso-unitary, iso-irreducible iso-representation of the Lie-Santilli iso-algebra $\widehat{\text{SU}}(2)$ under the condition of *iso-unimodularity*

$$\text{Det } \hat{I} = 1. \quad (12)$$

The above condition allowed Santilli to characterize the basic iso-unit of iso-mechanics in terms of Bohm's *hidden variable* λ [43] which was presented for the first time in [3, (6.8.19), page 248], according to the rules

$$\begin{aligned} \text{Det } \hat{I} &= \text{Det}[(UU^\dagger)] = \text{Det}[\text{Diag}(g_{11}, g_{22})] = 1, \\ g_{11} &= g_{22}^{-1} = \lambda \geq 0, \end{aligned} \quad (13)$$

yielding the *iso-Pauli matrices* first proposed in [3, (6.8.20), page 248]

$$\begin{aligned} \hat{\sigma}_k &= U\sigma_k U^\dagger, \\ UU^\dagger &= \hat{I} = \text{Diag}(\lambda^{-1}, \lambda), \quad \hat{T} = \text{Diag}(\lambda, \lambda^{-1}), \\ \hat{\sigma}_1 &= \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \end{aligned} \quad (14)$$

and then used in [14] for the verification of the EPR argument thanks to the evident inapplicability of Bell's theorem [44] due to the non-unitary structure of the theory.

It is easy to see that the iso-Pauli matrices verify the Lie-Santilli iso-commutation rules

$$\begin{aligned} [\hat{\sigma}_i, \hat{\sigma}_j]^* &= \hat{\sigma}_i \star \hat{\sigma}_j - \hat{\sigma}_j \star \hat{\sigma}_i = \\ &= \hat{\sigma}_i \hat{T} \hat{\sigma}_j - \hat{\sigma}_j \hat{T} \hat{\sigma}_i = i2\epsilon_{ijk} \hat{\sigma}_k, \end{aligned} \quad (15)$$

showing the clear iso-morphism $\widehat{\text{SU}}(2) \approx \text{SU}(2)$.

The representation of the spin 1/2 of nucleons despite its generalized structure is given by the iso-eigenvalues on an iso-state $|\hat{b}\rangle$ of the *Hilbert-Myung-Santilli iso-space* $\hat{\mathcal{H}}$ [45] over the iso-field of iso-complex iso-numbers $\hat{\mathbb{C}}$ [46]

$$\begin{aligned} \hat{S}_k &= \frac{\hat{I}}{2} \star \hat{\sigma}_k = \frac{1}{2} \hat{\sigma}_k, \\ \hat{\sigma}_3 \star |\hat{b}\rangle &= \hat{\sigma}_3 \hat{T} |\hat{b}\rangle = \pm |\hat{b}\rangle, \\ \hat{\sigma}^2 \star |\hat{b}\rangle &= (\hat{\sigma}_1 \hat{T} \hat{\sigma}_1 + \hat{\sigma}_2 \hat{T} \hat{\sigma}_2 + \hat{\sigma}_3 \hat{T} \hat{\sigma}_3) \hat{T} |\hat{b}\rangle = 3 |\hat{b}\rangle. \end{aligned} \quad (16)$$

As it is well known, *non-unitary theories violate causality*, and that is the case for the hadronic spin when considered in its projection on a conventional Hilbert space \mathcal{H} over a conventional field \mathbb{C} . Additionally, non-unitary transforms generally change the numeric value of the isotopic element which represent physical, measurable quantities (see next section). These and other problems are resolved by the reformulation of non-unitary time evolution (9) into the *iso-unitary iso-transformations* [47]

$$\begin{aligned} WW^\dagger &= \hat{I}, \quad W = \hat{W} \hat{T}^{1/2}, \\ WW^\dagger &= \hat{W} \star \hat{W}^\dagger = \hat{W}^\dagger \star \hat{W} = \hat{I}, \end{aligned} \quad (17)$$

under which reformulation the iso-unit, iso-product, Lie-Santilli iso-algebras, etc., are invariant,

$$\hat{I} \rightarrow \hat{W} \star \hat{I} \star \hat{W}^\dagger = \hat{I}' \equiv \hat{I}, \quad (18)$$

$$\begin{aligned} \hat{A} \star \hat{B} &\rightarrow \hat{W} \star (\hat{A} \star \hat{B}) \star \hat{W}^\dagger = \\ &= \hat{A}' \star \hat{B}' = \hat{A}' \hat{T}' \hat{B}', \quad \hat{T}' \equiv \hat{T}, \\ \hat{A}' &= \hat{W} \star \hat{A} \star \hat{W}^\dagger, \quad \hat{B}' = \hat{W} \star \hat{B} \star \hat{W}^\dagger, \\ \hat{T}' &= (\hat{W}^\dagger \star \hat{W})^{-1}. \end{aligned} \quad (19)$$

It should be noted that, by no means, hadronic spin solely characterizes the spin 1/2 because it was conceived [26, 27] for the characterization of the most general possible notion of spin for an extended particle such as a hadron in the core of a star with ensuing *non-local contributions from the star environment* (see [3, 14], [34]–[37]) according to the *de Broglie-Bohm non-local theory* [48]. The notion of hadronic spin was then specialized to the spin of nucleons because of clear experimental evidence, rather than popular views in nuclear physics, establishing its value 1/2.

The iso-representation of the Deuteron spin $J_D = 1$ in its true bound state with $L_D = 0$ via the hadronic spin is elementary. To see it, let us call for clarity *iso-protons*, *iso-neutron*, *iso-nucleons*, *iso-Deuteron* and *iso-Helium* (with corresponding symbols $\hat{p}, \hat{n}, \hat{N}, \hat{D}, \hat{He}$), the particles and nuclei characterized by the hadronic spin. With reference to [3, Section 2.11, page 265 on] on the addition of hadronic spins, the most stable hadronic bound state of the iso-Deuteron as a

hadronic bound state of an iso-proton and an iso-neutron is given by the *axial triplet state*. The axial triplet coupling first identified in the new chemical species of magnecules (see [49, Chapter 8, page 303 on] and [50, 51]) and then used for the new Intermediate Controlled Nuclear Fusion [52–54] with iso-representation (Fig. 1)

$$\hat{D} = \begin{pmatrix} \hat{p}_\uparrow \\ \star \\ \hat{n}_\uparrow \end{pmatrix}. \quad (20)$$

3 Iso-representation of the Deuteron magnetic moment

Another serious limitation of quantum mechanics in nuclear physics has been the inability, in about one century of studies, to achieve an exact representation of nuclear magnetic moments via the tabulated values for the magnetic moments of the proton and of the neutron in vacuum [55]

$$\mu_p = +2.79285 \mu_N, \quad \mu_n = -1.91304 \mu_N, \quad (21)$$

where μ_N represents the *nuclear magneton*.

As an example, the magnetic moment predicted by quantum mechanics (qm) from values (21) for the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = \mu_p + \mu_n = (2.79285 - 1.91304) \mu_N = 0.87981 \mu_N, \quad (22)$$

and does not represent the experimental value of the Deuteron magnetic moment

$$\mu_D^{ex} = 0.85647 \mu_N, \quad (23)$$

due to a deviation in *excess* of about 3%,

$$\mu_D^{qm} - \mu_D^{ex} = 0.02334 \mu_N \approx 2.95\% \mu_D^{ex}, \quad (24)$$

with larger deviations for heavier nuclei.

E. Fermi [56], V.F. Weisskopf [25] and other founders of nuclear physics formulated the hypothesis, hereon referred to as the *Fermi-Weisskopf hypothesis*, that in the transition from isolated particles in vacuum to members of a nuclear structure, protons and neutrons experience a deformation of their extended charge distribution with consequential change of their magnetic moments (21) while conserving their spin 1/2 (see the statement at the top of [25, page 31]).

The first numerically exact and time-invariant representation of the Deuteron magnetic moment (23) was achieved in 1994 by Santilli [57] (see also its subsequent extended study in [58]) thanks to the prior construction of the isotopic branch of hadronic mechanics for the representation of extended, thus deformable hadrons and related iso-symmetries [32]–[42] with the isotopic element

$$\hat{T} = \text{Diag} \left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2} \right), \quad (25)$$

in which n_k^2 , $k = 1, 2, 3$, represent the semi-axes of the *deformable* proton and of the nucleon under strong nuclear forces and n_4^2 represents their density. Under the assumption, for simplicity, that the proton and the neutron in the Deuteron structure have the same dimension, shape and density, [57] reached a numerically exact and time invariant representation of the magnetic moment of the Deuteron in [57, (3.6), page 124] with the following values of the characteristic n -quantities (that are denoted with the symbols $b_\mu = 1/n_\mu$ in [57])

$$\begin{aligned} b_1 &= \frac{1}{n_1} = b_2 = \frac{1}{n_2} = 1.0028, \\ b_3 &= \frac{1}{n_3} = 1.662, \quad b_4 = \frac{1}{n_4} \end{aligned} \quad (26)$$

(whose derivation is not reviewed here for brevity), by therefore confirming the 1981 preliminary experimental verification of the Fermi-Weisskopf hypothesis via neutron interferometry [59].

In this paper, we present, apparently for the first time, a second numerically exact and time invariant representation of the magnetic moment of the Deuteron (23), with spin $S_D = 1$ in its ground state via the representation of Santilli's iso-Pauli matrices (14) by using the Clifford's algebra representation of the conventional Pauli matrices [60]–[64], whose representation is here assumed to be known for brevity.

Note that, when formulated on their associative enveloping algebra, the iso-Pauli matrices satisfy all algebraic properties of the conventional Pauli matrices. Consequently, we can use the conventional representation in its entirety and introduce the *representation of iso-Pauli matrices (14) in terms of Clifford algebra* $\tilde{\mathbf{G}}_3 = \tilde{\mathbf{G}}_3(\mathbf{R}^3)$ with the iso-basis

$$\tilde{\mathbf{G}}_3 : \{1, \hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3, \hat{\sigma}_1 \hat{\sigma}_2, \hat{\sigma}_1 \hat{\sigma}_3, \hat{\sigma}_2 \hat{\sigma}_3, i := \hat{\sigma}_1 \hat{\sigma}_2 \hat{\sigma}_3\}, \quad (27)$$

and main properties

$$\begin{aligned} \hat{\sigma}_1^2 &= \hat{\sigma}_2^2 = \hat{\sigma}_3^2 = 1, \\ \hat{\sigma}_{12} &= \hat{\sigma}_1 \hat{\sigma}_2 = -\hat{\sigma}_{21}, \quad \hat{\sigma}_{13} = \hat{\sigma}_1 \hat{\sigma}_3, \quad \hat{\sigma}_{23} = \hat{\sigma}_2 \hat{\sigma}_3, \\ \hat{\sigma}_{12}^2 &= \hat{\sigma}_1 \hat{\sigma}_2 \hat{\sigma}_1 \hat{\sigma}_2 = -\hat{\sigma}_1 \hat{\sigma}_2 \hat{\sigma}_2 \hat{\sigma}_1 = -\hat{\sigma}_1^2 \hat{\sigma}_2^2 = -1. \end{aligned} \quad (28)$$

The *standard basis* of unit iso-vectors $\{\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3\}$ define the x, y, z iso-coordinate axes, respectively. The *iso-spectral basis* is

$$\begin{pmatrix} \hat{u}_+ & \hat{\sigma}_1 \hat{u}_- \\ \hat{\sigma}_1 \hat{u}_+ & \hat{u}_- \end{pmatrix}, \quad (29)$$

where $\hat{u}_\pm := \frac{1}{2}(1 \pm \hat{\sigma}_3)$ are *mutually annihilating iso-idempotents*. In the standard iso-basis of $\tilde{\mathbf{G}}_3$,

$$\begin{aligned} \{\hat{\sigma}_1, \hat{\sigma}_2 &= i\hat{\sigma}_1 \hat{\sigma}_3, \hat{\sigma}_3\}, \\ \hat{i} &= \hat{\sigma}_1 \hat{\sigma}_2 \hat{\sigma}_3, \end{aligned} \quad (30)$$

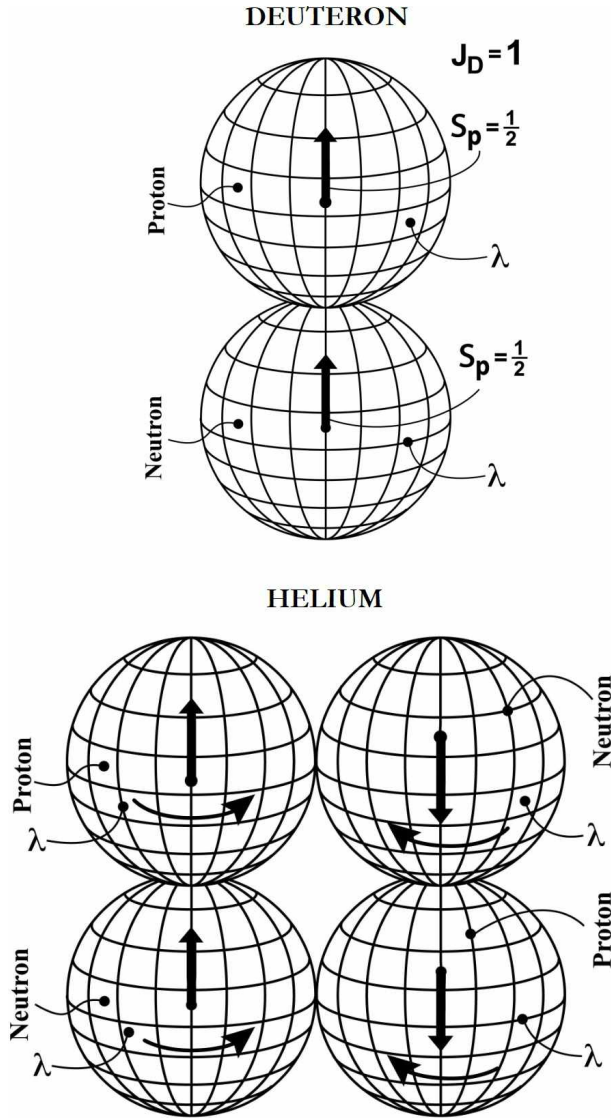


Fig. 1: In the top, we illustrate the structure of the iso-Deuteron as a hadronic bound state of an iso-proton and an iso-neutron in axial triplet coupling, thus representing for the first time the spin of the Deuteron $S_D = 1$ in its ground state, with null angular contributions $L_D = 0$. The prefix “iso” represents the novel hadronic spin characterized by the iso-Pauli matrices, (14), with an explicit and concrete realization of Bohm’s hidden variable λ . The axial triplet coupling was first identified in the new chemical species of mag-necules (see [49, Chapter 8, page 303 on] and [50, 51]) and then used for the new Intermediate Controlled Nuclear Fusion [52–54]. In the bottom, we illustrate the structure model of the iso-Helium as a hadronic bound state under strong interactions of two iso-Deuterons in singlet coupling, which allows a representation of the null spin and magnetic moment of Helium in its ground state. It should be noted that the above model is not necessarily extendable to heavier stable nuclei due to the prior need of resolving the problem of nuclear stability caused by the natural instability of the neutron, which problem is planned for study in a subsequent paper (see [65] for a preliminary study).

is the *conventional unit* of the associative algebra $\tilde{\mathbf{G}}_3$. It must be remembered that $\hat{\sigma}_k$ verify the property

$$\hat{\sigma}_k^2 = \hat{\sigma}_k \star \hat{\sigma}_k = 1, \quad (31)$$

for $k = 1, 2, 3$, where the \star denotes the iso-product.

We now show that the hidden variable λ of the iso-Pauli matrices (14) can provide a second representation of the deformation of the magnetic moment of nucleons of [57, 58] with consequential exact representation of nuclear magnetic moments.

By introducing the realization of the hidden variable λ

$$\lambda = e^{\phi} \geq 0, \quad (32)$$

with respect to the basis of the *standard unit* of the iso-Pauli matrices \hat{I} , the *iso-reciprocal* \hat{T} and the *iso-vector basis* $\{\hat{\sigma}_k\}$, are given by

$$\hat{I} = \cosh \phi + \sigma_3 \sinh \phi = e^{\phi \sigma_3}, \quad (33)$$

$$\hat{T} = \cosh \phi - \sigma_3 \sinh \phi = e^{-\phi \sigma_3},$$

Consequently

$$\begin{aligned} \hat{\sigma}_1 &= \sigma_1 \hat{I} = \hat{T} \sigma_1, \\ \hat{\sigma}_2 &= \sigma_2 \hat{I} = \hat{T} \sigma_2, \\ \hat{\sigma}_3 &= \sigma_3 \hat{I} = \hat{I} \sigma_3. \end{aligned} \quad (34)$$

By recalling that σ_3 characterizes the nucleon spin $S = 1/2$, we reach the result that the replacement of the standard basis of the Clifford algebra \mathbf{G}_3 for Pauli matrices with the iso-Pauli matrices (14) implies the EPR completion of $\hat{\sigma}_3$ into the expression

$$\hat{\sigma}_3 |\hat{b}\rangle = \sigma_3 \hat{I} |\hat{b}\rangle = \sigma_3 e^{\phi \sigma_3} |\hat{b}\rangle. \quad (35)$$

Recall that the quantum mechanical (qm) relationship between magnetic moments μ and spins S occurs via the gyro-magnetic factor g ,

$$\mu = gS, \quad (36)$$

and that the corresponding relation for the isotopic branch of hadronic mechanics (hm) is given by an expression of the type [57]

$$\mu_{hm} |\hat{b}\rangle = KgS |\hat{b}\rangle, \quad (37)$$

where K is an iso-renormalization constant of the gyromagnetic factor g created by the new notion of hadronic spin $1/2$. By using property (28), we reach the relation

$$\mu_{hm} |\hat{b}\rangle = e^{\phi \sigma_3} \mu_{qm} |\hat{b}\rangle = e^{\phi \sigma_3} gS |\hat{b}\rangle. \quad (38)$$

Recall also that: 1) Bohm’s hidden variable λ is associated with the *spin* of a particle according to (14); 2) The proton and the neutron have the same spin $1/2$ and essentially the same mass, thus being characterized by the same λ ; 3) The quantum mechanical representation of the magnetic moment

of the Deuteron is *in excess* of about 3% according to (24). By selecting the value for conformity with the selected spin orientation (Fig. 1)

$$\sigma_3 |\hat{b}\rangle = -|\hat{b}\rangle, \quad (39)$$

we can write the expression per each nucleon

$$\mu_{hm,k} \approx (1 + \phi\sigma_3)\mu_{qm,k} = (1 - \phi)\mu_{qm,k}, \quad k = p, n, \quad (40)$$

from which we obtain the iso-renormalized value of the magnetic moment of the proton and of the neutron

$$\hat{\mu}_p = +(1 - \phi) 2.79285 \mu_N, \quad \hat{\mu}_n = -(1 - \phi) 1.91304 \mu_N, \quad (41)$$

with corresponding value for the magnetic moment of the Deuteron

$$\begin{aligned} \mu_D^{hm} &= (1 - \phi) 2.79285 - (1 - \phi) 1.91304 \mu_N = \\ &= (1 - \phi) 0.87981 \mu_N = \mu_D^{ex} = 0.85647 \mu_N. \end{aligned} \quad (42)$$

From this, we obtain the numeric value

$$\phi = 1 - 0.87981/0.85647 = 1 - 0.02334 = 0.97666, \quad (43)$$

with corresponding *numeric value of Bohm's hidden variable for the Deuteron*

$$\lambda = e^\phi = e^{0.97666} = 2.65557, \quad (44)$$

by thereby achieving the desired exact representation of the magnetic moment of the Deuteron in terms of Bohm hidden variable λ . Its invariance over time follows from the derivation of iso-Pauli matrices (14) from the Lie-Santilli iso-symmetry $\hat{P}(3.1)$ [39]–[41].

The iso-representation of the magnetic moment of $4 - He - 2$ as the *iso-Helium* $\bar{He}(2)$ is a consequence (Fig. 1). The study of the iso-representation for heavier stable nuclei was initiated in [65], but its in-depth achievement requires the still missing consistent representation of nuclear stability against the natural instability of the neutron, which problem is planned for study in a subsequent paper.

We should finally note that in this section we have used the *standard Clifford algebra* and not the full isotopic Clifford algebra \hat{G} introduced by R. da Rocha and J. Vaz Jr. [66]. This is due to the fact that the full isotopy \hat{G}_3 of G_3 would have required the use of iso-product (6) with the isotopic element $\hat{T} = e^{-\phi\sigma_3} = 1/\hat{I}$, and the consequential lack of representation in (38) of the magnetic moment of the Deuteron for spin $S_D = 1$ in the ground state.

The understanding is however that the full iso-Clifford iso-algebra \hat{G}_{3N} is expected to be important for the numerically exact and time invariant representation of the spins and magnetic moments of nuclei with $A \geq 2$ nucleons.

In a nutshell, we can say that the Copenhagen interpretation of quantum mechanics deals with the simplest possible realization of quantum axioms, while the EPR completion of quantum into hadronic mechanics deals with progressively broader realizations of the same axioms for systems with progressively increasing complexity.

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Gödel Time Travel With Warp Drive Propulsion

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In the first part of this work, we recall the basic principles of the Alcubierre warp drive space-time within the extrinsic curvature formalism. In the created singular region, we consider a hollow object that carries a charged current all around its external shape which interacts with an electromagnetic potential. As a result, this comoving object placed inside the region will follow a Finslerian geodesic. This allows to re-define a new lapse function that contains the potential-charge interacting term which can be chosen arbitrarily large, in order to lower the energy density required for sustaining the space-time distortion. Ultimately, this new lapse function is adjusted so as to keep the warp drive energy tensor positive thus always satisfying the famous energy conditions. In the second part, we apply this result to the Gödel curves following our previous publication whereby it was shown that Gödel's metric is a physical model not bound to any astrophysical representation. In this perspective, we suggest a possible mode of time travel.

Notations

Space-time Greek indices α, β run from 0, 1, 2, 3.

Spatial Latin indices a, b run from 1, 2, 3.

Space-time signature is: +2 (Part I) and -2 (Part II).

PART I

1 The warp drive metric

1.1 The (3 + 1) formalism or ADM technique

Arnowitt, Deser and Misner (ADM) suggested a technique which leads to decompose the space-time into a family of spacelike hypersurfaces and parametrized by the value of an arbitrarily chosen time coordinate x^0 [1]. This *foliation* displays a proper time element dt between two nearby hypersurfaces labeled $x^0 = \text{const}$, $x^0 + dx^0 = \text{const}$ and the proper time element $cd\tau$ must be proportional to dx^0 , thus we write:

$$cd\tau = N(x^\alpha, x^0) dx^0, \quad (1.1)$$

where, according to the ADM terminology, N is called the *lapse function*.

Let us now evaluate the 3-vector whose spatial coordinates x^a are lying in the hypersurface $x^0 = \text{const}$ and which is normal to it, on the second hypersurface $x^0 + dx^0 = \text{const}$, where these coordinates now become $N^a dx^0$. The vector N^a is called the *shift vector*. The 4-metric tensor covariant components are

$$(g_{\alpha\beta})_{\text{ADM}} = \begin{pmatrix} -N^2 - N_a N_b g^{ab} & N_b \\ N_a & g_{ab} \end{pmatrix}. \quad (1.2)$$

The line element corresponding to the hypersurfaces separation is therefore written as

$$\begin{aligned} (ds^2)_{\text{ADM}} &= \\ &= -N^2 (dx^0)^2 + g_{ab} (N^a dx^0 + dx^a)(N^b dx^0 + dx^b) = \\ &= (-N^2 + N_a N^a)(dx^0)^2 + 2N_b dx^0 dx^b + g_{ab} dx^a dx^b, \end{aligned} \quad (1.3)$$

where g_{ab} is the 3-metric of the hypersurfaces. The contravariant components of the ADM metric tensor are

$$(g^{\alpha\beta})_{\text{ADM}} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N^b}{N^2} \\ \frac{N^a}{N^2} & g^{ab} - \frac{N^a N^b}{N^2} \end{pmatrix}. \quad (1.4)$$

As a result, the hypersurfaces have a unit time-like normal with contravariant components:

$$u^\alpha = N^{-1} (1, -N^a). \quad (1.5)$$

If the universe is approximated to a Minkowski space within an orthonormal coordinates frame of reference and where the fundamental 3-tensor satisfies $g^{ab} = \delta^{ab}$, the metric (1.3) becomes

$$ds^2 = -(N^2 - N_a N^a) c^2 dt^2 + 2N^a dx c dt + dx^a dx^b \quad (1.6)$$

or, in another notation,

$$ds^2 = -N^2 dt^2 + (dx + N^a c dt)^2 + dy^2 + dz^2. \quad (1.6\text{bis})$$

The Einstein action can be written in terms of the metric tensor $(g_{\alpha\beta})_{\text{ADM}}$ as [2]

$$S_{\text{ADM}} = \int c dt \int N \left({}^{(3)}R - K_b^a K_a^b + K^2 \right) \sqrt{{}^{(3)}g} dx^3 + \text{boundary terms},$$

where $K_a^b = K^b_a$, and ${}^{(3)}R$ is the 3-Ricci scalar and stands for the *intrinsic curvature* of the hypersurface

$$x^0 = \text{const}, \quad \sqrt{{}^{(3)}g} = \sqrt{\det \|g_{ab}\|} \leftrightarrow \sqrt{{}^{(-4)}g} = N \sqrt{{}^{(3)}g}$$

so that

$$K_{ab} = (2N)^{-1} (-N_{a;b} - N_{a;b} + \partial_0 g_{ab}) \quad (1.7)$$

represents the *extrinsic curvature*, and as such describes the manner in which the hypersurface $x^0 = \text{const}$ is embedded in the surrounding space-time. The rate of change of the 3-metric tensor g_{ab} with respect to the time label can be decomposed into “normal” and “tangential” contributions:

- The normal change is proportional to the extrinsic curvature $2K_{ab}/N$ of the hypersurface;
- The tangential change is given by the Lie derivative of g_{ab} along the shift vector N^a , namely:

$$\mathcal{L}_N g_{ab} = 2N_{(a;b)}. \quad (1.8)$$

With the choice of $N^a = 0$, we have a particular coordinate frame called *normal coordinates* according to (1.5) which is called an *Eulerian gauge*. Inspection shows that

$$K_{ab} = -u_{a;b} \quad (1.9)$$

which is sometimes called the *second fundamental form* of the 3-space. Six of the ten Einstein equations imply for K_b^a to evolve according to

$$\begin{aligned} \frac{\partial K_b^a}{c \partial t} + \mathcal{L}_N K_b^a &= \nabla^a \nabla_b N + \\ &+ N \left[R_b^a + K_a^c K_b^c + 4\pi(T - C) \delta_b^a - 8\pi T_b^a \right], \end{aligned} \quad (1.10)$$

$$C = T_{\alpha\beta} u^\alpha u^\beta, \quad (1.11)$$

where C is the matter energy density in the rest frame of normal congruence (time-like vector field) with $T = T_a^a$. Using the Gauss-Codazzi relations [3] one can express the Einstein tensor as a function of both the intrinsic and extrinsic curvatures. It is convenient here to introduce the 3-momentum current density $I_a = -u_c T_a^c$. So the remaining four equations finally form the so-called *constraint equations*

$$H = \frac{1}{2} \left({}^{(3)}R - K_b^a K_a^b + K^2 \right) - 8\pi C = 0, \quad (1.12)$$

$$H_b = \nabla_a \left(K_b^a - K \delta_b^a \right) - 8\pi I_b = 0. \quad (1.13)$$

Therefore, another way of writing (1.11) eventually leads to the formula

$$C = \frac{1}{16\pi} \left({}^{(3)}R - K_{ab} K^{ab} + K^2 \right). \quad (1.14)$$

1.2 Salient features of Alcubierre’s theory

1.2.1 The Alcubierre metric

In 1994, M. Alcubierre showed that an arbitrary large velocity (superluminal) can be achieved by building a so-called *space-time warped region (bubble-like region)* progressing along the x -direction which is a time-like trajectory, without violating the law of relativity [4]. Inside the bubble, the proper time element $d\tau$ is equal to the coordinate time dt which is also the

proper time of a distant observer, so any object in the bubble does not suffer any time dilation as it moves. Outside and inside the bubble, space-time remains flat. In the classical interpretation, the warp drive requires *contraction* of the front space, and *expansion* behind the same bubble in the chosen direction, quite in analogy to the inflationary phase of the expanding universe.

In terms of the ADM formalism, the Alcubierre metric is defined from a flat space-time, while the lapse function and the shift functions are chosen as

$$\left. \begin{aligned} N &= 1 \\ N^1 &= -v_s(t) f(r_s, t) \\ N^2 &= N^3 = 0 \end{aligned} \right\}. \quad (1.15)$$

Next, we define

$$r_s(t) = \sqrt{(x - x_s(t))^2 + y^2 + z^2} \quad (1.16)$$

as the distance outward from the center of a spaceship placed in the bubble, variable until R_B , which is the *radius of the bubble*. With respect to a distant observer, the apparent velocity of the ship (thus the bubble), is given by:

$$v_s(t) = \frac{dx_s(t)}{dt}, \quad (1.17)$$

where $x_s(t)$ is the trajectory of the bubble along the x -direction. Such a region is transported forward with respect to distant observers, along the x -direction, and any spacecraft placed at rest inside, has no local velocity, but always moves along a time-like curve, regardless of $v_s(t)$. We then have the line element of the *Alcubierre metric*

$$(ds^2)_{\text{AL}} = -c^2 dt^2 + [dx - v_s f(r_s, t) c dt]^2 + dy^2 + dz^2, \quad (1.18)$$

$$d\tau = dt, \quad (1.19)$$

Inside the spacecraft, the occupants will never suffer acceleration and so it is not difficult to show that the 4-velocity of a distant observer called *Eulerian observer* [5], has the following components:

$$(u^\alpha)_E = \{c, v_s c f(r_s, t), 0, 0\}, \quad (1.20)$$

$$(u_\alpha)_E = \{-c, 0, 0, 0\}. \quad (1.21)$$

The Eulerian observer is a special type of observer which refers to the Eulerian gauge defined above but with $N^1 \neq 0$, and as such, it follows time-like geodesic orthogonal to euclidean hypersurfaces. This observer starts out just inside the bubble shell at its first equator with zero initial velocity. Once during his stay inside the bubble, this observer travels along a time-like curve: $x = x_s(t)$ with a constant velocity nearing the ship’s velocity: $v_s = dx_s/dt$. The Eulerian observer’s velocity will always be less than the bubble’s velocity unless $r_s = 0$,

i.e., when this observer is at the center of the spaceship located inside. After reaching the second region's equator, this observer decelerates and is left at rest while going out at the rear edge of the bubble.

The Eulerian observer's velocity is needed to evaluate the energy density required to create the bubble. (see below) The function $f(r_s, t)$ is so defined as to cause space-time to contract on the forward edge and equally expanding on the trailing edge of the *bubble* as stated above. This is easily verified by using the expansion of the volume elements $\theta = (u^\alpha)_{E;\alpha}$ given by

$$\theta = \frac{v_s df}{(dx)_{AL}}. \quad (1.22)$$

1.2.2 The Alcubierre function

The function $f(r_s, t)$ is often referred to as a *top hat function* and Alcubierre originally chose the following form

$$f(r_s, t) = \frac{\tanh\{\sigma(r_s + R_B)\} - \tanh\{\sigma(r_s - R_B)\}}{2 \tanh\{\sigma R_B\}}, \quad (1.23)$$

where $R_B > 0$ is the *radius of the bubble*, and σ is a *bump parameter* which can be used to "tune" the wall thickness of the bubble. The larger this parameter, the greater the contained energy density, for its shell thickness decreases. Moreover the absolute increase of σ means a faster approach of the condition

$$\lim f(r_s, t) = 1, \text{ for } r_s \in (-R_B, R_B), \text{ otherwise } \sigma \rightarrow \infty.$$

In the ADM formalism the expansion scalar is shown to be

$$\theta = \partial_1 N^1 = -\text{Trace } K_{ab}, \quad (1.24)$$

which, with (1.13), becomes

$$\theta = v_s \frac{df}{dr_s} \frac{x_s}{r_s}. \quad (1.24\text{bis})$$

Note that the Natário warp drive evades the problem of contraction/expansion, by imposing the divergence free constraint to the shift vector $\nabla[v_s^2 f^2(r_s, t)] = 0$ [6].

Obviously, the shape of the function f induces both a volume contraction and expansion ahead and behind of the bubble. Let us now write down the Alcubierre metric in the equivalent form

$$(ds^2)_{Al} = -\left[1 - v_s^2 f^2(r_s, t)\right] c^2 dt^2 - 2v_s f c dt dx + dx^2 + dy^2 + dz^2, \quad (1.25)$$

which puts in evidence the covariant components of the metric tensor

$$\left. \begin{aligned} (g_{00})_{Al} &= -[1 - v_s^2 f^2(r_s, t)] \\ (g_{01})_{Al} &= (g_{10})_{Al} = -v_s f(r_s, t) \\ (g_{11})_{Al} &= (g_{22})_{Al} = (g_{33})_{Al} = 1 \end{aligned} \right\}. \quad (1.26)$$

1.2.3 Energy conditions

With the components (1.26), the Einstein-Alcubierre tensor is written

$$(G^{\alpha\beta})_{Al} = (R^{\alpha\beta})_{Al} - \frac{1}{2} (g^{\alpha\beta})_{Al} R, \quad (1.27)$$

$$(T^{\alpha\beta})_{Al} = \frac{c^4}{8\pi} (G^{\alpha\beta})_{Al}. \quad (1.28)$$

The *weak energy condition* (WEC) stipulates [7] that we must always have

$$C_{Al} = (T^{\alpha\beta})_{Al} (u_\alpha)_E (u_\beta)_E \geq 0 \quad (1.29)$$

From (1.14) we see that there in the Alcubierre space-time ${}^{(3)}R = 0$. Thus we get

$$C_{Al} = \frac{1}{16\pi} (K^2 - K_{ab} K^{ab}), \quad (1.30)$$

$$C_{Al} = \frac{1}{16\pi} \left[(\partial_1 N^1)^2 - (\partial_1 N^1)^2 - 2(\partial_2 N^1)^2 - 2(\partial_3 N^1)^2 \right], \quad (1.31)$$

$$(T^{00})_{Al} (u_0)_E (u_0)_E = (T^{00})_{Al} = -\frac{c^4}{32\pi} v_s^2 \left[\left(\frac{\partial f}{\partial y} \right)^2 + \left(\frac{\partial f}{\partial z} \right)^2 \right] < 0. \quad (1.32)$$

By taking into account the form of (1.23) we find the energy density:

$$(T^{00})_{Al} = -\frac{c^4}{32\pi} (v_s)^2 \left(\frac{df}{dr_s} \right)^2 \frac{y^2 + z^2}{r_s^2}. \quad (1.33)$$

This expression is unfortunately negative as measured by the Eulerian observer, and therefore it violates the weak energy conditions.

2 Reducing the energy density

2.1 A new configuration

Inside this bubble a spacecraft is engineered with a surrounding "shell" of thickness, $R_e - R_i$, where R_e is the outer radius, and R_i the inner radius. Now, let us consider a fluid of density ρ carrying a charge μ which fills this shell. By applying an electromagnetic field with a 4-potential A_α inside the shell, the whole spacecraft surrounded by the charge density will follow a specific *Finslerian geodesic* [8] provided the ratio μ/ρ remains constant all along the trajectory

$$ds_{\text{shell}} = ds + \frac{\mu}{\rho} A_\alpha dx^\alpha, \quad (2.1)$$

where $ds = \sqrt{\eta_{\alpha\beta} dx^\alpha dx^\beta}$.

Therefore we may write the metric (neglecting the non-quadratic term)

$$(ds^2)_{\text{shell}} = ds^2 + \left(\frac{\mu}{\rho} A_\alpha dx^\alpha \right)^2. \quad (2.2)$$

Now, the shell containing the charge μ which is acted upon by the potential A_a , must be included in the formulation of the metric (1.25). This can be achieved in a manner not too dissimilar to the one chosen in [9, 10]. First we have for the time component of the interaction term

$$\frac{\mu}{\rho} i A_0 dx^0 = \frac{\mu}{\rho} \Phi c dt, \quad (2.3)$$

where Φ is the scalar potential. The metric tensor time component in (2.2) becomes

$$g_{00} = -\left(1 + \frac{\mu}{\rho} \Phi\right)^2. \quad (2.4)$$

The remaining spatial components $(\mu/\rho)A_a dx^a$ can be neglected if the 3-velocity of the global charges carrier (spacecraft) is low, since in this case the 3-density current is equal to $j_a = \mu v_a \approx 0$. Hence, the metric (2.2) would reduce to

$$ds^2 = -\left(1 + \frac{\mu}{\rho} \Phi\right)^2 + dz^2 + dx^2 + dy^2. \quad (2.5)$$

In the framework of the Alcubierre metric, the spaceship shell is part of the warp drive bubble and as such the interaction term should be a function of r_s , R_B , σ , and the thickness $(R_e - R_i)$ but not the speed v_s .

Therefore we are led to define the lapse function as

$$N = \sqrt{1 + iS^2}, \quad (2.6)$$

where

$$S = \frac{1}{2} \left\{ 1 + \tanh \left[\sigma(r_s + R_e)^2 \right] \right\}^{-\frac{\sigma\Phi\mu}{\rho}}. \quad (2.7)$$

The dimensionless factor a delimits the shell thickness

$$a = (R_e - R_i)^{-1} \int_{R_i}^{R_e} dR, \quad (2.7bis)$$

and (2.7) is verified from the center of the spacecraft location to the ext. bubble wall R_e , where $f = 1$.

The Alcubierre metric (1.25) can then be re-written as

$$ds^2 = -\left[N^2 - v_s^2 f^2(r_s)\right] c^2 dt^2 - 2v_s f(r_s) c dt dx + dz^2 + dx^2 + dy^2. \quad (2.8)$$

From the internal radius R_i throughout the spacecraft interior, there is no charge, and we see that the space is Minkowskian so that the spacecraft and its occupants will never suffer any tidal forces nor time dilation as per (1.10bis).

From the metric (2.8), it is now easy to infer the Eulerian observer's velocity components. We have

$$c^2 = -c^2 (N^2 - v_s^2 f^2) \left(\frac{dt}{d\tau} \right)^2 - 2v_s f c \frac{dt}{d\tau} u_E + u_E^2. \quad (2.9)$$

The Eulerian observer travels along the geodesic where he "sees"

$$\frac{dt}{d\tau} = N^{-1}, \quad (2.10)$$

which yields

$$0 = u_E^2 - 2v_s f c N^{-1} u_E + v_s^2 f^2 c^2 N^{-2} \quad (2.11)$$

and finally we obtain

$$u_E = v_s f c N^{-1}, \quad (2.12)$$

$$(u^\mu)_E = \{cN^{-1}, v_s f c N^{-1}, 0, 0\}, \quad (2.13)$$

$$(u_\mu)_E = \{-cN, 0, 0, 0\}. \quad (2.14)$$

2.2 The energy required for the propulsion

If we insert N into (1.30), the formula

$$C_{Al} = (u_0)_E (u_0)_E T^{00} \quad (2.15)$$

yields the new energy density requirement

$$T^{00} = -\frac{c^4}{32\pi} \frac{v_s^2 (y^2 + z^2)}{N^4 r_s^2} \left(\frac{df}{dr_s} \right)^2. \quad (2.16)$$

Now, recalling the form (2.6) for N , we have

$$N^4 = (1 + iS^2)^2 < 0. \quad (2.17)$$

Thus the energy conditions $T^{00} \geq 0$ are obviously always satisfied. Therefore we may choose the factor N (thereby S) arbitrarily large so as to substantially reduce the required energy density for the ship frame.

The higher the charge and the potential, the lower the energy requirement.

In the closed volume V of the spacecraft shell one can inject a flow of electrons according to the constant ratios

$$\frac{\mu}{\rho} = \frac{\sum_V e}{\sum_V m}. \quad (2.18)$$

We see that the leptonic lightweight would have the capacity to lower the negative energy even further. The splitting shell-inner part of the spacecraft frame, is really the hallmark of the theory here: it implies that the proper time τ of the inner part of the spacecraft is not affected by the term N .

PART II

In "The Time Machine" (1895), the novel by H. G. Wells, an English scientist constructs a machine which allows him to travel back and forth in time. The history of fascinating idea of time travel can be traced back to Kurt Gödel who found a solution of Einstein's field equations that contains closed time-like curves (CTCs) [11]. Those make it theoretically

feasible to go on journey into one's own past. In our previous publication [12], we formally demonstrated that Gödel's model was not just a mere (speculative) cosmological model as it was always accepted, but an ordinary metric with own physical properties.

Upon these results we develop here the bases for a possible time travel mode of displacement.

3 Reformulation of Gödel's metric (reminder)

The classical Gödel line element is generically given by the interval

$$ds^2 = a^2 \left(dx_0^2 - dx_1^2 + dx_2^2 \frac{e^{2x_1}}{2} - dx_3^2 + 2e^{x_1} dx_0 dx_2 \right) \quad (3.1)$$

or, equivalently,

$$ds^2 = a^2 \left[-dx_1^2 - dx_3^2 - dx_2^2 \frac{e^{2x_1}}{2} + (e^{x_1} dx_2 + dx_0)^2 \right], \quad (3.2)$$

where $a > 0$ is a constant.

In our theory, we assumed that a is slightly space-time variable and we set

$$a^2 = e^{2U}. \quad (3.3)$$

As a result, the Gödel metric tensor components are conformal to the real Gödel metric tensor $g_{\mu\nu}$

$$(g_{\mu\nu})' = e^{2U} g_{\mu\nu}, \quad (g^{\mu\nu})' = e^{-2U} g^{\mu\nu}. \quad (3.4)$$

The exact Gödel metric reads now

$$(ds^2)' = e^{2U} \left[dx_0^2 - dx_1^2 + dx_2^2 \frac{e^{2x_1}}{2} - dx_3^2 + 2e^{x_1} (dx_0 dx_2) \right]. \quad (3.5)$$

This implies that this metric is a solution of the field equations describing a peculiar perfect fluid [13–15]

$$G_{\mu\beta} = \kappa \left[(\rho + P) u_\mu u_\beta - P g_{\mu\beta} \right]. \quad (3.6)$$

The model is likened to a fluid in rotation with mass density ρ and pressure P . The positive scalar U is shown to be:

$$U(x^\mu) = \int \frac{dP}{\rho + P}. \quad (3.7)$$

From (3.4) and (3.6) one formally infers that the flow lines of matter of the fluid follow conformal geodesics given by

$$s' = \int e^U ds. \quad (3.8)$$

The hallmark of the theory is the substitution (3.3): the Gödel space-time is no longer the representation of a cosmological model but it is relegated to the rank of an ordinary metric where its physical properties could allow for a possible replication.

4 Closed time-like curves

With Gödel one defines new coordinates (t, r, ϕ) which in the reformulated version lead to the line element

$$ds^2 = 4e^{2U} \left[dt_G^2 - dr^2 + (\sinh^4 r - \sinh^2 r) d\phi^2 + 2\sqrt{2} \sinh^2 r d\phi dt \right]. \quad (4.1)$$

This metric exhibits the rotational symmetry of the solution about the chosen Gödel t_G -time axis where $r = 0$ orthogonal to the hyperplane (x, y, z) , since we clearly see that the spatial components of the metric tensor and its covariant derivative do not depend on f . For $r \geq 0$, we have $0 \leq \phi \leq 2\pi$. If a curve r_G is defined by $\sinh^4 r = 1$, that is

$$r_G = \ln(1 + \sqrt{2}), \quad (4.2)$$

the circle $r > \ln(1 + \sqrt{2})$, i.e. $(\sinh^4 r - \sinh^2 r) > 0$ in the "hyperplane" $t_G = 0$, is a *closed time-like curve* (which is not a *geodesic line*!). Here r_G is referred to as the *Gödel radius*.

The circle of radius r_G is a *light-like curve*, where the light cones are tangential to the hyperplane (x, y, z) of zero t_G . Photons trajectories reaching this radius are closing up, therefore r_G constitutes a *chronal horizon* beyond which an observer located at the origin ($r = 0$) cannot detect them. The following quantity corresponds to r_G , it is $(ds^2)' = e^{2U} ds^2 = 0$ with $e^{2U} \neq 0$.

For $r > r_G$ the light cone opens up and tips over until its future part reaches the negative values of t_G . In this an *achronal domain*, any closed curve is a time-like curve. The conformal line $s' = \int e^U ds$, the integral of which is performed over the curve length is always a time-like geodesic provided the following transformation is applied

$$t = t_G + \tanh\left(\frac{r - r_G}{r_G}\right) \sqrt{x^2 + y^2}, \quad (4.3)$$

where $r - r_G$ measures the distance from the Gödel radius onward. So long as $r < r_G$, then t coincides with the Gödel time axis t_G . When $r > r_G$, then $t_G = 0$ and the time coordinate t becomes space-like as viewed from within the Gödel space-time. The Gödel space coordinates should then be transformed as follows

$$x \text{ (resp. } y, z) = x_G - (x_G + x_N) \tanh\left(\frac{r - r_G}{r_G}\right). \quad (4.4)$$

For $r < r_G$, x (resp. y, z) coincides with the Gödel space-time coordinates x_G (resp. y_G, z_G) of the hyperplane (x, y, z) . For $r > r_G$, x (resp. y, z) coincides with a new coordinate x_N (resp. y_N, z_N) distinct from x_G (resp. y_G, z_G).

5 Time displacement mode

5.1 Creating a "bubble" along a Gödel curve

As we demonstrated, the conformal factor e^{2U} is not related to the hypothetical cosmological constant Λ .

It is therefore possible to adjust the factor U in order to create a pressureless singularity within the new Gödel space-time. In the following such a singular region is likened to the warp drive “bubble” which is bound to move along a Gödel curve.

The bubble follows the trajectory $x_s(t)$ where the time coordinate t satisfies here (4.3). Therefore for $R \leq R_B$, the bubble is assumed to be ruled by the new Alcubierre metric (2.8) expressed with the signature -2

$$ds^2 = (N^2 - v_s^2 f^2) c^2 dt^2 - 2v_s f(r_s) c dt dx - dz^2 - dx^2 - dy^2. \quad (5.1)$$

This space-time is thus regarded as *globally hyperbolic* and the bubble will never know whether it moves along a CTC. As a result, the bubble is seen by a specific observer (see below) as being transported forward along the x -direction *tangent* to a CTC beyond the Gödel radius r_G . In the absence of charge outside of the bubble ($R > R_B \rightarrow \infty$), there is $f = 0$ and we retrieve Gödel’s metric (2.1).

5.2 Gödel chronal horizon

At the origin of the coordinate system, the axis of the light cone is orthogonal to the (x, y, z) hyperplane as described above by the metric (2.1). The circle of radius r_G is a *light-like curve*, where the light cones are tangential to the plane of constant (or zero) t and photons trajectories reaching this radius are closing up, therefore r_G constitutes a *chronal horizon*. Such an horizon is a special type of the *Cauchy horizon* beyond which an observer located at the origin ($r = 0$) cannot detect them. With increasing $r > r_G$ the light cones continue to keel over and their opening angles widen until their future parts reach the negative values of t . In this an *achronal domain*, any closed curve is a time-like curve. As a result, the bubble follows a reversed chronological sequence with respect to the coordinate t .

The bubble moves backwards in time and travels into the past of a specific observer resting at $r = 0$ whose proper time satisfies $\tau = t$. After regressing, once $r < r_G$, the bubble can return to the original causal domain at the departing coordinate time t , thus slightly aging with respect to the rest observer depending on its trip own time duration.

Concluding remarks

Without going into details of a sound engineering, we have just briefly sketched the basic principle of the existing theory using electromagnetism and charged current to suit the warp drive propulsion. Our approach heavily relies on a specific configuration describing a spacecraft located inside a warp drive bubble, which certainly deserves further scrutiny. In order to avoid an additional heavy treatment of the warp drive subject we have skipped some of the important aspects of the topic, as for example the causally separation of the bubble center to the outer edge of the bubble wall and beyond.

For further rigorous studies of classical warp drive physics, one can refer to [16–19]. Unlike our concept all of these theories rely on negative energy contributions also referred to as “exotic energy” or “exotic matter” [20]. Such form of energy has never been detected so far, although its theoretical production based on a L. de Broglie’s publication [21] has been suggested in [22]. By introducing a “complex” potential, our warp drive concept does not require any form of exotic matter.

As a space-time short-cut Morris, Thorne et al. [23] derived a specific static wormhole comparable to the Einstein-Rosen-bridge. Combining two wormholes with a distorted one the authors could produce a time lag which would act as a time machine. Of particular interest is the recent paper published by Tippet and Tsang [24] where the Alcubierre warp is applied to a CTC. Like in our theory, a bubble of curvature travels along a closed trajectory and is ruled by a Rindler geometry. At any rate Exotic matter is still required.

Natário investigated an “optimal time travel” in the Gödel universe for a particle bound to accelerate along a CTC [25]. For this purpose, the well known Rocket Equation trajectories in general relativity are here applied to a CTC. Natário however keeps the factor $a = 1$ (and the cosmological constant $\Lambda = -\frac{1}{2}$), which necessarily restricts again this field of research to a finely tuned universe space-time. In contrast to all those attempts and related theories, the model we suggest here is derived from a reformulated Gödel metric that exhibits consistent physical properties which are known to exist. Because of this reformulation, new physical conditions render plausible a system which may accommodate a potential time machine.

The basic engineering we presented in here, pre-suppose a high level of technological accuracy, which is far from being reached by today’s knowledge.

Billions of billions of distant galaxies must certainly harbour quite a great number of inhabitable worlds where advanced civilizations have certainly developed capabilities to allow for such interstellar propulsion modes. Indeed, our universe is 13.7 billion years old compared to the 4 billion years of our (marginal) Earth. Given this scale, an evolution difference of just one million years only between us and other extraterrestrial forms of thinking beings, is not unrealistic, and it implicitly means an incredible exponential degree of superior knowledge which is certainly beyond our common understanding.

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Twin Universes Confirmed by General Relativity

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The twin universes hypothesis was initially proposed by A. D. Sakharov followed by several astrophysicists in order to explain some unsolved questions mainly the current dark matter issue. However, no one could provide a physical justification as to the origin and existence of the second universe. We show here that general relativity formally yields two coupled field equations exhibiting an opposite sign, which lends support to Sakharov's conjecture. To this end, we use Cartan's calculus, in order to derive the differential form of Einstein's field equations. This procedure readily leads to a particular representation whereby the Einstein's field equation is classically inferred from the "Landau-Lifshitz superpotential". Since this superpotential is a fourth rank tensor (density-like), a second field equation naturally arises from the derivation, a result which has been so far totally obscured and overlooked in all classical treatments.

Notations

Space-time Greek indices α, β run from 0, 1, 2, 3 for local coordinates.

Space-time Latin indices a, b run from 0, 1, 2, 3 for a general basis.

Space-time signature is -2 .

Einstein's constant is denoted by κ .

We assume here that $c = 1$.

1 Differential calculus

1.1 The field equations in GR (short overview)

In General Relativity, the line element on the 4-dimensional pseudo-Riemannian manifold (M, g) is given by the interval $ds^2 = g_{ab} dx^a dx^b$. By varying the action $\mathcal{S} = \mathcal{L}_E d^4x$ with respect to g_{ab} where the Lagrangian density is given by

$$\mathcal{L}_E = g^{ab} \sqrt{-g} \left(\left\{ \begin{smallmatrix} e \\ ab \end{smallmatrix} \right\} \left\{ \begin{smallmatrix} d \\ de \end{smallmatrix} \right\} - \left\{ \begin{smallmatrix} d \\ ae \end{smallmatrix} \right\} \left\{ \begin{smallmatrix} e \\ bd \end{smallmatrix} \right\} \right), \quad (1)$$

one infers the symmetric Einstein tensor

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R, \quad (2)$$

where, as is well-known,

$$R_{bc} = \partial_a \left\{ \begin{smallmatrix} a \\ bc \end{smallmatrix} \right\} - \partial_c \left\{ \begin{smallmatrix} a \\ ba \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} d \\ bc \end{smallmatrix} \right\} \left\{ \begin{smallmatrix} a \\ da \end{smallmatrix} \right\} - \left\{ \begin{smallmatrix} d \\ ba \end{smallmatrix} \right\} \left\{ \begin{smallmatrix} a \\ dc \end{smallmatrix} \right\} \quad (3)$$

is the (symmetric) Ricci tensor whose contraction gives the curvature scalar R , and $\left\{ \begin{smallmatrix} e \\ ab \end{smallmatrix} \right\}$ denote the Christoffel symbols of the second kind.

The source free field equations are

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R + \Lambda g_{ab} = 0, \quad (4)$$

where Λ is usually called the cosmological constant. The second rank tensor G_{ab} is symmetric and is only function of the metric tensor components g_{ab} and their first and second order

derivatives. Due to the Bianchi's identities the Einstein tensor is conceptually conserved

$$\nabla_a G_b^a = 0, \quad (5)$$

where ∇_a is the Riemann covariant derivative.

When a massive source is present, the field equations become

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} (R - 2\Lambda) = \kappa T_{ab}. \quad (6)$$

If ρ is the matter density, T_{ab} is here the tensor describing the pressure of a free fluid

$$T_{ab} = \rho u_a u_b. \quad (7)$$

1.2 The general structures on a manifold

Let us now consider a 4-manifold M referred to a vector basis e_a . A locally defined set of four linearly independent vector fields, determined by the dual basis θ^a of the local coordinates

$$\theta^a = a_a^\beta dx^\beta \quad (8)$$

is called a *tetrad field* or *vierbein* [1].

On this manifold, it is well known that the connection coefficients $\Gamma_{\alpha\beta}^\gamma$ can be decomposed in the most general sense as

$$\Gamma_{\alpha\beta}^\gamma = \left\{ \begin{smallmatrix} \gamma \\ \alpha\beta \end{smallmatrix} \right\} + K_{\alpha\beta}^\gamma + (\Gamma_{\alpha\beta}^\gamma)_S, \quad (9)$$

where $K_{\alpha\beta}^\gamma$ is the *contorsion tensor* which is built from the *torsion tensor* $T_{\alpha\beta}^\gamma = \frac{1}{2} (\Gamma_{[\beta\alpha]}^\gamma - \Gamma_{\alpha\beta}^\gamma)$, and

$$(\Gamma_{\alpha\beta}^\gamma)_S = \frac{1}{2} g^{\gamma\mu} (D_\beta g_{\alpha\mu} + D_\alpha g_{\beta\mu} - D_\mu g_{\alpha\beta}) \quad (10)$$

is the *segment connection* formed with the general covariant derivatives of the metric tensor (denoted here by D instead of the Riemann symbol ∇)

$$D_\gamma g_{\alpha\beta} = \partial_\gamma g_{\alpha\beta} - \Gamma_{\alpha\gamma\beta} - \Gamma_{\beta\gamma\alpha} \neq 0. \quad (11)$$

The connection $(\Gamma_{\alpha\beta}^\gamma)_s$ characterizes a particular property of the manifold related to a second type of structure, called the *segment curvature*. This additional curvature results from the variation of the parallel transported vector around a small closed path.

In a dual basis θ^α , the following 2-forms can be associated with any parallel transported vector along the closed path:

— a rotation curvature form

$$\Omega_\beta^\alpha = \frac{1}{2} R_{\beta\gamma\delta}^\alpha \theta^\gamma \wedge \theta^\delta, \quad (12)$$

— a torsion form

$$\Omega^\alpha = \frac{1}{2} T_{\gamma\delta}^\alpha \theta^\gamma \wedge \theta^\delta, \quad (13)$$

— a segment curvature form

$$\Omega = -\frac{1}{2} R_{\alpha\gamma\delta}^\alpha \theta^\gamma \wedge \theta^\delta. \quad (14)$$

These are the maximum admissible mathematical structures defining a general manifold.

1.3 The Cartan structure equations

We now introduce the *Cartan procedure*. This is a powerful coordinate calculus extensively used in the foregoing.

Let us first define the connection forms

$$\Gamma_\beta^\alpha = \left\{ \begin{smallmatrix} \alpha \\ \gamma\beta \end{smallmatrix} \right\} \theta^\gamma. \quad (15)$$

The *first Cartan structure equation* is related to the torsion by [2, p. 40]

$$\Omega^\alpha = \frac{1}{2} T_{\gamma\delta}^\alpha \theta^\gamma \wedge \theta^\delta = d\theta^\alpha + \Gamma_\gamma^\alpha \wedge \theta^\gamma. \quad (16)$$

and the *second Cartan structure equation* is [2, p. 42]

$$\Omega_\beta^\alpha = \frac{1}{2} R_{\beta\gamma\delta}^\alpha \theta^\gamma \wedge \theta^\delta = d\Gamma_\beta^\alpha + \Gamma_\gamma^\alpha \wedge \Gamma_\beta^\gamma. \quad (17)$$

and $R_{\beta\gamma\delta}^\alpha$ are here the components of the curvature tensor in the most general sense.

Within the Riemannian framework alone (torsion free), $R_{\beta\gamma\delta}^\alpha$ reduces to the Riemann curvature tensor components and the first structure equation (16) becomes

$$d\theta^\alpha = -\Gamma_\gamma^\alpha \wedge \theta^\gamma. \quad (18)$$

We shall now define the absolute exterior differential D of a tensor valued p -form of type (r, s)

$$(D\phi)_{j_1 \dots j_s}^{i_1 \dots i_r} = d\phi_{j_1 \dots j_s}^{i_1 \dots i_r} + \Gamma_k^{i_1} \wedge \phi_{j_1 \dots j_s}^{k i_2 \dots i_r} + \dots - \Gamma_{j_1}^k \wedge \phi_{k j_2 \dots j_s}^{i_1 i_2 \dots i_r} - \dots$$

As a simple example, the Bianchi identities can be simply written with the exterior differential as

$$D\Omega^\alpha = \Omega_\beta^\alpha \wedge \theta^\beta \quad (1\text{st Bianchi identity}),$$

$$D\Omega_\beta^\alpha = 0 \quad (2\text{nd Bianchi identity}).$$

2 The differential Einstein equations

2.1 The Einstein action

We first recall the definition of the Hodge star operator for an oriented n -dimensional pseudo-Riemannian manifold (M, g) , wherein the volume element is determined by g

$$\eta = \sqrt{-g} \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3.$$

Let $\Lambda_k(E)$ be the subspace of completely antisymmetric multilinear forms on the real vector space E . The *Hodge star operator* $*$ is a linear isomorphism $\Lambda_k(M) \rightarrow \Lambda_{n-k}(M)$, where $k \leq n$. If $\{\theta^0, \theta^1, \theta^2, \theta^3\}$ is an oriented basis of 1-forms, this operator is defined by

$$\begin{aligned} *(\theta^{i_1} \wedge \theta^{i_2} \wedge \dots \wedge \theta^{i_k}) &= \\ &= \frac{\sqrt{-g}}{(n-k)!} \varepsilon_{j_1 \dots j_n} g^{j_1 i_1} \dots g^{j_k i_k} \theta^{j_{k+1}} \wedge \dots \wedge \theta^{j_n}. \end{aligned}$$

With this preparation, the Einstein action simply reads

$$*R = R\eta. \quad (19)$$

To show this, we express this action in terms of tetrads. With $\eta^{\mu\nu} = *(\theta^\mu \wedge \theta^\nu)$ and taking into account (17) we have

$$\eta_{\beta\gamma} \wedge \Omega^{\beta\gamma} = \frac{1}{2} \eta_{\beta\gamma} R_{\mu\nu}^{\beta\gamma} \theta^\mu \wedge \theta^\nu,$$

$$*(\theta^\mu \wedge \theta^\nu) = \frac{1}{2} \eta_{\beta\gamma\sigma\rho} g^{\beta\gamma} \theta^\sigma \wedge \theta^\rho,$$

i.e., we have

$$\eta_{\beta\gamma} = \frac{1}{2} \eta_{\beta\gamma\sigma\rho} \theta^\sigma \wedge \theta^\rho. \quad (20)$$

Thus, we have

$$\eta_{\beta\gamma} \wedge \theta^\mu \wedge \theta^\nu = \frac{1}{2} \eta_{\beta\gamma\sigma\rho} \theta^\sigma \wedge \theta^\rho \wedge \theta^\mu \wedge \theta^\nu = (\delta_\beta^\mu \delta_\gamma^\nu - \delta_\gamma^\mu \delta_\beta^\nu) \eta,$$

$$\eta_{\beta\gamma} \wedge \Omega^{\beta\gamma} = \frac{1}{2} (\delta_\beta^\mu \delta_\gamma^\nu - \delta_\gamma^\mu \delta_\beta^\nu) R_{\mu\nu}^{\beta\gamma} \eta = R\eta = *R.$$

Taking also into account (20), we compute the absolute exterior differential $D\eta_{\beta\gamma} = \frac{1}{2} D(\eta_{\beta\gamma\sigma\rho} \theta^\sigma \wedge \theta^\rho)$. In an orthonormal frame $\eta_{\beta\gamma\sigma\rho}$ is constant and $D\eta_{\beta\gamma\sigma\rho} = 0$. This manifests the fact that in the *Riemannian framework* (metric connection), orthonormality is preserved under parallel transfer. Therefore, $D\eta_{\beta\gamma} = \eta_{\beta\gamma\sigma\rho} D\theta^\sigma \wedge \theta^\rho$.

Now, keeping in mind that the basis θ^σ is a tensor 1-form of the type (1,0), the first structure equation reads

$$D\theta^\sigma = \Omega^\sigma,$$

$$D\eta_{\beta\gamma} = \eta_{\beta\gamma\sigma\rho} \Omega^\sigma \wedge \theta^\rho = \Omega^\sigma \wedge \eta_{\beta\gamma\sigma\rho}.$$

The latter equation is zero for the Riemannian connection $D\eta_{\beta\gamma} = 0$. In the same way, we can show that

$$D\eta_\alpha^{\beta\gamma} = d\eta_\alpha^{\beta\gamma} + \Gamma_\delta^\beta \wedge \eta_\alpha^{\delta\gamma} + \Gamma_\delta^\gamma \wedge \eta_\alpha^{\beta\delta} - \Gamma_\alpha^\delta \wedge \eta_\delta^{\beta\gamma} = 0 \quad (21)$$

with $\eta_\alpha^{\beta\gamma} = *(\theta^\beta \wedge \theta^\gamma \wedge \theta_\alpha)$ (all indices are raised or lowered with $g_{\alpha\beta}$ from $g = g_{\alpha\beta} \theta^\alpha \otimes \theta^\beta$).

2.2 The Einstein field equations

From (20), we infer that

$$\eta_{\beta\gamma\delta} = \eta_{\beta\gamma\delta\lambda} \theta^\lambda. \quad (22)$$

Under the variation $\delta\theta^\beta$ of the orthonormal tetrad fields θ^β , we have

$$\delta(\eta_{\beta\gamma} \wedge \Omega^{\beta\gamma}) = \delta\eta_{\beta\gamma} \wedge \Omega^{\beta\gamma} + \eta_{\beta\gamma\delta} \wedge \delta\Omega^{\beta\gamma\delta}.$$

Now, using (20) and (22) yields

$$\delta\eta_{\beta\gamma} = \frac{1}{2} \delta(\eta_{\beta\gamma\delta\lambda} \theta^\delta \wedge \theta^\lambda) = \delta\theta^\delta \wedge \eta_{\beta\gamma\delta}.$$

Hence, applying the varied second structure equation

$$\delta\Omega^{\beta\gamma} = d\delta\Gamma^{\beta\gamma} + \delta\Gamma_\eta^\beta \wedge \Gamma^{\eta\gamma} + \Gamma_\eta^\beta \wedge \delta\Gamma^{\eta\gamma},$$

we obtain

$$\begin{aligned} \delta(\eta_{\beta\gamma} \wedge \Omega^{\beta\gamma}) &= \delta\theta^\delta \wedge (\eta_{\beta\gamma\delta} \wedge \Omega^{\beta\gamma}) + d(\eta_{\beta\gamma} \wedge \delta\Gamma^{\beta\gamma}) - \\ &- d\eta_{\beta\gamma} \wedge \delta\Gamma^{\beta\gamma} + \eta_{\beta\gamma} \wedge (\delta\Gamma_\eta^\beta \wedge \Gamma^{\eta\gamma} + \Gamma_\eta^\beta \wedge \delta\Gamma^{\eta\gamma}), \end{aligned} \quad (23)$$

and from the second line we extract $d\eta_{\beta\gamma} + \eta_{\beta\gamma} \wedge (\Gamma_\gamma^\eta + \Gamma_{\beta\eta})$, which is just $D\eta_{\beta\gamma}$.

However, we know that $D\eta_{\beta\gamma} = 0$, and finally, the varied Einstein action is

$$\begin{aligned} \delta(\eta_{\beta\gamma} \wedge \Omega^{\beta\gamma}) &= \delta\theta^\beta \wedge (\eta_{\beta\gamma\delta} \wedge \Omega^{\gamma\delta}) + d(\eta_{\beta\gamma} \wedge \delta\Gamma^{\beta\gamma}) + \\ &+ (\text{exact differential}). \end{aligned} \quad (24)$$

The global Lagrangian density \mathcal{L} in the presence of matter is written as

$$\mathcal{L} = -\frac{1}{2\kappa} {}^*R + \mathcal{L}_{\text{matter}}.$$

Setting up ${}^*T_\beta$ as the energy-momentum 3-form for *bare matter* we have the Lagrangian density for the varied matter

$$\delta\mathcal{L}_{\text{matter}} = -\delta\theta^\beta \wedge {}^*T_\beta$$

and taking into account (24), the global variation is

$$\delta\mathcal{L} = -\delta\theta^\beta \wedge \left(\frac{1}{2\kappa} \eta_{\beta\gamma\delta} \wedge \Omega^{\gamma\delta} + {}^*T_\beta \right) + (\text{exact differential}).$$

We eventually arrive at the field equations in the differential form

$$-\frac{1}{2} \eta_{\beta\gamma\delta} \wedge \Omega^{\gamma\delta} = \kappa {}^*T_\beta, \quad (25)$$

where T_α is related to the energy-momentum tensor $T_{\alpha\beta}$ by $T_\alpha = T_{\alpha\beta} \theta^\beta$.

In the same manner, we can obtain $G_\alpha = G_{\alpha\beta} \theta^\beta$ for the Einstein tensor $G_{\alpha\beta}$ (see Appendix A).

2.3 The energy-momentum tensor

It is well known however, that $G_{\alpha\beta}$ is intrinsically conserved while the massive tensor $T_{\alpha\beta}$ is not. This is because the gravitational field is not included in $T_{\alpha\beta}$. To restore conservation for the energy-momentum tensor, we start by reformulating (25) in the form

$$-\frac{1}{2} \Omega_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha = \kappa {}^*T_\alpha. \quad (25\text{bis})$$

Then, we use the second structure equation under the following form

$$\Omega_{\beta\gamma} = d\Gamma_{\beta\gamma} - \Gamma_{\mu\beta} \wedge \Gamma_\gamma^\mu \quad (26)$$

so that we obtain

$$d\Gamma_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha = d(\Gamma_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha) + \Gamma_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha. \quad (27)$$

Then, using (21) in (26), we infer

$$\begin{aligned} d\Gamma_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha &= d(\Gamma_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha) + \\ &+ \Gamma_{\beta\gamma} \wedge (-\Gamma_\delta^\beta \wedge \eta^{\delta\gamma}_\alpha - \Gamma_\delta^\gamma \wedge \eta^{\beta\delta}_\alpha + \Gamma_\alpha^\delta \wedge \eta^{\beta\gamma}_\delta). \end{aligned} \quad (28)$$

Adding the second contribution of (26) to (28), we obtain the Einstein field equations in a new form, which is

$$-\frac{1}{2} d(\Gamma_{\beta\gamma} \wedge \eta^{\beta\gamma}_\alpha) = \kappa ({}^*T_\alpha + {}^*t_\alpha), \quad (29)$$

where we denote

$${}^*t_\alpha = -\frac{1}{2\kappa} \Gamma_{\beta\gamma} \wedge (\Gamma_{\delta\alpha} \wedge \eta^{\beta\gamma\delta} - \Gamma_\delta^\gamma \wedge \eta^{\beta\delta}_\alpha), \quad (30)$$

and the quantity ${}^*t_\alpha$ is interpreted as the energy-momentum (pseudo-tensor) of the gravitational field generated by this distributed matter.

Equation (29) readily implies the conservation law

$$d({}^*T_\alpha + {}^*t_\alpha) = 0. \quad (31)$$

Writing

$$t_\alpha = t_{\alpha\beta} \theta^\beta, \quad (32)$$

we see that $t_{\alpha\beta}$ describes the gravitational field, which can be expressed, for example, by the Einstein-Dirac pseudo-tensor [3, p. 61].

From (30) we verify that $t_{\alpha\beta}$ is not symmetric. To correct this problem, we shall not apply the Belinfante symmetrization procedure [4]. Instead, we will modify the field differential equations. We first revert to the field equations (25) in which we insert $\eta^{\alpha\beta\gamma} = \eta^{\alpha\beta\gamma\delta} \theta_\delta$. With (26) this yields

$$-\frac{1}{2} \eta^{\alpha\beta\gamma\delta} \theta_\delta \wedge (d\Gamma_{\beta\gamma} - \Gamma_{\mu\beta} \wedge \Gamma_\gamma^\mu) = \kappa {}^*T^\alpha \quad (33)$$

leading to

$$-\frac{1}{2} \eta^{\alpha\beta\gamma\delta} d(\Gamma_{\beta\gamma} \wedge \theta_\delta) = \kappa ({}^*T^\alpha + {}^*t^\alpha), \quad (34)$$

where

$${}^*t^\alpha = -\frac{1}{2}\kappa\eta^{\alpha\beta\gamma\delta}(\Gamma_{\mu\beta}\wedge\Gamma_\gamma^\mu\wedge\theta_\delta - \Gamma_{\beta\gamma}\wedge\Gamma_{\mu\delta}\wedge\theta^\mu). \quad (35)$$

We see that ${}^*t^\alpha$ is unaffected by the exterior product terms in the bracket, therefore $t_{\alpha\beta}$ is now symmetric. In that case, we identify ${}^*t^\alpha$ with the Landau-Lifshitz 3-form ${}^*t_{L-L}^\alpha$, which yields the corresponding pseudo-tensor $t_{L-L}^{\alpha\beta}$.

3 The 4th rank tensor equation

3.1 The first set of Einstein's field equations

Multiply (34) by $\sqrt{-g}$. Then, taking $\eta_{\alpha\beta\gamma\delta} = -\frac{1}{2}\sqrt{-g}\varepsilon_{\alpha\beta\gamma\delta}$ into account, we find a new form for the field equations

$$-d(\sqrt{-g}\eta^{\alpha\beta\gamma\delta}\Gamma_{\beta\gamma}\wedge\theta_\delta) = 2\kappa\sqrt{-g}({}^*T^\alpha + {}^*t_{L-L}^\alpha) \quad (36)$$

or

$$d(\sqrt{-g}\Gamma^{\beta\gamma}\wedge\eta_{\beta\gamma}^\alpha) = 2\sqrt{-g}({}^*T^\alpha + {}^*t_{L-L}^\alpha). \quad (37)$$

From these equations follows immediately the differential conservation law

$$d[\sqrt{-g}({}^*T^\alpha + {}^*t_{L-L}^\alpha)] = 0. \quad (38)$$

A tedious calculation eventually shows that

$$d(\sqrt{-g}\Gamma^{\beta\gamma}\wedge\eta_{\beta\gamma}^\alpha) = \frac{1}{\sqrt{-g}}H^{\alpha\beta\gamma\gamma}_{,\beta\gamma}\eta_\gamma, \quad (39)$$

where

$$H^{\alpha\beta\gamma\gamma} = -g(g^{\alpha\nu}g^{\beta\gamma} - g^{\beta\nu}g^{\gamma\alpha}) \quad (40)$$

is the “Landau-Lifshitz superpotential” [5, eq. 101.2]. Therefore the field equations read here

$$H^{\alpha\beta\gamma\gamma}_{,\beta\gamma} = 2\kappa[-g(T^{\alpha\nu} + t_{L-L}^{\alpha\nu})]. \quad (41)$$

Explicitly, we have

$$H^{\alpha\beta\gamma\gamma}_{,\beta\gamma} = \partial_\beta\{\partial_\gamma[-g(g^{\alpha\nu}g^{\beta\gamma} - g^{\beta\nu}g^{\gamma\alpha})]\}. \quad (42)$$

REMARK: It is essential to note that the quantities $t_{L-L}^{\alpha\nu}$ do not represent a true tensor. Indeed, the gravitational field can be transformed away at any point and its energy is not localizable. This is why the left hand side of (41) and (42) exhibits ordinary derivatives instead of covariant ones.

The 4th rank tensor $H^{\alpha\beta\gamma\gamma}_{,\beta\gamma}$ can be regarded as a special choice of $R^{\alpha\nu}$ — the Ricci tensor, where all first derivatives of the metric tensor cancel out at this given point.

The Landau-Lifshitz pseudo-tensor has the form

$$\begin{aligned} (-g)t_{L-L}^{\alpha\nu} = & \frac{1}{2\kappa}\left\{\#g^{\alpha\nu}_{,\lambda}\#g^{\lambda\mu}_{,\mu} - \#g^{\alpha\lambda}_{,\lambda}\#g^{\nu\mu}_{,\mu} + \right. \\ & + \frac{1}{2}g^{\alpha\nu}g_{\lambda\mu}\#g^{\lambda\theta}_{,\rho}\#g^{\rho\mu}_{,\theta} + g_{\mu\lambda}g^{\theta\rho}\#g^{\alpha\lambda}_{,\theta}\#g^{\nu\mu}_{,\rho} - \\ & - (g^{\alpha\lambda}g_{\mu\theta}\#g^{\nu\theta}_{,\rho}\#g^{\mu\rho}_{,\lambda} + g^{\nu\lambda}g_{\mu\theta}\#g^{\alpha\theta}_{,\rho}\#g^{\mu\rho}_{,\lambda}) + \\ & \left. + \frac{1}{8}(2g^{\alpha\lambda}g^{\nu\mu} - g^{\alpha\nu}g^{\lambda\mu})(2g_{\theta\rho}g_{\delta\tau} - g_{\rho\delta}g_{\theta\tau})\#g^{\theta\tau}_{,\lambda}\#g^{\rho\delta}_{,\mu}\right\}, \end{aligned} \quad (43)$$

where $\#g^{\alpha\nu} = \sqrt{-g}g^{\alpha\nu}$.

When velocities are low and the gravitational field is weak (42) reduces to

$$H^{0i0j}_{,ij} = \partial_i\{\partial_j[-g(g^{00}g^{ij} - g^{i0}g^{j0})]\}, \quad (44)$$

where $i, j, \dots = 1, 2, 3$ are the spatial indices. We can write this equation in mixed indices by lowering one of the space indices

$$H^{0i0j}_{,ij} = \partial_i\partial_j(-gg^{00}\delta_i^j). \quad (45)$$

When $i = j$, the Newton law is retrieved through the weak potential $g^{00} = 1 + 2\psi$ as (45) reduces to the Laplacian

$$\partial_i\partial_i g^{00} = \Delta\psi, \quad (46)$$

so that we obtain the well-known Poisson equation

$$\Delta\psi = G\rho,$$

where G is Newton's constant.

Therefore, at the Newtonian approximation, we can write the *generalized Poisson equation*, which has the form

$$H^{0i0j}_{,ij} = 2\kappa\sqrt{-g}(T^{00} + t_{L-L}^{00}), \quad (47)$$

where the Newtonian pseudo-tensor t_{L-L}^{00} reads

$$\begin{aligned} (-g)t_{L-L}^{00} = & \frac{1}{2\kappa}\left\{\#g^{00}_{,k}\#g^{kn}_{,n} - \#g^{0k}_{,k}\#g^{0n}_{,n} + \right. \\ & + \frac{1}{2}g^{00}g_{kn}\#g^{kr}_{,l}\#g^{ln}_{,r} + g_{nk}g^{rl}\#g^{0k}_{,r}\#g^{0l}_{,l} - \\ & - (g^{0k}g_{nr}\#g^{0r}_{,l}\#g^{nl}_{,k} + g^{0k}g_{nr}\#g^{0r}_{,l}\#g^{nl}_{,k}) + \\ & \left. + \frac{1}{8}(2g^{0k}g^{0n} - g^{00}g^{kn})(2g_{rl}g_{sm} - g_{ls}g_{rm})\#g^{rm}_{,k}\#g^{ls}_{,n}\right\}. \end{aligned} \quad (48)$$

Unlike the classical Newtonian theory, the static bare mass density generally produces a gravitational field, which is described by t_{L-L}^{00} at the considered point.

REMARK: A slightly variable cosmological term L term induces a stress energy tensor of vacuum which restores a conserved property of the r.h.s. of equation (6) thus avoiding the use of the ill-defined gravitational field pseudo-tensor as shown in [6, 7].

3.2 The second set of Einstein's field equations

The *second rank* tensor field equations have been inferred from a *fourth rank* tensor density like. It is then natural to consider a second set of field equations which is contained in the former.

A close inspection of the “Landau-Lifshitz superpotential” (40) leads to the obvious choice for this second field equation

$$d(\sqrt{-g}\Gamma^{\gamma\alpha}\wedge\eta_{\gamma\alpha}^\beta) = \frac{1}{\sqrt{-g}}H^{\alpha\beta\gamma\gamma}_{,\gamma\alpha}\eta_\gamma, \quad (49)$$

$$H^{\alpha\beta\gamma\gamma}_{,\gamma\alpha} = 2\kappa\sqrt{-g}(T^{\beta\gamma} + t_{L-L}^{\beta\gamma}). \quad (50)$$

Note that (41) and (50) are linked using a common index. Furthermore, each set of field equations differ from a sign.

PROOF: Let us label the “negative” equation as

$${}^{(-)}H^{\alpha\beta\gamma\gamma}_{,\gamma\alpha} = \partial_\alpha \left\{ \partial_\gamma [-g(g^{\alpha\gamma}g^{\beta\gamma} - g^{\beta\gamma}g^{\gamma\alpha})] \right\}. \quad (51)$$

Now in the same manner as for (44), equation (51) reduces to

$${}^{(-)}H^{i00}_{,ij} = \partial_i \left\{ \partial_j [-g(g^{i0}g^{0j} - g^{00}g^{ij})] \right\}. \quad (52)$$

Lowering one of the space indices we obtain

$${}^{(-)}H^{i00}_{j,ij} = \partial_i (\partial_j g g^{00} \delta_i^j), \quad (53)$$

which is just the opposite to $H^{00j}_{i,ij} = \partial_i \partial_j (-g g^{00} \delta_i^j)$ (45).

Had we set $i = j$, we would have found

$${}^{(-)}\Delta\psi = -\Delta\psi. \quad (54)$$

As a consequence, the right member of the Poisson equation (in our orthonormal frame) should also reverse sign

$${}^{(-)}(G\rho) = -G\rho. \quad (55)$$

Since the Einstein constant is here a common factor, we infer that mass densities of each field equations differ from a sign as well as the gravitation potential ψ .

Therefore, in the framework of the Newtonian approximation we find two opposite field tensors which induce two opposite energy density tensors which we label as

$${}^{(+)}(T^{00} + t^{00}_{L-L}) \quad \text{and} \quad {}^{(-)}(T^{00} + t^{00}_{L-L}). \quad (56)$$

3.3 Two antagonist manifolds

Conservation properties lead to the following evident corresponding equivalences

$$H^{\alpha\beta\gamma\gamma}_{,\beta\gamma} \rightarrow {}^{(+)}G^{\alpha\gamma} = \kappa \left[{}^{(+)}(T^{\alpha\gamma} + t^{\alpha\gamma}_{L-L}) \right]. \quad (57)$$

$$H^{\alpha\beta\gamma\gamma}_{,\gamma\alpha} \rightarrow {}^{(-)}G^{\beta\gamma} = \kappa \left[{}^{(-)}(T^{\beta\gamma} + t^{\beta\gamma}_{L-L}) \right]. \quad (58)$$

Hence, the field equation (57) can be regarded as being defined on a “positive” manifold with respect to the “negative” manifold on which is defined the field equation (58).

REMARK: One should always bear in mind that both ${}^{(+)}G^{\alpha\gamma}$ and ${}^{(-)}G^{\beta\gamma}$ are coupled through the 4th rank tensor $H_{\beta\alpha\gamma\mu}$, which necessarily imposes that indices must keep their respective label. The “intertwined” metrics are then

$${}^{(+)}ds^2 = {}^{(+)}g_{\alpha\gamma} dx^\alpha dx^\gamma, \quad {}^{(-)}ds^2 = {}^{(-)}g_{\beta\gamma} dx^\beta dx^\gamma, \quad (59)$$

and, in the “vierbein” (tetrad) formalism, we have

$${}^{(+)}g_{\alpha\gamma} = e_\alpha^a e_\gamma^b \eta_{ab}, \quad {}^{(-)}g_{\beta\gamma} = e_\beta^a e_\gamma^b \eta_{ab}, \quad (60)$$

where η_{ab} is the Minkowski tensor.

One thus writes the *Pfaffian metrics* as

$${}^{(+)}ds^2 = \eta_{ab} {}^{(+)}\eta^a \eta^b, \quad {}^{(-)}ds^2 = \eta_{ab} {}^{(-)}\theta^a \theta^b, \quad (61)$$

$${}^{(+)}\theta^\alpha = e_\alpha^a dx^a, \quad {}^{(-)}\theta^a = e_\beta^a dx^\beta, \quad (62)$$

The common basis 1-form $\theta^b = e_\nu^b dx^\nu$ outlines the coupling between the metrics.

Obviously, in a flat space-time, ${}^{(+)}g_{\alpha\gamma}$ and ${}^{(-)}g_{\beta\gamma}$ coincide with η_{ab} meaning that the twin universes emerge from curvature.

Conclusions and outlook

The twin universe hypothesis recently saw a revived interest.

Several astrophysicists conjectured that it could provide an appropriate explanation to the puzzle of the dark energy and dark matter issues and other unsolved observational data questions [8–15]. However, all these theories do not justify the origin of the double universe which remains a pure arbitrary statement, not relying on any sound physical grounds. In here we showed that General Relativity formally confirms the existence of two coupled Einstein’s field equations characterizing two co-existing antagonist manifolds.

General Relativity further shows that there exists at most two such field equations [16].

We hope that this formal demonstration will help to substantiate the current research in astrophysics.

Appendix. Classical Einstein tensor retrieved from the differential equations

Using (12), the field equations

$$-\frac{1}{2} \eta_{\beta\gamma\delta} \wedge \Omega^{\gamma\delta} = \kappa {}^*T_\beta \quad (A1)$$

can be written in the form

$$-\frac{1}{4} \eta_\alpha^\mu \theta^\sigma \wedge \theta^\delta R^\nu_{\mu\sigma\delta} = \kappa T_{\alpha\beta} \eta^\beta. \quad (A2)$$

We first use the following relations

$$\eta^\alpha \wedge {}^*\theta^\alpha, \quad (A3)$$

$$\eta_\alpha = \frac{1}{3!} (\eta_{\alpha\beta\gamma\delta} \theta^\beta \wedge \theta^\gamma \wedge \theta^\delta) = \frac{1}{3!} \theta^\beta \wedge \eta_{\alpha\beta}. \quad (A4)$$

Then, applying the following Riemannian identities

$$\theta^\beta \wedge \eta_\alpha = \delta_\alpha^\beta \eta,$$

$$\theta^\gamma \wedge \eta_{\alpha\beta} = \delta_\beta^\gamma \eta_\alpha - \delta_\alpha^\gamma \eta_\beta,$$

$$\theta^\delta \wedge \eta_{\alpha\beta\gamma} = \delta_\gamma^\delta \eta_{\alpha\beta} + \delta_\beta^\delta \eta_{\gamma\alpha} + \delta_\alpha^\delta \eta_{\beta\gamma},$$

$$\theta^\varepsilon \wedge \eta_{\alpha\beta\gamma\delta} = \delta_\delta^\varepsilon \eta_{\alpha\beta\gamma} - \delta_\gamma^\varepsilon \eta_{\delta\alpha\beta} + \delta_\beta^\varepsilon \eta_{\gamma\delta\alpha} - \delta_\alpha^\varepsilon \eta_{\beta\gamma\delta},$$

we obtain

$$\begin{aligned}
 & -\frac{1}{4} R^{\mu\nu}_{\sigma\tau} \left[\delta^\tau_\nu (\delta^\sigma_\mu \eta_\alpha - \delta^\sigma_\nu \eta_\mu) + \right. \\
 & \quad \left. + \delta^\tau_\mu (\delta^\sigma_\alpha \eta_\nu - \delta^\sigma_\nu \eta_\alpha) + \delta^\tau_\alpha (\delta^\sigma_\nu \eta_\mu - \delta^\sigma_\mu \eta_\nu) \right] = \\
 & = -\frac{1}{2} R^{\mu\nu}_{\mu\nu} \eta_\alpha + R^{\mu\nu}_{\alpha\nu} \eta_\mu = \\
 & = -\frac{1}{2} R^{\beta\nu}_{\alpha\nu} \eta_\beta - \frac{1}{2} \eta^\beta_\alpha R^{\mu\nu}_{\mu\nu} \eta_\beta = \\
 & = \left(R^\beta_\alpha - \frac{1}{2} \delta^\beta_\alpha R \right) \eta_\beta.
 \end{aligned} \tag{A5}$$

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